Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.2-d-x-^m-a+b-arctan-c-x^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [166]. This is test number [148].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 94.58 (157)	% 5.42 (9)
Mathematica	% 98.19 (163)	% 1.81 (3)
Maple	% 86.75 (144)	% 13.25 (22)
Maxima	% 56.02 (93)	% 43.98 (73)
Fricas	% 55.42 (92)	% 44.58 (74)
Sympy	% 54.82 (91)	% 45.18 (75)
Giac	% 48.19 (80)	% 51.81 (86)
Mupad	% 65.06 (108)	% 34.94 (58)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

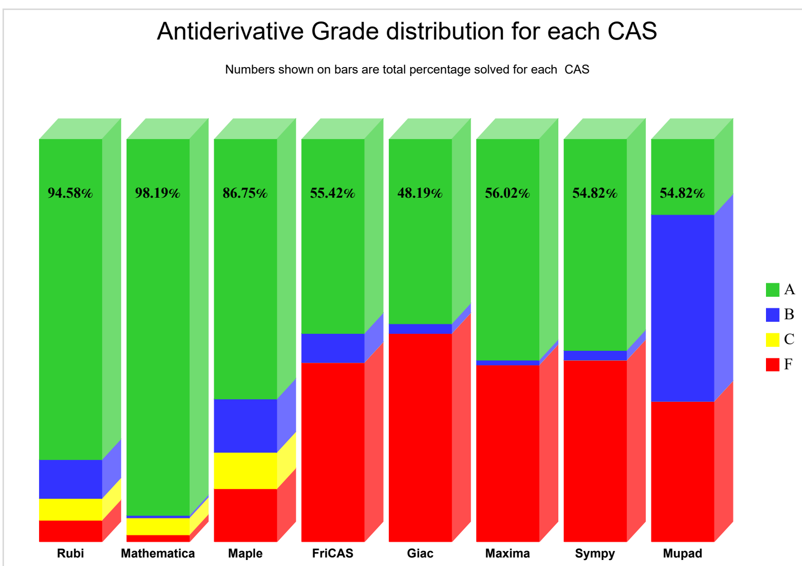
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

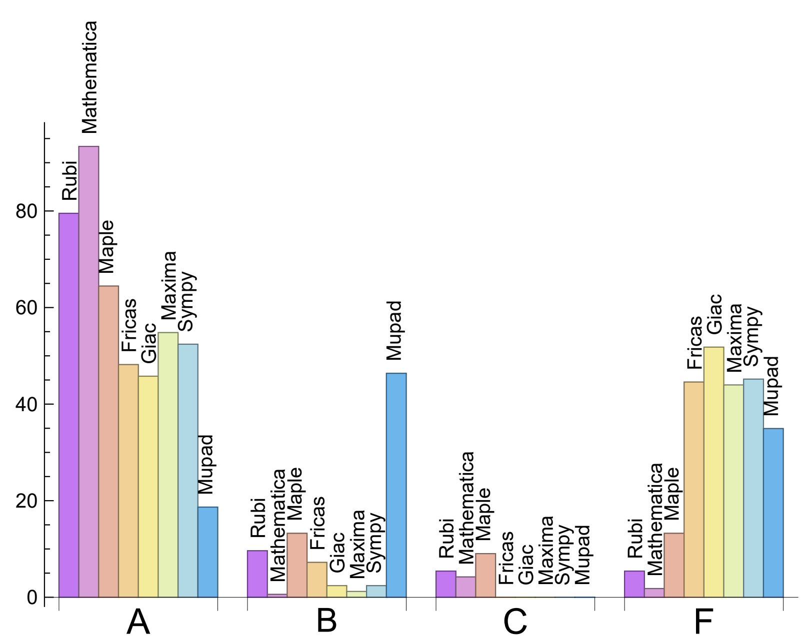
System	% A grade	% B grade	% C grade	% F grade
Rubi	79.52	9.64	5.42	5.42
Mathematica	93.37	0.60	4.22	1.81
Maple	64.46	13.25	9.04	13.25
Maxima	54.82	1.20	0.00	43.98
Fricas	48.19	7.23	0.00	44.58
Sympy	52.41	2.41	0.00	45.18
Giac	45.78	2.41	0.00	51.81
Mupad	18.67	46.39	0.00	34.94

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	9	100.00 %	0.00 %	0.00 %
Mathematica	3	100.00 %	0.00 %	0.00 %
Maple	22	90.91 %	0.00 %	9.09 %
Maxima	73	68.49 %	15.07 %	16.44 %
Fricas	74	83.78 %	0.00 %	16.22 %
Sympy	75	77.33 %	22.67 %	0.00 %
Giac	86	82.56 %	17.44 %	0.00 %
Mupad	58	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

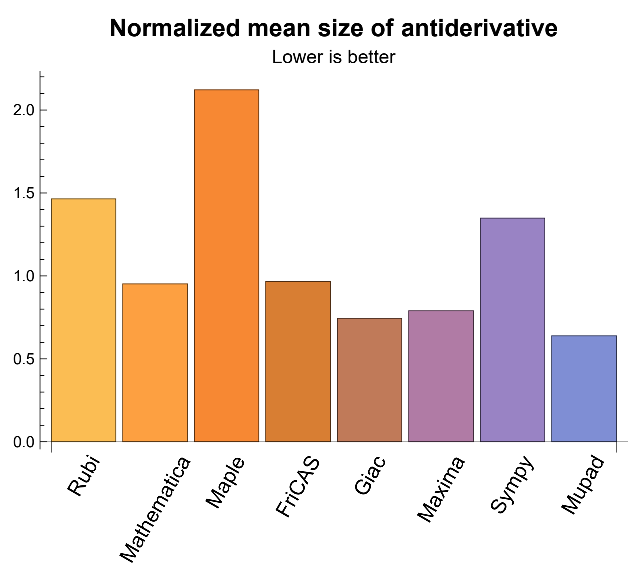
1.3 Performance

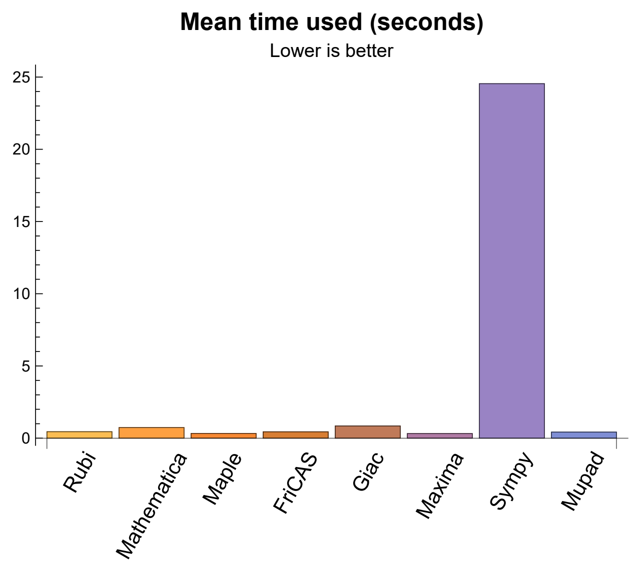
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.45	199.46	1.46	59.00	1.00
Mathematica	0.73	163.80	0.95	65.00	1.07
Maple	0.33	309.46	2.12	56.00	0.95
Maxima	0.32	59.97	0.79	43.00	0.91
Fricas	0.44	91.00	0.97	42.00	0.96
Sympy	24.54	112.75	1.35	58.00	1.09
Giac	0.85	54.98	0.75	41.50	0.93
Mupad	0.42	46.49	0.64	38.00	0.81

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {74, 75, 76, 77, 79, 80, 86, 87, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 140, 141, 142, 143, 145, 146, 152}

Mathematica {14, 16, 18, 20, 22, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 75, 77, 79, 82, 83, 86, 87, 89, 90, 114, 116, 118, 120, 121, 122, 123, 125, 126, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

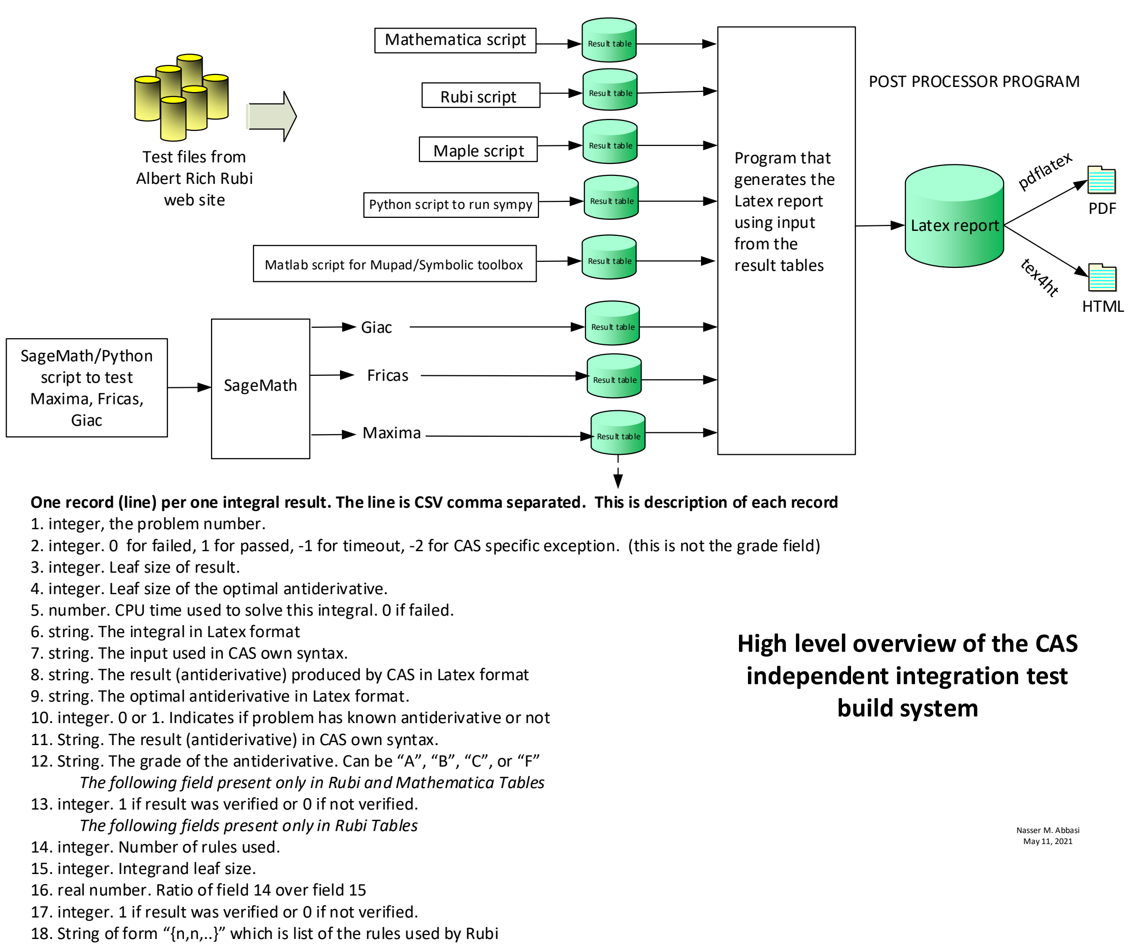
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 78, 81, 82, 83, 84, 85, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 117, 124, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade: { 75, 77, 79, 86, 87, 114, 116, 118, 120, 121, 122, 123, 141, 143, 145, 152 }

C grade: { 74, 76, 80, 113, 115, 119, 140, 142, 146 }

F grade: { 89, 90, 125, 126, 147, 148, 149, 150, 153 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166 }

B grade: { 83 }

C grade: { 9, 11, 66, 82, 102, 158, 159 }

F grade: { 81, 84, 85 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 18, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 145, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 7, 14, 16, 20, 22, 24, 26, 28, 29, 32, 34, 87, 123, 136, 141, 143, 147, 149, 152, 153, 157, 166 }

C grade: { 19, 25, 27, 30, 31, 33, 64, 86, 100, 122, 144, 148, 150, 151, 165 }

F grade: { 56, 75, 78, 79, 81, 82, 83, 84, 85, 88, 89, 90, 93, 114, 117, 118, 120, 121, 124, 125, 126, 129 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 157, 165 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 166 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 74, 76, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 105, 107, 109, 111, 113, 115, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 68, 69, 70, 71, 72, 73, 104, 106, 108, 110, 112, 166 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 97, 98, 99, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 161, 162, 163 }

B grade: { 158, 159, 160, 164 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 59, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 100, 103, 113, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

2.1.7 Giac

A grade: { 6, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 47, 48, 49, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 91, 92, 94, 95, 96, 97, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 127, 128, 130, 131, 132, 133, 135, 137, 138, 139, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 66, 102, 134, 136 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 43, 46, 50, 51, 52, 56, 64, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

2.1.8 Mupad

A grade: { 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 146, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165 }

C grade: { }

F grade: { 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 161, 166 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	53	57	54	63	0	52
normalized size	1	1.00	1.08	0.90	0.97	0.92	1.07	0.00	0.88
time (sec)	N/A	0.032	0.003	0.008	0.431	0.455	1.425	0.000	0.442

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	52	56	59	60	0	54
normalized size	1	1.00	1.09	0.93	1.00	1.05	1.07	0.00	0.96
time (sec)	N/A	0.042	0.014	0.007	0.330	0.448	1.042	0.000	0.417

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	44	48	47	53	0	44
normalized size	1	1.00	1.10	0.92	1.00	0.98	1.10	0.00	0.92
time (sec)	N/A	0.027	0.003	0.006	0.470	0.467	0.817	0.000	0.210

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	43	46	49	49	0	42
normalized size	1	1.00	1.11	0.96	1.02	1.09	1.09	0.00	0.93
time (sec)	N/A	0.032	0.009	0.006	0.321	0.435	0.577	0.000	0.370

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	35	37	34	42	0	34
normalized size	1	1.00	1.14	0.95	1.00	0.92	1.14	0.00	0.92
time (sec)	N/A	0.016	0.002	0.006	0.469	0.420	0.423	0.000	0.123

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	31	33	26	31	27
normalized size	1	1.00	1.00	0.97	1.07	1.14	0.90	1.07	0.93
time (sec)	N/A	0.011	0.003	0.004	0.324	0.422	0.205	2.958	0.098

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	74	0	0	0	0	28
normalized size	1	1.00	1.00	2.11	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.029	0.003	0.029	0.000	0.414	0.000	0.000	0.287

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	38	39	39	37	37	0	36
normalized size	1	1.00	1.09	1.11	1.11	1.06	1.06	0.00	1.03
time (sec)	N/A	0.023	0.003	0.011	0.327	0.425	0.631	0.000	0.319

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	46	35	31	26	37	0	42
normalized size	1	1.00	1.24	0.95	0.84	0.70	1.00	0.00	1.14
time (sec)	N/A	0.020	0.003	0.009	0.435	0.441	0.528	0.000	0.332

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	51	51	50	61	0	46
normalized size	1	1.00	1.02	0.96	0.96	0.94	1.15	0.00	0.87
time (sec)	N/A	0.033	0.016	0.013	0.340	0.434	1.059	0.000	0.119

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	44	46	41	46	0	42
normalized size	1	1.00	0.96	0.92	0.96	0.85	0.96	0.00	0.88
time (sec)	N/A	0.025	0.003	0.009	0.427	0.423	0.827	0.000	0.363

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	60	62	59	71	0	56
normalized size	1	1.00	1.00	0.94	0.97	0.92	1.11	0.00	0.88
time (sec)	N/A	0.038	0.018	0.011	0.335	0.426	1.673	0.000	0.371

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	138	171	163	152	199	0	171
normalized size	1	1.00	0.96	1.19	1.13	1.06	1.38	0.00	1.19
time (sec)	N/A	0.310	0.139	0.015	0.458	0.428	2.393	0.000	0.667

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	169	334	0	0	0	0	-1
normalized size	1	1.00	0.99	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.514	0.095	0.000	0.422	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	111	135	136	121	155	0	134
normalized size	1	1.00	0.99	1.21	1.21	1.08	1.38	0.00	1.20
time (sec)	N/A	0.209	0.085	0.014	0.432	0.432	1.444	0.000	0.316

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	131	298	0	0	0	0	-1
normalized size	1	1.00	0.95	2.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.295	0.016	0.000	0.437	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	97	104	83	107	0	88
normalized size	1	1.00	0.99	1.28	1.37	1.09	1.41	0.00	1.16
time (sec)	N/A	0.106	0.069	0.022	0.474	0.430	0.734	0.000	0.410

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	90	128	0	0	0	0	-1
normalized size	1	1.00	1.08	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.081	0.192	0.000	0.429	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	144	1128	0	0	0	0	-1
normalized size	1	1.00	1.09	8.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.078	0.513	0.000	0.436	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	102	323	0	0	0	0	-1
normalized size	1	1.00	1.24	3.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.149	0.018	0.000	0.452	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	90	110	98	94	119	0	140
normalized size	1	1.00	1.14	1.39	1.24	1.19	1.51	0.00	1.77
time (sec)	N/A	0.128	0.066	0.013	0.442	0.511	1.015	0.000	2.310

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	153	399	0	0	0	0	-1
normalized size	1	1.00	1.09	2.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.389	0.019	0.000	0.459	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	128	147	152	135	170	0	171
normalized size	1	1.00	1.10	1.27	1.31	1.16	1.47	0.00	1.47
time (sec)	N/A	0.219	0.093	0.018	0.419	0.453	1.778	0.000	2.257

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	291	528	0	0	0	0	-1
normalized size	1	1.00	1.14	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.948	0.829	0.023	0.000	0.427	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	396	3053	0	0	0	0	-1
normalized size	1	1.00	1.46	11.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.756	0.900	5.524	0.000	0.425	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	225	445	0	0	0	0	-1
normalized size	1	1.00	1.16	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.545	0.559	0.019	0.000	0.469	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	269	2020	0	0	0	0	-1
normalized size	1	1.00	1.31	9.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.618	2.526	0.000	0.416	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	152	352	0	0	0	0	-1
normalized size	1	1.00	1.16	2.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.303	0.018	0.000	0.425	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	192	270	0	0	0	0	-1
normalized size	1	1.00	1.61	2.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.098	0.227	0.000	0.428	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	212	2309	0	0	0	0	-1
normalized size	1	1.00	1.03	11.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.151	0.243	0.000	0.420	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	214	2159	0	0	0	0	-1
normalized size	1	1.00	1.84	18.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.399	0.291	0.000	0.422	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	176	457	0	0	0	0	-1
normalized size	1	1.00	1.32	3.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.341	0.021	0.000	0.444	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	321	5974	0	0	0	0	-1
normalized size	1	1.00	1.51	28.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.881	3.190	0.000	0.421	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	265	550	0	0	0	0	-1
normalized size	1	1.00	1.34	2.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.601	0.724	0.024	0.000	0.429	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.008	0.652	0.466	0.000	0.418	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.003	0.012	0.269	0.000	0.416	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.015	0.448	0.321	0.000	0.420	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.008	0.619	0.708	0.000	0.403	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.003	0.914	0.264	0.000	0.417	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	1.063	0.382	0.000	0.404	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	2.107	0.966	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.003	1.928	0.488	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	1.349	0.999	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	1.022	0.952	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.003	2.371	0.448	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	1.172	1.136	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	1.340	0.977	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.003	0.019	0.452	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	1.218	0.716	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	1.582	0.977	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.003	2.717	0.508	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	2.951	1.138	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	108	74	86	118	104	86	49
normalized size	1	1.00	0.92	0.63	0.74	1.01	0.89	0.74	0.42
time (sec)	N/A	0.067	0.028	0.025	0.411	0.476	3.224	0.995	0.321

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	4.115	3.371	0.000	0.451	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	2.655	2.928	0.000	0.430	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.033	2.771	0.000	0.426	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	0.280	1.127	0.000	0.430	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.005	0.461	0.696	0.000	0.435	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	0.360	1.359	0.000	0.444	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	54	51	58	60	49
normalized size	1	1.00	1.09	0.93	1.00	0.94	1.07	1.11	0.91
time (sec)	N/A	0.035	0.008	0.031	0.413	0.426	65.098	0.164	0.357

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	80	47	44
normalized size	1	1.00	1.11	0.96	1.02	1.09	1.70	1.00	0.94
time (sec)	N/A	0.031	0.013	0.035	0.307	0.434	46.372	0.171	0.349

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
normalized size	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.027	0.006	0.029	0.411	0.429	23.237	0.200	0.327

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	38	39	66	40	35
normalized size	1	1.00	1.14	1.00	1.06	1.08	1.83	1.11	0.97
time (sec)	N/A	0.014	0.007	0.023	0.308	0.417	13.502	0.151	0.314

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0	32
normalized size	1	1.00	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.045	0.005	0.109	0.000	0.409	0.000	0.000	0.327

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	41	43	75	60	38
normalized size	1	1.00	1.13	1.00	1.05	1.10	1.92	1.54	0.97
time (sec)	N/A	0.024	0.007	0.030	0.317	0.437	30.285	0.174	0.343

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	35	30	42	74	41
normalized size	1	1.00	1.17	0.95	0.85	0.73	1.02	1.80	1.00
time (sec)	N/A	0.025	0.007	0.032	0.418	0.403	24.120	2.810	0.356

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	51	53	54	92	69	50
normalized size	1	1.00	1.09	0.93	0.96	0.98	1.67	1.25	0.91
time (sec)	N/A	0.032	0.013	0.033	0.324	0.438	81.386	0.142	0.365

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	179	140	147	372	184	169	64
normalized size	1	1.00	1.11	0.87	0.91	2.31	1.14	1.05	0.40
time (sec)	N/A	0.113	0.064	0.039	0.445	0.462	33.026	2.852	0.363

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	177	138	145	337	173	165	62
normalized size	1	1.00	1.11	0.87	0.91	2.12	1.09	1.04	0.39
time (sec)	N/A	0.101	0.037	0.027	0.433	0.442	17.684	3.075	0.392

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	107	125	127	319	930	149	49
normalized size	1	1.00	0.76	0.89	0.91	2.28	6.64	1.06	0.35
time (sec)	N/A	0.105	0.046	0.029	0.428	0.443	10.228	0.180	0.390

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	158	125	132	322	151	138	55
normalized size	1	1.00	1.10	0.87	0.92	2.25	1.06	0.97	0.38
time (sec)	N/A	0.086	0.046	0.026	0.421	0.448	20.443	0.209	0.201

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	177	132	142	389	590	159	63
normalized size	1	1.00	1.11	0.83	0.89	2.45	3.71	1.00	0.40
time (sec)	N/A	0.104	0.057	0.033	0.413	0.449	38.521	5.357	0.428

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	177	138	138	350	185	150	63
normalized size	1	1.00	1.11	0.87	0.87	2.20	1.16	0.94	0.40
time (sec)	N/A	0.103	0.054	0.032	0.414	0.467	63.234	0.245	0.452

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	731	121	151	169	137	199	145	150
normalized size	1	5.90	0.98	1.22	1.36	1.10	1.60	1.17	1.21
time (sec)	N/A	1.627	0.125	0.051	0.490	0.449	98.996	0.177	1.016

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F(-2)	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	647	141	0	0	0	0	0	-1
normalized size	1	4.20	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.366	0.335	180.000	0.000	0.422	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	612	85	113	126	100	155	100	112
normalized size	1	6.80	0.94	1.26	1.40	1.11	1.72	1.11	1.24
time (sec)	N/A	1.042	0.070	0.035	0.488	0.445	34.914	1.333	0.683

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	255	107	146	0	0	0	0	-1
normalized size	1	2.52	1.06	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.554	0.074	0.206	0.000	0.429	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	165	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.321	0.112	0.356	0.000	0.424	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	290	127	0	0	0	0	0	-1
normalized size	1	2.99	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.653	0.157	0.595	0.000	0.427	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	419	98	118	110	115	167	0	152
normalized size	1	4.82	1.13	1.36	1.26	1.32	1.92	0.00	1.75
time (sec)	N/A	1.136	0.083	0.056	0.519	0.440	52.408	0.000	0.613

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1393	1393	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.602	5.175	0.357	0.000	0.438	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1191	1191	5504	0	0	0	0	0	-1
normalized size	1	1.00	4.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.778	31.540	0.307	0.000	0.425	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1164	1164	5535	0	0	0	0	0	-1
normalized size	1	1.00	4.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.622	30.755	0.326	0.000	0.439	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1360	1360	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.227	2.942	0.333	0.000	0.417	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1444	1444	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.427	2.891	0.346	0.000	0.430	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	951	170	690	0	0	0	0	-1
normalized size	1	6.38	1.14	4.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	4.736	0.172	1.560	0.000	0.416	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	545	224	306	0	0	0	0	-1
normalized size	1	3.78	1.56	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.545	0.126	0.243	0.000	0.421	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	245	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	0.207	0.403	0.000	0.417	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	239	0	0	0	0	0	-1
normalized size	1	0.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.831	0.461	0.559	0.000	0.431	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	0	196	0	0	0	0	0	-1
normalized size	1	0.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.659	0.349	1.182	0.000	0.423	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	1.962	0.265	0.000	0.434	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.284	0.250	0.000	0.424	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.066	0.241	0.000	0.435	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	0.335	0.145	0.000	0.408	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.357	0.189	0.000	0.430	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	54	51	0	60	49
normalized size	1	1.00	1.09	0.93	1.00	0.94	0.00	1.11	0.91
time (sec)	N/A	0.039	0.009	0.031	0.417	0.414	0.000	0.163	0.454

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	133	47	44
normalized size	1	1.00	1.11	0.96	1.02	1.09	2.83	1.00	0.94
time (sec)	N/A	0.036	0.014	0.033	0.315	0.417	177.260	0.807	0.379

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
normalized size	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.029	0.006	0.028	0.413	0.424	89.853	0.169	0.405

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	38	39	117	40	35
normalized size	1	1.00	1.14	1.00	1.06	1.08	3.25	1.11	0.97
time (sec)	N/A	0.021	0.007	0.026	0.313	0.416	47.589	0.709	0.100

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0	32
normalized size	1	1.00	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.050	0.005	0.103	0.000	0.426	0.000	0.000	0.351

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	41	43	126	60	38
normalized size	1	1.00	1.13	1.00	1.05	1.10	3.23	1.54	0.97
time (sec)	N/A	0.025	0.007	0.031	0.318	0.469	96.086	2.010	0.377

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	35	30	42	74	41
normalized size	1	1.00	1.17	0.95	0.85	0.73	1.02	1.80	1.00
time (sec)	N/A	0.027	0.007	0.037	0.424	0.410	88.005	0.170	0.386

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	51	53	54	0	69	50
normalized size	1	1.00	1.09	0.93	0.96	0.98	0.00	1.25	0.91
time (sec)	N/A	0.035	0.014	0.034	0.326	0.444	0.000	0.156	0.401

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	179	165	148	399	311	167	114
normalized size	1	1.00	1.03	0.95	0.85	2.29	1.79	0.96	0.66
time (sec)	N/A	0.325	0.057	0.156	0.428	0.459	58.103	3.785	1.164

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	131	98	92	234	898	95	91
normalized size	1	1.00	1.30	0.97	0.91	2.32	8.89	0.94	0.90
time (sec)	N/A	0.098	0.045	0.026	0.421	0.459	31.243	0.160	2.295

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	170	148	137	505	301	137	107
normalized size	1	1.00	1.03	0.90	0.83	3.06	1.82	0.83	0.65
time (sec)	N/A	0.294	0.052	0.108	0.416	0.480	73.747	0.179	0.909

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	183	105	102	121	352	108	118
normalized size	1	1.00	1.59	0.91	0.89	1.05	3.06	0.94	1.03
time (sec)	N/A	0.091	0.052	0.038	0.413	0.430	144.506	4.327	2.567

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	181	167	152	439	320	171	122
normalized size	1	1.00	1.03	0.95	0.86	2.49	1.82	0.97	0.69
time (sec)	N/A	0.431	0.086	0.108	0.423	0.455	149.056	2.989	1.002

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	185	113	106	137	359	119	106
normalized size	1	1.00	1.58	0.97	0.91	1.17	3.07	1.02	0.91
time (sec)	N/A	0.096	0.029	0.030	0.424	0.448	71.605	3.953	1.936

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	170	154	137	408	303	157	113
normalized size	1	1.00	1.03	0.93	0.83	2.47	1.84	0.95	0.68
time (sec)	N/A	0.393	0.028	0.108	0.431	0.467	39.660	2.110	0.689

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	170	104	98	90	328	91	99
normalized size	1	1.00	1.63	1.00	0.94	0.87	3.15	0.88	0.95
time (sec)	N/A	0.085	0.040	0.026	0.421	0.439	58.274	3.822	1.828

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	179	159	147	595	320	161	120
normalized size	1	1.00	1.03	0.91	0.84	3.42	1.84	0.93	0.69
time (sec)	N/A	0.413	0.054	0.105	0.420	0.516	109.838	2.985	0.706

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	F(-1)	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	731	121	151	169	129	0	145	150
normalized size	1	5.90	0.98	1.22	1.36	1.04	0.00	1.17	1.21
time (sec)	N/A	1.655	0.102	0.047	0.518	0.471	0.000	0.189	1.140

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	647	141	0	0	0	0	0	-1
normalized size	1	4.20	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.394	0.314	180.000	0.000	0.427	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	612	85	113	126	91	209	100	112
normalized size	1	6.80	0.94	1.26	1.40	1.01	2.32	1.11	1.24
time (sec)	N/A	1.052	0.068	0.043	0.515	0.445	122.135	1.601	0.724

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	255	107	148	0	0	0	0	-1
normalized size	1	2.45	1.03	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.644	0.074	0.234	0.000	0.419	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	167	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.316	0.124	0.400	0.000	0.416	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	290	125	0	0	0	0	0	-1
normalized size	1	2.90	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.661	0.168	0.592	0.000	0.442	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	419	98	118	110	102	223	0	152
normalized size	1	4.82	1.13	1.36	1.26	1.17	2.56	0.00	1.75
time (sec)	N/A	1.122	0.083	0.045	0.540	0.461	173.366	0.000	0.691

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	536	167	0	0	0	0	0	-1
normalized size	1	3.48	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.450	0.412	0.418	0.000	0.433	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	1867	346	0	0	0	0	0	-1
normalized size	1	7.78	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.034	0.562	0.411	0.000	0.426	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	951	170	867	0	0	0	0	-1
normalized size	1	6.47	1.16	5.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	4.733	0.176	1.776	0.000	0.445	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	545	224	303	0	0	0	0	-1
normalized size	1	3.92	1.61	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.702	0.100	0.244	0.000	0.428	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	248	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.217	0.394	0.000	0.418	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	0	240	0	0	0	0	0	-1
normalized size	1	0.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.860	0.424	0.570	0.000	0.451	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	0	196	0	0	0	0	0	-1
normalized size	1	0.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.670	0.352	0.885	0.000	0.445	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.982	0.268	0.000	0.436	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.298	0.243	0.000	0.438	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.067	0.227	0.000	0.435	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	0.341	0.151	0.000	0.408	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.363	0.181	0.000	0.420	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	46	45	41	46	84	45
normalized size	1	1.00	1.10	0.92	0.90	0.82	0.92	1.68	0.90
time (sec)	N/A	0.031	0.011	0.042	0.416	0.401	0.495	4.012	0.411

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	55	43	40	41	69	40
normalized size	1	1.00	1.12	1.28	1.00	0.93	0.95	1.60	0.93
time (sec)	N/A	0.030	0.008	0.047	0.435	0.416	0.368	1.998	0.352

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	37	36	31	36	74	36
normalized size	1	1.00	1.13	0.95	0.92	0.79	0.92	1.90	0.92
time (sec)	N/A	0.017	0.008	0.037	0.416	0.416	0.289	0.182	0.344

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	38	27	25	22	46	25
normalized size	1	1.00	1.00	1.41	1.00	0.93	0.81	1.70	0.93
time (sec)	N/A	0.012	0.003	0.036	0.318	0.420	0.178	1.759	0.303

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	94	0	0	0	74	32
normalized size	1	1.00	1.00	2.41	0.00	0.00	0.00	1.90	0.82
time (sec)	N/A	0.045	0.006	0.052	0.000	0.433	0.000	3.960	0.338

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	37	36	38	41	36	39	43
normalized size	1	1.00	1.09	1.06	1.12	1.21	1.06	1.15	1.26
time (sec)	N/A	0.019	0.012	0.024	0.322	0.429	0.646	0.167	0.344

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	42	37	44	67	50
normalized size	1	1.00	1.12	0.95	0.98	0.86	1.02	1.56	1.16
time (sec)	N/A	0.024	0.011	0.033	0.415	0.440	0.851	1.796	0.384

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	45	54	55	60	51	56
normalized size	1	1.00	1.09	0.82	0.98	1.00	1.09	0.93	1.02
time (sec)	N/A	0.036	0.010	0.033	0.314	0.433	1.095	0.204	0.369

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	862	111	157	134	125	144	0	140
normalized size	1	7.07	0.91	1.29	1.10	1.02	1.18	0.00	1.15
time (sec)	N/A	1.791	0.101	0.056	0.431	0.434	0.814	0.000	0.473

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	787	152	445	0	0	0	0	-1
normalized size	1	5.18	1.00	2.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.490	0.355	0.128	0.000	0.426	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	663	73	116	104	88	97	0	98
normalized size	1	8.09	0.89	1.41	1.27	1.07	1.18	0.00	1.20
time (sec)	N/A	1.126	0.058	0.051	0.425	0.422	0.445	0.000	0.404

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	478	105	357	0	0	0	0	-1
normalized size	1	5.76	1.27	4.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.440	0.115	0.104	0.000	1.103	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	148	1249	0	0	0	0	-1
normalized size	1	1.00	1.00	8.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.092	0.340	0.000	0.559	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	F(-1)	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	259	107	147	0	0	0	0	-1
normalized size	1	2.70	1.11	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.528	0.120	0.207	0.000	0.435	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	836	99	110	120	109	117	140	143
normalized size	1	9.95	1.18	1.31	1.43	1.30	1.39	1.67	1.70
time (sec)	N/A	1.294	0.076	0.046	0.429	0.468	1.044	0.232	2.749

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	0	253	608	0	0	0	0	-1
normalized size	1	0.00	1.18	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.629	0.653	0.127	0.000	0.434	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	0	330	6441	0	0	0	0	-1
normalized size	1	0.00	1.44	28.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.520	0.731	1.376	0.000	1.467	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	174	507	0	0	0	0	-1
normalized size	1	0.00	1.20	3.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.352	0.289	0.125	0.000	0.419	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	215	2363	0	0	0	0	-1
normalized size	1	0.00	1.81	19.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.739	0.271	0.352	0.000	0.408	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	219	2542	0	0	0	0	-1
normalized size	1	1.00	0.95	11.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	0.220	0.236	0.000	0.413	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F(-1)	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	551	189	306	0	0	0	0	-1
normalized size	1	4.05	1.39	2.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.359	0.189	0.253	0.000	0.521	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	F(-1)	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	0	178	396	0	0	0	0	-1
normalized size	1	0.00	1.21	2.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.255	0.293	0.102	0.000	0.469	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	32	31	27	39	31	31
normalized size	1	1.00	0.67	0.63	0.61	0.53	0.76	0.61	0.61
time (sec)	N/A	0.015	0.015	0.029	0.413	0.469	1.969	0.138	0.349

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	26	20	32	26	26
normalized size	1	1.00	0.67	0.64	0.62	0.48	0.76	0.62	0.62
time (sec)	N/A	0.010	0.011	0.025	0.413	0.484	1.261	0.161	0.365

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	16	14	19	16	16
normalized size	1	1.00	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.006	0.007	0.023	0.414	0.412	1.147	0.350	0.074

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	61	35	0	0	0	24
normalized size	1	1.00	1.00	1.97	1.13	0.00	0.00	0.00	0.77
time (sec)	N/A	0.035	0.005	0.040	0.418	0.528	0.000	0.000	0.302

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	22	21	17	94	21	21
normalized size	1	1.00	1.11	0.81	0.78	0.63	3.48	0.78	0.78
time (sec)	N/A	0.012	0.010	0.031	0.411	0.434	1.917	0.303	0.348

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	34	27	26	26	160	26	24
normalized size	1	1.00	0.81	0.64	0.62	0.62	3.81	0.62	0.57
time (sec)	N/A	0.014	0.012	0.030	0.408	0.459	4.919	0.165	0.353

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	24	85	24	24
normalized size	1	1.00	0.83	0.69	0.67	0.67	2.36	0.67	0.67
time (sec)	N/A	0.015	0.016	0.025	0.306	0.481	4.793	0.175	0.351

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	19	24	19	-1
normalized size	1	1.00	0.86	0.69	0.66	0.66	0.83	0.66	-0.03
time (sec)	N/A	0.012	0.011	0.031	0.308	0.468	1.239	0.177	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.80
time (sec)	N/A	0.008	0.008	0.025	0.307	0.437	0.325	0.179	0.346

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	20	18	22
normalized size	1	1.00	1.00	0.86	0.82	1.18	0.91	0.82	1.00
time (sec)	N/A	0.009	0.012	0.036	0.313	0.431	1.155	0.174	0.358

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	26	25	33	143	28	27
normalized size	1	1.00	0.84	0.70	0.68	0.89	3.86	0.76	0.73
time (sec)	N/A	0.014	0.020	0.033	0.310	0.479	4.897	0.169	0.354

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	57	50	0	0	0	25
normalized size	1	1.00	1.00	1.73	1.52	0.00	0.00	0.00	0.76
time (sec)	N/A	0.036	0.007	0.091	0.424	0.451	0.000	0.000	0.336

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	32	94	0	63	0	0	-1
normalized size	1	1.00	0.82	2.41	0.00	1.62	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.014	0.046	0.000	0.473	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [142] had the largest ratio of [2.286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	12	0.250
2	A	4	3	1.00	12	0.250
3	A	4	3	1.00	12	0.250
4	A	4	3	1.00	12	0.250
5	A	3	3	1.00	10	0.300
6	A	3	2	1.00	8	0.250
7	A	3	2	1.00	12	0.167
8	A	5	5	1.00	12	0.417
9	A	3	3	1.00	12	0.250
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	12	0.250
13	A	16	7	1.00	14	0.500
14	A	14	9	1.00	14	0.643
15	A	11	7	1.00	14	0.500
16	A	9	8	1.00	14	0.571
17	A	6	5	1.00	12	0.417
18	A	5	5	1.00	10	0.500
19	A	6	5	1.00	14	0.357
20	A	4	4	1.00	14	0.286
21	A	8	7	1.00	14	0.500
22	A	8	7	1.00	14	0.500
23	A	13	8	1.00	14	0.571
24	A	33	11	1.00	14	0.786
25	A	24	11	1.00	14	0.786
26	A	18	10	1.00	14	0.714
27	A	12	9	1.00	14	0.643
28	A	8	8	1.00	12	0.667
29	A	5	6	1.00	10	0.600
30	A	8	6	1.00	14	0.429
31	A	5	6	1.00	14	0.429
32	A	7	6	1.00	14	0.429
33	A	14	11	1.00	14	0.786
34	A	16	8	1.00	14	0.571
35	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	0	0	0.00	0	0.000
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	0	0	0.00	0	0.000
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	0	0	0.00	0	0.000
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	0	0	0.00	0	0.000
53	A	12	9	1.00	8	1.125
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	2	2	1.00	14	0.143
57	A	0	0	0.00	0	0.000
58	A	0	0	0.00	0	0.000
59	A	0	0	0.00	0	0.000
60	A	5	4	1.00	14	0.286
61	A	4	3	1.00	14	0.214
62	A	4	4	1.00	14	0.286
63	A	2	2	1.00	12	0.167
64	A	4	3	1.00	14	0.214
65	A	5	5	1.00	14	0.357
66	A	4	4	1.00	14	0.286
67	A	4	3	1.00	14	0.214
68	A	11	8	1.00	14	0.571
69	A	11	8	1.00	14	0.571
70	A	11	7	1.00	10	0.700
71	A	10	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	11	8	1.00	14	0.571
73	A	11	8	1.00	14	0.571
74	C	62	19	5.90	16	1.187
75	B	53	19	4.20	16	1.187
76	C	44	16	6.80	16	1.000
77	B	28	12	2.52	14	0.857
78	A	7	6	1.00	16	0.375
79	B	24	13	2.99	16	0.812
80	C	46	23	4.82	16	1.438
81	A	86	27	1.00	16	1.687
82	A	69	23	1.00	12	1.917
83	A	47	23	1.00	16	1.438
84	A	64	25	1.00	16	1.562
85	A	77	25	1.00	16	1.562
86	B	155	30	6.38	16	1.875
87	B	82	23	3.78	14	1.643
88	A	9	7	1.00	16	0.438
89	F	0	0	N/A	0	N/A
90	F	0	0	N/A	0	N/A
91	A	0	0	0.00	0	0.000
92	A	0	0	0.00	0	0.000
93	A	3	3	1.00	16	0.188
94	A	0	0	0.00	0	0.000
95	A	0	0	0.00	0	0.000
96	A	5	4	1.00	14	0.286
97	A	4	3	1.00	14	0.214
98	A	4	4	1.00	14	0.286
99	A	2	2	1.00	14	0.143
100	A	4	3	1.00	14	0.214
101	A	5	5	1.00	14	0.357
102	A	4	4	1.00	14	0.286
103	A	4	3	1.00	14	0.214
104	A	12	8	1.00	14	0.571
105	A	9	8	1.00	10	0.800
106	A	11	7	1.00	14	0.500
107	A	9	9	1.00	14	0.643

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
108	A	12	8	1.00	14	0.571
109	A	9	9	1.00	14	0.643
110	A	11	7	1.00	12	0.583
111	A	8	8	1.00	14	0.571
112	A	12	8	1.00	14	0.571
113	C	62	19	5.90	16	1.187
114	B	53	19	4.20	16	1.187
115	C	44	16	6.80	16	1.000
116	B	28	12	2.45	16	0.750
117	A	7	6	1.00	16	0.375
118	B	24	13	2.90	16	0.812
119	C	46	23	4.82	16	1.438
120	B	59	24	3.48	16	1.500
121	B	239	32	7.78	16	2.000
122	B	155	30	6.47	16	1.875
123	B	82	23	3.92	16	1.438
124	A	9	7	1.00	16	0.438
125	F	0	0	N/A	0	N/A
126	F	0	0	N/A	0	N/A
127	A	0	0	0.00	0	0.000
128	A	0	0	0.00	0	0.000
129	A	3	3	1.00	16	0.188
130	A	0	0	0.00	0	0.000
131	A	0	0	0.00	0	0.000
132	A	5	4	1.00	14	0.286
133	A	5	4	1.00	14	0.286
134	A	4	4	1.00	12	0.333
135	A	4	3	1.00	10	0.300
136	A	4	3	1.00	14	0.214
137	A	2	2	1.00	14	0.143
138	A	4	4	1.00	14	0.286
139	A	5	4	1.00	14	0.286
140	C	88	34	7.07	16	2.125
141	B	73	34	5.18	16	2.125
142	C	58	32	8.09	14	2.286
143	B	31	14	5.76	12	1.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	7	6	1.00	16	0.375
145	B	28	12	2.70	16	0.750
146	C	66	23	9.95	16	1.438
147	F	0	0	N/A	0	N/A
148	F	0	0	N/A	0	N/A
149	F	0	0	N/A	0	N/A
150	F	0	0	N/A	0	N/A
151	A	9	7	1.00	16	0.438
152	B	82	23	4.05	16	1.438
153	F	0	0	N/A	0	N/A
154	A	6	4	1.00	10	0.400
155	A	5	4	1.00	8	0.500
156	A	4	4	1.00	6	0.667
157	A	4	3	1.00	10	0.300
158	A	4	4	1.00	10	0.400
159	A	5	4	1.00	10	0.400
160	A	3	2	1.00	12	0.167
161	A	3	2	1.00	12	0.167
162	A	2	2	1.00	12	0.167
163	A	4	4	1.00	12	0.333
164	A	3	2	1.00	12	0.167
165	A	4	3	1.00	10	0.300
166	A	4	3	1.00	10	0.300

Chapter 3

Listing of integrals

3.1 $\int x^5 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)+\frac{b\tan^{-1}(cx)}{6c^6}-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}$$

```
[Out] -1/6*b*x/c^5+1/18*b*x^3/c^3-1/30*b*x^5/c+1/6*b*arctan(c*x)/c^6+1/6*x^6*(a+b
*arctan(c*x))
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 302, 203}

$$\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)+\frac{bx^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\tan^{-1}(cx)}{6c^6}-\frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x)/(6*c^5) + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (b*ArcTan[c*x])/(6*c^6
) + (x^6*(a + b*ArcTan[c*x]))/6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}(cx)\right) dx = \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right) - \frac{1}{6}(bc)\int \frac{x^6}{1+c^2x^2}\,dx$$
$$= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right) - \frac{1}{6}(bc)\int \left(\frac{1}{c^6}-\frac{x^2}{c^4}+\frac{x^4}{c^2}-\frac{1}{c^6\left(1+c^2x^2\right)}\right) dx$$
$$= -\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)+\frac{b\int \frac{1}{1+c^2x^2}\,dx}{6c^5}$$
$$= -\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{b\tan^{-1}(cx)}{6c^6}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)$$

Mathematica [A] time = 0.00, size = 64, normalized size = 1.08

$$\frac{ax^6}{6}+\frac{b\tan^{-1}(cx)}{6c^6}-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{1}{6}bx^6\tan^{-1}(cx)-\frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x)/c^5 + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (a*x^6)/6 + (b*ArcTan[
c*x])/(6*c^6) + (b*x^6*ArcTan[c*x])/6
```

fricas [A] time = 0.46, size = 54, normalized size = 0.92

$$\frac{15\,ac^6x^6-3\,bc^5x^5+5\,bc^3x^3-15\,bcx+15\left(bc^6x^6+b\right)\arctan\left(cx\right)}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 - 15*b*c*x + 15*(b*c^6*x^6 +
 b)*arctan(c*x))/c^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 53, normalized size = 0.90

$$\frac{x^6a}{6}+\frac{b\,x^6\arctan\left(cx\right)}{6}-\frac{b\,x^5}{30c}+\frac{b\,x^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\arctan\left(cx\right)}{6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctan(c*x)-1/30*b*x^5/c+1/18*b*x^3/c^3-1/6*b*x/c^5+1/6
*b*arctan(c*x)/c^6
```

maxima [A] time = 0.43, size = 57, normalized size = 0.97

$$\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^
6 - 15*arctan(c*x)/c^7))*b
```

mupad [B] time = 0.44, size = 52, normalized size = 0.88

$$\frac{\frac{b\,\mathrm{atan}(c\,x)}{6}+\frac{b\,c^3\,x^3}{18}-\frac{b\,c^5\,x^5}{30}-\frac{b\,c\,x}{6}}{c^6}+\frac{a\,x^6}{6}+\frac{b\,x^6\,\mathrm{atan}\left(c\,x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x)),x)
```

```
[Out] ((b*atan(c*x))/6 + (b*c^3*x^3)/18 - (b*c^5*x^5)/30 - (b*c*x)/6)/c^6 + (a*x^
6)/6 + (b*x^6*atan(c*x))/6
```

sympy [A] time = 1.42, size = 63, normalized size = 1.07

$$\begin{cases} \frac{ax^6}{6}+\frac{bx^6\,\mathrm{atan}\,(cx)}{6}-\frac{bx^5}{30c}+\frac{bx^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\,\mathrm{atan}\,(cx)}{6c^6} & \text{for}\ c\neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x)/6 - b*x**5/(30*c) + b*x**3/(18*c**3)
 - b*x/(6*c**5) + b*atan(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

3.2 $\int x^4 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)+\frac{bx^2}{10c^3}-\frac{b\log\left(c^2x^2+1\right)}{10c^5}-\frac{bx^4}{20c}$$

```
[Out] 1/10*b*x^2/c^3-1/20*b*x^4/c+1/5*x^5*(a+b*arctan(c*x))-1/10*b*ln(c^2*x^2+1)/
c^5
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 266, 43}

$$\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)+\frac{bx^2}{10c^3}-\frac{b\log\left(c^2x^2+1\right)}{10c^5}-\frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (x^5*(a + b*ArcTan[c*x]))/5 - (b*Log[1
+ c^2*x^2])/(10*c^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\tan^{-1}(cx)\right)dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}(bc)\int\frac{x^5}{1+c^2x^2}\,dx\\
&= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int\frac{x^2}{1+c^2x}\,dx,x,x^2\right)\\
&= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x}{c^2}+\frac{1}{c^4\left(1+c^2x\right)}\right)dx,x,x^2\right)\\
&= \frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{b\log\left(1+c^2x^2\right)}{10c^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.09

$$\frac{ax^5}{5}+\frac{bx^2}{10c^3}-\frac{b\log\left(c^2x^2+1\right)}{10c^5}+\frac{1}{5}bx^5\tan^{-1}(cx)-\frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x])/5 - (b*
Log[1 + c^2*x^2])/(10*c^5)
```

fricas [A] time = 0.45, size = 59, normalized size = 1.05

$$\frac{4\,bc^5x^5\arctan\left(cx\right)+4\,ac^5x^5-bc^4x^4+2\,bc^2x^2-2\,b\log\left(c^2x^2+1\right)}{20\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b
*log(c^2*x^2 + 1))/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 52, normalized size = 0.93

$$\frac{a\,x^5}{5}+\frac{x^5b\arctan\left(cx\right)}{5}-\frac{b\,x^4}{20c}+\frac{b\,x^2}{10c^3}-\frac{b\ln\left(c^2x^2+1\right)}{10c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x)),x)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctan(c*x)-1/20*b*x^4/c+1/10*b*x^2/c^3-1/10*b*ln(c^2*x
^2+1)/c^5
```

maxima [A] time = 0.33, size = 56, normalized size = 1.00

$$\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*
x^2 + 1)/c^6))*b
```

mupad [B] time = 0.42, size = 54, normalized size = 0.96

$$\frac{a\,x^5}{5}-\frac{\frac{b\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atan}\left(c\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x)),x)
```

```
[Out] (a*x^5)/5 - ((b*log(c^2*x^2 + 1))/10 - (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^5
 + (b*x^5*atan(c*x))/5
```

sympy [A] time = 1.04, size = 60, normalized size = 1.07

$$\begin{cases} \frac{ax^5}{5} + \frac{bx^5 \operatorname{atan}(cx)}{5} - \frac{bx^4}{20c} + \frac{bx^2}{10c^3} - \frac{b \log\left(x^2 + \frac{1}{c^2}\right)}{10c^5} & \text{for } c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x)/5 - b*x**4/(20*c) + b*x**2/(10*c**3)
 - b*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*x**5/5, True))
```

3.3 $\int x^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=48

$$\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)-\frac{b\tan^{-1}(cx)}{4c^4}+\frac{bx}{4c^3}-\frac{bx^3}{12c}$$

```
[Out] 1/4*b*x/c^3-1/12*b*x^3/c-1/4*b*arctan(c*x)/c^4+1/4*x^4*(a+b*arctan(c*x))
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 302, 203}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)+\frac{bx}{4c^3}-\frac{b\tan^{-1}(cx)}{4c^4}-\frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) - (b*ArcTan[c*x])/(4*c^4) + (x^4*(a + b*ArcT
an[c*x]))/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}(cx)\right)dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}(bc)\int\frac{x^4}{1+c^2x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}(bc)\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)dx\\
&= \frac{bx}{4c^3}-\frac{bx^3}{12c}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)-\frac{b\int\frac{1}{1+c^2x^2}\,dx}{4c^3}\\
&= \frac{bx}{4c^3}-\frac{bx^3}{12c}-\frac{b\tan^{-1}(cx)}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 53, normalized size = 1.10

$$\frac{ax^4}{4}-\frac{b\tan^{-1}(cx)}{4c^4}+\frac{bx}{4c^3}+\frac{1}{4}bx^4\tan^{-1}(cx)-\frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) + (a*x^4)/4 - (b*ArcTan[c*x])/(4*c^4) + (b*x
^4*ArcTan[c*x])/4
```

fricas [A] time = 0.47, size = 47, normalized size = 0.98

$$\frac{3\,ac^4x^4 - bc^3x^3 + 3\,bcx + 3\,\left(bc^4x^4 - b\right)\arctan\left(cx\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x))/c^
4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 44, normalized size = 0.92

$$\frac{x^4a}{4} + \frac{x^4b\arctan\left(cx\right)}{4} - \frac{b\,x^3}{12c} + \frac{bx}{4c^3} - \frac{b\arctan\left(cx\right)}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x)),x)
```

```
[Out] 1/4*x^4*a+1/4*x^4*b*arctan(c*x)-1/12*b*x^3/c+1/4*b*x/c^3-1/4*b*arctan(c*x)/
c^4
```

maxima [A] time = 0.47, size = 48, normalized size = 1.00

$$\frac{1}{4}\,ax^4 + \frac{1}{12}\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x
)/c^5))*b
```

mupad [B] time = 0.21, size = 44, normalized size = 0.92

$$\frac{a\,x^4}{4} - \frac{\frac{b\,\mathrm{atan}(c\,x)}{4} + \frac{b\,c^3\,x^3}{12} - \frac{b\,c\,x}{4}}{c^4} + \frac{b\,x^4\,\mathrm{atan}\left(c\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x)),x)
```

```
[Out] (a*x^4)/4 - ((b*atan(c*x))/4 + (b*c^3*x^3)/12 - (b*c*x)/4)/c^4 + (b*x^4*ata
n(c*x))/4
```

sympy [A] time = 0.82, size = 53, normalized size = 1.10

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atan}(cx)}{4} - \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b \operatorname{atan}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x)/4 - b*x**3/(12*c) + b*x/(4*c**3) - b
*atan(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

3.4 $\int x^2 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=45

$$\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\log\left(c^2x^2+1\right)}{6c^3}-\frac{bx^2}{6c}$$

```
[Out] -1/6*b*x^2/c+1/3*x^3*(a+b*arctan(c*x))+1/6*b*ln(c^2*x^2+1)/c^3
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 266, 43}

$$\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\log\left(c^2x^2+1\right)}{6c^3}-\frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x^2)/(6*c) + (x^3*(a + b*ArcTan[c*x]))/3 + (b*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tan^{-1}(cx)\right)dx&=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}(bc)\int\frac{x^3}{1+c^2x^2}\,dx\\&=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)\\&=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^2\right)\\&=-\frac{bx^2}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\log\left(1+c^2x^2\right)}{6c^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.11

$$\frac{ax^3}{3}+\frac{b\log\left(c^2x^2+1\right)}{6c^3}+\frac{1}{3}bx^3\tan^{-1}(cx)-\frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x^2)/c + (a*x^3)/3 + (b*x^3*ArcTan[c*x])/3 + (b*Log[1 + c^2*x^2])/(
6*c^3)
```

fricas [A] time = 0.44, size = 49, normalized size = 1.09

$$\frac{2\,bc^{3}x^{3}\arctan\left(cx\right)+2\,ac^{3}x^{3}-bc^{2}x^{2}+b\log\left(c^{2}x^{2}+1\right)}{6\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1)
)/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 43, normalized size = 0.96

$$\frac{x^{3}a}{3}+\frac{b\,x^{3}\arctan\left(cx\right)}{3}-\frac{b\,x^{2}}{6c}+\frac{b\ln\left(c^{2}x^{2}+1\right)}{6c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c*x)-1/6*b*x^2/c+1/6*b*ln(c^2*x^2+1)/c^3
```

maxima [A] time = 0.32, size = 46, normalized size = 1.02

$$\frac{1}{3}\,ax^{3}+\frac{1}{6}\left(2\,x^{3}\arctan\left(cx\right)-c\left(\frac{x^{2}}{c^{2}}-\frac{\log\left(c^{2}x^{2}+1\right)}{c^{4}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b
```

mupad [B] time = 0.37, size = 42, normalized size = 0.93

$$\frac{a\,x^{3}}{3}+\frac{b\,x^{3}\,\mathrm{atan}\left(c\,x\right)}{3}+\frac{b\,\ln\left(c^{2}\,x^{2}+1\right)}{6\,c^{3}}-\frac{b\,x^{2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x)),x)
```

```
[Out] (a*x^3)/3 + (b*x^3*atan(c*x))/3 + (b*log(c^2*x^2 + 1))/(6*c^3) - (b*x^2)/(6
*c)
```

sympy [A] time = 0.58, size = 49, normalized size = 1.09

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{atan}(cx)}{3} - \frac{bx^2}{6c} + \frac{b \log\left(x^2 + \frac{1}{c^2}\right)}{6c^3} & \text{for } c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x)/3 - b*x**2/(6*c) + b*log(x**2 + c**(
-2))/(6*c**3), Ne(c, 0)), (a*x**3/3, True))
```

3.5 $\int x\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)+\frac{b\tan^{-1}(cx)}{2c^2}-\frac{bx}{2c}$$

```
[Out] -1/2*b*x/c+1/2*b*arctan(c*x)/c^2+1/2*x^2*(a+b*arctan(c*x))
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)+\frac{b\tan^{-1}(cx)}{2c^2}-\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x)/(2*c) + (b*ArcTan[c*x])/(2*c^2) + (x^2*(a + b*ArcTan[c*x]))/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tan^{-1}(cx)\right)dx &= \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\int\frac{x^2}{1+c^2x^2}\,dx\\ &= -\frac{bx}{2c}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)+\frac{b\int\frac{1}{1+c^2x^2}\,dx}{2c}\\ &= -\frac{bx}{2c}+\frac{b\tan^{-1}(cx)}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.14

$$\frac{ax^2}{2}+\frac{b\tan^{-1}(cx)}{2c^2}+\frac{1}{2}bx^2\tan^{-1}(cx)-\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*x)/c + (a*x^2)/2 + (b*ArcTan[c*x])/(2*c^2) + (b*x^2*ArcTan[c*x])/2
```

fricas [A] time = 0.42, size = 34, normalized size = 0.92

$$\frac{ac^2x^2 - bcx + \left(bc^2x^2 + b\right)\arctan\left(cx\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*x^2 - b*c*x + (b*c^2*x^2 + b)*arctan(c*x))/c^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 35, normalized size = 0.95

$$\frac{a\,x^2}{2} + \frac{b\,x^2\arctan\left(cx\right)}{2} - \frac{bx}{2c} + \frac{b\arctan\left(cx\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctan(c*x)-1/2*b*x/c+1/2*b*arctan(c*x)/c^2
```

maxima [A] time = 0.47, size = 37, normalized size = 1.00

$$\frac{1}{2}ax^2 + \frac{1}{2}\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
```

mupad [B] time = 0.12, size = 34, normalized size = 0.92

$$\frac{a\,x^2}{2} + \frac{b\,\mathrm{atan}\left(c\,x\right)}{2\,c^2} + \frac{b\,x^2\,\mathrm{atan}\left(c\,x\right)}{2} - \frac{b\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x)),x)
```

```
[Out] (a*x^2)/2 + (b*atan(c*x))/(2*c^2) + (b*x^2*atan(c*x))/2 - (b*x)/(2*c)
```

sympy [A] time = 0.42, size = 42, normalized size = 1.14

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2\,\mathrm{atan}\left(cx\right)}{2} - \frac{bx}{2c} + \frac{b\,\mathrm{atan}\left(cx\right)}{2c^2} & \text{for}\ c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x)/2 - b*x/(2*c) + b*atan(c*x)/(2*c**2)
, Ne(c, 0)), (a*x**2/2, True))
```

3.6 $\int \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=29

$$ax-\frac{b\log\left(c^2x^2+1\right)}{2c}+bx\tan^{-1}(cx)$$

```
[Out] a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4846, 260}

$$ax-\frac{b\log\left(c^2x^2+1\right)}{2c}+bx\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x],x]
```

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^{-1}(cx)\right)\,dx &= ax+b\int \tan^{-1}(cx)\,dx\\ &= ax+bx\tan^{-1}(cx)-(bc)\int \frac{x}{1+c^2x^2}\,dx\\ &= ax+bx\tan^{-1}(cx)-\frac{b\log\left(1+c^2x^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 29, normalized size = 1.00

$$ax-\frac{b\log\left(c^2x^2+1\right)}{2c}+bx\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x],x]
```

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

fricas [A] time = 0.42, size = 33, normalized size = 1.14

$$\frac{2\,bcx\arctan\left(cx\right)+2\,acx-b\log\left(c^2x^2+1\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="fricas")
[Out] 1/2*(2*b*c*x*arctan(c*x) + 2*a*c*x - b*log(c^2*x^2 + 1))/c
```

giac [A] time = 2.96, size = 31, normalized size = 1.07

$$ax + \frac{\left(2\,cx\arctan\left(cx\right) - \log\left(c^{2}x^{2} + 1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="giac")
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

maple [A] time = 0.00, size = 28, normalized size = 0.97

$$ax + bx\arctan\left(cx\right) - \frac{b\ln\left(c^{2}x^{2} + 1\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x),x)
[Out] a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

maxima [A] time = 0.32, size = 31, normalized size = 1.07

$$ax + \frac{\left(2\,cx\arctan\left(cx\right) - \log\left(c^{2}x^{2} + 1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="maxima")
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

mupad [B] time = 0.10, size = 27, normalized size = 0.93

$$a\,x - \frac{b\,\ln\left(c^{2}\,x^{2} + 1\right)}{2\,c} + b\,x\,\mathrm{atan}\left(c\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c*x),x)
[Out] a*x - (b*log(c^2*x^2 + 1))/(2*c) + b*x*atan(c*x)
```

sympy [A] time = 0.21, size = 26, normalized size = 0.90

$$ax + b\left(\begin{cases} x\,\mathrm{atan}\left(cx\right) - \frac{\log\left(c^{2}x^{2}+1\right)}{2c} & \text{for}\; c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x),x)
[Out] a*x + b*Piecewise((x*atan(c*x) - log(c**2*x**2 + 1)/(2*c), Ne(c, 0)), (0, T
rue))
```

3.7 $\int \frac{a+b\tan^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=35

$$a\log(x)+\frac{1}{2}ib\mathrm{Li}_2(-icx)-\frac{1}{2}ib\mathrm{Li}_2(icx)$$

```
[Out] a*ln(x)+1/2*I*b*polylog(2,-I*c*x)-1/2*I*b*polylog(2,I*c*x)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4848, 2391}

$$\frac{1}{2}ib\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\mathrm{PolyLog}(2,icx)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x,x]
```

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{x}\,dx &= a\log(x)+\frac{1}{2}(ib)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ib)\int \frac{\log(1+icx)}{x}\,dx\\ &= a\log(x)+\frac{1}{2}ib\mathrm{Li}_2(-icx)-\frac{1}{2}ib\mathrm{Li}_2(icx)\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a\log(x)+\frac{1}{2}ib\mathrm{Li}_2(-icx)-\frac{1}{2}ib\mathrm{Li}_2(icx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x,x]
```

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\arctan(cx)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="giac")
[Out] sage0*x
```

maple [B] time = 0.03, size = 74, normalized size = 2.11

$$a\ln(cx)+b\ln(cx)\arctan(cx)+\frac{ib\ln(cx)\ln(icx+1)}{2}-\frac{ib\ln(cx)\ln(-icx+1)}{2}+\frac{ib\,\mathrm{dilog}(icx+1)}{2}-\frac{ib\,\mathrm{dilog}(-icx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x,x)
[Out] a*ln(c*x)+b*ln(c*x)*arctan(c*x)+1/2*I*b*ln(c*x)*ln(1+I*c*x)-1/2*I*b*ln(c*x)
*ln(1-I*c*x)+1/2*I*b*dilog(1+I*c*x)-1/2*I*b*dilog(1-I*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\arctan(cx)}{x}\,dx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="maxima")
[Out] b*integrate(arctan(c*x)/x, x) + a*log(x)
```

mupad [B] time = 0.29, size = 28, normalized size = 0.80

$$a\,\ln(x)-\frac{b\,\left(\mathrm{Li}_2\left(1-c\,x\,1\mathrm{i}\right)-\mathrm{Li}_2\left(1+c\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x,x)
[Out] a*log(x) - (b*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{atan}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x,x)
[Out] Integral((a + b*atan(c*x))/x, x)
```

3.8 $\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{a+b\tan^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(c^2x^2+1\right)+bc\log(x)$$

```
[Out] (-a-b*arctan(c*x))/x+b*c*ln(x)-1/2*b*c*ln(c^2*x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4852, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(c^2x^2+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x])/x) + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx = -\frac{a+b\tan^{-1}(cx)}{x} + (bc)\int \frac{1}{x\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tan^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tan^{-1}(cx)}{x} + bc\log(x) - \frac{1}{2}bc\log\left(1+c^2x^2\right)$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.09

$$-\frac{a}{x} - \frac{1}{2}bc\log\left(c^2x^2+1\right) + bc\log(x) - \frac{b\tan^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^2,x]
[Out] -(a/x) - (b*ArcTan[c*x])/x + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

fricas [A] time = 0.43, size = 37, normalized size = 1.06

$$-\frac{bcx\log\left(c^2x^2+1\right) - 2\,bcx\log(x) + 2\,b\arctan(cx) + 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="fricas")
[Out] -1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 39, normalized size = 1.11

$$-\frac{a}{x} - \frac{b\arctan(cx)}{x} + cb\ln(cx) - \frac{bc\ln\left(c^2x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2,x)
[Out] -a/x-b/x*arctan(c*x)+c*b*ln(c*x)-1/2*b*c*ln(c^2*x^2+1)
```

maxima [A] time = 0.33, size = 39, normalized size = 1.11

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b - a/x
```

mupad [B] time = 0.32, size = 36, normalized size = 1.03

$$b\,c\,\ln(x)-\frac{a}{x}-\frac{b\,\mathrm{atan}\left(c\,x\right)}{x}-\frac{b\,c\,\ln\left(c^2\,x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^2,x)
```

```
[Out] b*c*log(x) - a/x - (b*atan(c*x))/x - (b*c*log(c^2*x^2 + 1))/2
```

sympy [A] time = 0.63, size = 37, normalized size = 1.06

$$\begin{cases} -\frac{a}{x}+bc\log\left(x\right)-\frac{bc\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b\,\mathrm{atan}\,(cx)}{x} & \text{for } c\neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*log(x) - b*c*log(x**2 + c**(-2))/2 - b*atan(c*x)/x, N
e(c, 0)), (-a/x, True))
```

3.9 $\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{a+b\tan^{-1}(cx)}{2x^2}-\frac{1}{2}bc^2\tan^{-1}(cx)-\frac{bc}{2x}$$

```
[Out] -1/2*b*c/x-1/2*b*c^2*arctan(c*x)+1/2*(-a-b*arctan(c*x))/x^2
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 325, 203}

$$-\frac{a+b\tan^{-1}(cx)}{2x^2}-\frac{1}{2}bc^2\tan^{-1}(cx)-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^3,x]
```

```
[Out] -(b*c)/(2*x) - (b*c^2*ArcTan[c*x])/2 - (a + b*ArcTan[c*x])/(2*x^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx &= -\frac{a+b\tan^{-1}(cx)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx\\ &= -\frac{bc}{2x}-\frac{a+b\tan^{-1}(cx)}{2x^2}-\frac{1}{2}\left(bc^3\right)\int \frac{1}{1+c^2x^2}\,dx\\ &= -\frac{bc}{2x}-\frac{1}{2}bc^2\tan^{-1}(cx)-\frac{a+b\tan^{-1}(cx)}{2x^2}\end{aligned}$$

Mathematica [C] time = 0.00, size = 46, normalized size = 1.24

$$-\frac{a}{2x^2}-\frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^2\right)}{2x}-\frac{b\tan^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*ArcTan[c*x])/(2*x^2) - (b*c*Hypergeometric2F1[-1/2, 1, 1/2,
 -(c^2*x^2)])/(2*x)
```

fricas [A] time = 0.44, size = 26, normalized size = 0.70

$$-\frac{bcx + \left(bc^2x^2 + b\right)\arctan\left(cx\right) + a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 35, normalized size = 0.95

$$-\frac{a}{2x^2} - \frac{b\arctan\left(cx\right)}{2x^2} - \frac{bc}{2x} - \frac{b\,c^2\arctan\left(cx\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x)-1/2*b*c/x-1/2*b*c^2*arctan(c*x)
```

maxima [A] time = 0.44, size = 31, normalized size = 0.84

$$-\frac{1}{2}\left(\left(c\arctan\left(cx\right) + \frac{1}{x}\right)c + \frac{\arctan\left(cx\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b - 1/2*a/x^2
```

mupad [B] time = 0.33, size = 42, normalized size = 1.14

$$-\frac{\frac{a}{2} + \frac{b\,\mathrm{atan}(c\,x)}{2} + \frac{b\,c\,x}{2}}{x^2} - \frac{b\,c\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,\sqrt{c^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^3,x)
```

```
[Out] - (a/2 + (b*atan(c*x))/2 + (b*c*x)/2)/x^2 - (b*c*atan((c^2*x)/(c^2)^(1/2))*
(c^2)^(1/2))/2
```

sympy [A] time = 0.53, size = 37, normalized size = 1.00

$$-\frac{a}{2x^2} - \frac{bc^2\,\mathrm{atan}\left(cx\right)}{2} - \frac{bc}{2x} - \frac{b\,\mathrm{atan}\left(cx\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*c**2*atan(c*x)/2 - b*c/(2*x) - b*atan(c*x)/(2*x**2)
```

3.10 $\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=53

$$-\frac{a+b\tan^{-1}(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(c^2x^2+1\right)-\frac{bc}{6x^2}$$

```
[Out] -1/6*b*c/x^2+1/3*(-a-b*arctan(c*x))/x^3-1/3*b*c^3*ln(x)+1/6*b*c^3*ln(c^2*x^
2+1)
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 266, 44}

$$-\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{6}bc^3\log\left(c^2x^2+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -(b*c)/(6*x^2) - (a + b*ArcTan[c*x])/(3*x^3) - (b*c^3*Log[x])/3 + (b*c^3*Lo
g[1 + c^2*x^2])/6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx &= -\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{3}(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)}\,dx\\
&= -\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^2\right)\\
&= -\frac{bc}{6x^2}-\frac{a+b\tan^{-1}(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+c^2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.02

$$-\frac{a}{3x^3}+\frac{1}{6}bc\left(c^2\log\left(c^2x^2+1\right)-2c^2\log(x)-\frac{1}{x^2}\right)-\frac{b\tan^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTan[c*x])/(3*x^3) + (b*c*(-x^(-2) - 2*c^2*Log[x] + c^2*L
og[1 + c^2*x^2]))/6
```

fricas [A] time = 0.43, size = 50, normalized size = 0.94

$$\frac{bc^3x^3\log\left(c^2x^2+1\right)-2\,bc^3x^3\log(x)-bcx-2\,b\arctan\left(cx\right)-2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(c
*x) - 2*a)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 51, normalized size = 0.96

$$-\frac{a}{3x^3}-\frac{b\arctan\left(cx\right)}{3x^3}-\frac{bc}{6x^2}-\frac{c^3b\ln\left(cx\right)}{3}+\frac{b\,c^3\ln\left(c^2x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x)-1/6*b*c/x^2-1/3*c^3*b*ln(c*x)+1/6*b*c^3*ln
(c^2*x^2+1)
```

maxima [A] time = 0.34, size = 51, normalized size = 0.96

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
 - 1/3*a/x^3
```

mupad [B] time = 0.12, size = 46, normalized size = 0.87

$$\frac{b\,c^3\,\ln\left(c^2\,x^2+1\right)}{6}-\frac{\frac{a}{3}+\frac{b\,\mathrm{atan}(c\,x)}{3}+\frac{b\,c\,x}{6}}{x^3}-\frac{b\,c^3\,\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^4,x)
```

```
[Out] (b*c^3*log(c^2*x^2 + 1))/6 - (a/3 + (b*atan(c*x))/3 + (b*c*x)/6)/x^3 - (b*c
^3*log(x))/3
```

sympy [A] time = 1.06, size = 61, normalized size = 1.15

$$\begin{cases} -\frac{a}{3x^3} - \frac{bc^3\log(x)}{3} + \frac{bc^3\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bc}{6x^2} - \frac{b\,\mathrm{atan}(cx)}{3x^3} & \text{for } c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*c**3*log(x)/3 + b*c**3*log(x**2 + c**(-2))/6 - b
*c/(6*x**2) - b*atan(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

3.11 $\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=48

$$-\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{1}{4}bc^4\tan^{-1}(cx)+\frac{bc^3}{4x}-\frac{bc}{12x^3}$$

```
[Out] -1/12*b*c/x^3+1/4*b*c^3/x+1/4*b*c^4*arctan(c*x)+1/4*(-a-b*arctan(c*x))/x^4
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 325, 203}

$$-\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)-\frac{bc}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^5,x]
```

```
[Out] -(b*c)/(12*x^3) + (b*c^3)/(4*x) + (b*c^4*ArcTan[c*x])/4 - (a + b*ArcTan[c*x
])/(4*x^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx &= -\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{1}{4}(bc)\int \frac{1}{x^4\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc}{12x^3}-\frac{a+b\tan^{-1}(cx)}{4x^4}-\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc}{12x^3}+\frac{bc^3}{4x}-\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{1+c^2x^2}\,dx\\
&= -\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)-\frac{a+b\tan^{-1}(cx)}{4x^4}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4}-\frac{bc\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-c^2x^2\right)}{12x^3}-\frac{b\tan^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*ArcTan[c*x])/(4*x^4) - (b*c*Hypergeometric2F1[-3/2, 1, -1/2
, -(c^2*x^2)])/(12*x^3)
```

fricas [A] time = 0.42, size = 41, normalized size = 0.85

$$\frac{3\,bc^3x^3-bcx+3\left(bc^4x^4-b\right)\arctan\left(cx\right)-3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^3*x^3 - b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x) - 3*a)/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 44, normalized size = 0.92

$$-\frac{a}{4x^4}-\frac{b\arctan\left(cx\right)}{4x^4}-\frac{bc}{12x^3}+\frac{b\,c^3}{4x}+\frac{b\,c^4\arctan\left(cx\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x)-1/12*b*c/x^3+1/4*b*c^3/x+1/4*b*c^4*arctan(
c*x)
```

maxima [A] time = 0.43, size = 46, normalized size = 0.96

$$\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b -
1/4*a/x^4
```

mupad [B] time = 0.36, size = 42, normalized size = 0.88

$$\frac{b\,c^4\,\mathrm{atan}\left(c\,x\right)}{4}-\frac{-b\,c^3\,x^3+\frac{b\,c\,x}{3}+a}{4\,x^4}-\frac{b\,\mathrm{atan}\left(c\,x\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^5,x)
```

```
[Out] (b*c^4*atan(c*x))/4 - (a - b*c^3*x^3 + (b*c*x)/3)/(4*x^4) - (b*atan(c*x))/(
4*x^4)
```

sympy [**A**] time = 0.83, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4}+\frac{bc^4\operatorname{atan}(cx)}{4}+\frac{bc^3}{4x}-\frac{bc}{12x^3}-\frac{b\operatorname{atan}(cx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**4*atan(c*x)/4 + b*c**3/(4*x) - b*c/(12*x**3) - b*atan(c*
x)/(4*x**4)
```

3.12 $\int \frac{a+b\tan^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=64

$$-\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)+\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(c^2x^2+1\right)-\frac{bc}{20x^4}$$

```
[Out] -1/20*b*c/x^4+1/10*b*c^3/x^2+1/5*(-a-b*arctan(c*x))/x^5+1/5*b*c^5*ln(x)-1/1
0*b*c^5*ln(c^2*x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4852, 266, 44}

$$-\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(c^2x^2+1\right)+\frac{1}{5}bc^5\log(x)-\frac{bc}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^6,x]
```

```
[Out] -(b*c)/(20*x^4) + (b*c^3)/(10*x^2) - (a + b*ArcTan[c*x])/(5*x^5) + (b*c^5*L
og[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^6}\,dx &= -\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{5}(bc)\int \frac{1}{x^5\left(1+c^2x^2\right)}\,dx\\
&= -\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{1}{x^3\left(1+c^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^3}-\frac{c^2}{x^2}+\frac{c^4}{x}-\frac{c^6}{1+c^2x}\right)dx,x,x^2\right)\\
&= -\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 1.00

$$-\frac{a}{5x^5}+\frac{1}{10}bc\left(2c^4\log(x)+\frac{c^2}{x^2}-c^4\log\left(c^2x^2+1\right)-\frac{1}{2x^4}\right)-\frac{b\tan^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (b*ArcTan[c*x])/(5*x^5) + (b*c*(-1/2*1/x^4 + c^2/x^2 + 2*c^4*L
og[x] - c^4*Log[1 + c^2*x^2]))/10
```

fricas [A] time = 0.43, size = 59, normalized size = 0.92

$$-\frac{2\,bc^5x^5\log\left(c^2x^2+1\right)-4\,bc^5x^5\log(x)-2\,bc^3x^3+bcx+4\,b\arctan\left(cx\right)+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b*
c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 60, normalized size = 0.94

$$-\frac{a}{5x^5}-\frac{b\arctan\left(cx\right)}{5x^5}-\frac{bc}{20x^4}+\frac{c^5b\ln\left(cx\right)}{5}+\frac{b\,c^3}{10x^2}-\frac{b\,c^5\ln\left(c^2x^2+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x)-1/20*b*c/x^4+1/5*c^5*b*ln(c*x)+1/10*b*c^3/
x^2-1/10*b*c^5*ln(c^2*x^2+1)
```

maxima [A] time = 0.33, size = 62, normalized size = 0.97

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b - 1/5*a/x^5
```

mupad [B] time = 0.37, size = 56, normalized size = 0.88

$$\frac{b\,c^5\,\ln(x)}{5}-\frac{b\,\mathrm{atan}\left(c\,x\right)}{5\,x^5}-\frac{b\,c^5\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{-\frac{b\,c^3\,x^3}{2}+\frac{b\,c\,x}{4}+a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^6,x)
```

```
[Out] (b*c^5*log(x))/5 - (b*atan(c*x))/(5*x^5) - (b*c^5*log(c^2*x^2 + 1))/10 - (a
 - (b*c^3*x^3)/2 + (b*c*x)/4)/(5*x^5)
```

sympy [A] time = 1.67, size = 71, normalized size = 1.11

$$\begin{cases} -\frac{a}{5x^5} + \frac{bc^5 \log(x)}{5} - \frac{bc^5 \log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3}{10x^2} - \frac{bc}{20x^4} - \frac{b \operatorname{atan}(cx)}{5x^5} & \text{for } c \neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x**2 + c**(-2))/10 +
b*c**3/(10*x**2) - b*c/(20*x**4) - b*atan(c*x)/(5*x**5), Ne(c, 0)), (-a/(5*
x**5), True))
```

3.13 $\int x^5 \left(a + b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=144

$$\frac{\left(a+b\tan^{-1}(cx)\right)^2}{6c^6}-\frac{abx}{3c^5}+\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c}-\frac{b^2x\tan^{-1}(cx)}{3c^5}$$

```
[Out] -1/3*a*b*x/c^5-4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2-1/3*b^2*x*arctan(c*x)/c^5+
1/9*b*x^3*(a+b*arctan(c*x))/c^3-1/15*b*x^5*(a+b*arctan(c*x))/c+1/6*(a+b*arc
tan(c*x))^2/c^6+1/6*x^6*(a+b*arctan(c*x))^2+23/90*b^2*ln(c^2*x^2+1)/c^6
```

Rubi [A] time = 0.31, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{abx}{3c^5}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{b^2x^4}{60c^2}-\frac{4b^2x^2}{45c^4}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -(a*b*x)/(3*c^5) - (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) - (b^2*x*ArcTa
n[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*x^5*(a + b*ArcTa
n[c*x]))/(15*c) + (a + b*ArcTan[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTan[c*x])^
2)/6 + (23*b^2*Log[1 + c^2*x^2])/(90*c^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}(cx)\right)^2dx=\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(bc)\int\frac{x^6\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$

$$=\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int x^4\left(a+b\tan^{-1}(cx)\right)dx}{3c}+\frac{b\int\frac{x^4\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx}{3c}$$

$$=-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{15}b^2\int\frac{x^5}{1+c^2x^2}dx+\frac{b\int x^2\left(a\right.}{}$$

$$=\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{30}b^2\,\mathrm{Sub}$$

$$=-\frac{abx}{3c^5}+\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6$$

$$=-\frac{abx}{3c^5}-\frac{b^2x^2}{30c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\tan^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{bx^5\left(a+b\tan^{-1}(c\right.}{15c}$$

$$=-\frac{abx}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\tan^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{bx^5\left(a+b\tan^{-1}(\right.}{15c}$$

Mathematica [A] time = 0.14, size = 138, normalized size = 0.96

$$\frac{cx\left(30a^2c^5x^5-4ab\left(3c^4x^4-5c^2x^2+15\right)+b^2cx\left(3c^2x^2-16\right)\right)+4b\tan^{-1}(cx)\left(15a\left(c^6x^6+1\right)+bcx\left(-3c^4x^4+5c^2\right.\right.}{180c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(30*a^2*c^5*x^5 + b^2*c*x*(-16 + 3*c^2*x^2) - 4*a*b*(15 - 5*c^2*x^2 +
3*c^4*x^4)) + 4*b*(b*c*x*(-15 + 5*c^2*x^2 - 3*c^4*x^4) + 15*a*(1 + c^6*x^6)
)*ArcTan[c*x] + 30*b^2*(1 + c^6*x^6)*ArcTan[c*x]^2 + 46*b^2*Log[1 + c^2*x^2
])/(180*c^6)
```

fricas [A] time = 0.43, size = 152, normalized size = 1.06

$$\frac{30\,a^2c^6x^6-12\,abc^5x^5+3\,b^2c^4x^4+20\,abc^3x^3-16\,b^2c^2x^2-60\,abcx+30\left(b^2c^6x^6+b^2\right)\arctan(cx)^2+46\,b^2\log\left(c\right.}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*a*b*c^3*x^3 - 1
6*b^2*c^2*x^2 - 60*a*b*c*x + 30*(b^2*c^6*x^6 + b^2)*arctan(c*x)^2 + 46*b^2*
log(c^2*x^2 + 1) + 4*(15*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 15*b
^2*c*x + 15*a*b)*arctan(c*x))/c^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 171, normalized size = 1.19

$$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(cx)^2}{6}-\frac{b^2\arctan(cx)x^5}{15c}+\frac{b^2\arctan(cx)x^3}{9c^3}-\frac{b^2x\arctan(cx)}{3c^5}+\frac{b^2\arctan(cx)^2}{6c^6}+\frac{b^2x^4}{60c^2}-\frac{4b^2x^2}{45c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctan(c*x)^2-1/15/c*b^2*arctan(c*x)*x^5+1/9/c^3*b^
2*arctan(c*x)*x^3-1/3*b^2*x*arctan(c*x)/c^5+1/6/c^6*b^2*arctan(c*x)^2+1/60*
b^2*x^4/c^2-4/45*b^2*x^2/c^4+23/90*b^2*ln(c^2*x^2+1)/c^6+1/3*a*b*x^6*arctan
(c*x)-1/15/c*x^5*a*b+1/9*a*b*x^3/c^3-1/3*a*b*x/c^5+1/3/c^6*a*b*arctan(c*x)
```

maxima [A] time = 0.46, size = 163, normalized size = 1.13

$$\frac{1}{6}b^2x^6\arctan(cx)^2+\frac{1}{6}a^2x^6+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)ab-\frac{1}{180}\left(4c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x)^2 + 1/6*a^2*x^6 + 1/45*(15*x^6*arctan(c*x) - c*((3*
c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b - 1/180*(4*c*((3
*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*c^4
*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*b^2
```

mupad [B] time = 0.67, size = 171, normalized size = 1.19

$$\frac{30b^2\operatorname{atan}(cx)^2+46b^2\ln\left(c^2x^2+1\right)+30a^2c^6x^6-16b^2c^2x^2+3b^2c^4x^4+60ab\operatorname{atan}(cx)+20b^2c^3x^3\operatorname{at}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x))^2,x)
```

```
[Out] (30*b^2*atan(c*x)^2 + 46*b^2*log(c^2*x^2 + 1) + 30*a^2*c^6*x^6 - 16*b^2*c^2
*x^2 + 3*b^2*c^4*x^4 + 60*a*b*atan(c*x) + 20*b^2*c^3*x^3*atan(c*x) - 12*b^2
*c^5*x^5*atan(c*x) - 60*b^2*c*x*atan(c*x) + 30*b^2*c^6*x^6*atan(c*x)^2 + 20
*a*b*c^3*x^3 - 12*a*b*c^5*x^5 - 60*a*b*c*x + 60*a*b*c^6*x^6*atan(c*x))/(180
*c^6)
```

sympy [A] time = 2.39, size = 199, normalized size = 1.38

$$\begin{cases} \frac{a^2x^6}{6}+\frac{abx^6\operatorname{atan}(cx)}{3}-\frac{abx^5}{15c}+\frac{abx^3}{9c^3}-\frac{abx}{3c^5}+\frac{ab\operatorname{atan}(cx)}{3c^6}+\frac{b^2x^6\operatorname{atan}^2(cx)}{6}-\frac{b^2x^5\operatorname{atan}(cx)}{15c}+\frac{b^2x^4}{60c^2}+\frac{b^2x^3\operatorname{atan}(cx)}{9c^3}-\frac{4b^2x^2}{45c^4}-\frac{b^2}{} \\ \frac{a^2x^6}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x)/3 - a*b*x**5/(15*c) + a*b*x**3/
(9*c**3) - a*b*x/(3*c**5) + a*b*atan(c*x)/(3*c**6) + b**2*x**6*atan(c*x)**2
/6 - b**2*x**5*atan(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atan(c*x)
/(9*c**3) - 4*b**2*x**2/(45*c**4) - b**2*x*atan(c*x)/(3*c**5) + 23*b**2*log
(x**2 + c**(-2))/(90*c**6) + b**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*x
**6/6, True))
```

3.14 $\int x^4 \left(a + b \tan^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=170

$$\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{5c^5}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^4\left(a+\right.}{}$$

```
[Out] -3/10*b^2*x/c^4+1/30*b^2*x^3/c^2+3/10*b^2*arctan(c*x)/c^5+1/5*b*x^2*(a+b*ar
ctan(c*x))/c^3-1/10*b*x^4*(a+b*arctan(c*x))/c+1/5*I*(a+b*arctan(c*x))^2/c^5
+1/5*x^5*(a+b*arctan(c*x))^2+2/5*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5+1/
5*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5
```

Rubi [A] time = 0.29, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {4852, 4916, 302, 203, 321, 4920, 4854, 2402, 2315}

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{5c^5}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}+\frac{1}{5}x^5$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) + (3*b^2*ArcTan[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*x^4*(a + b*ArcTan[c*x]))/(10*c) + (
(I/5)*(a + b*ArcTan[c*x])^2)/c^5 + (x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + ((I/5)*b^2*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}(cx)\right)^2dx=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}(2bc)\int\frac{x^5\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$

$$=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{(2b)\int x^3\left(a+b\tan^{-1}(cx)\right)dx}{5c}+\frac{(2b)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx}{5c}$$

$$=-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{10}b^2\int\frac{x^4}{1+c^2x^2}dx+\frac{(2b)\int x\,(\ldots}{\ldots}$$

$$=\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{5c^5}+\frac{1}{5}x^5\left(a+b\,\mathrm{t}\ldots\right.$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{i\left(a+b\tan^{-1}(cx)\ldots\right.}{5c^5}$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\tan^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\ldots$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\tan^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\ldots$$

Mathematica [A] time = 0.51, size = 169, normalized size = 0.99

$$\frac{6a^2c^5x^5 - 3abc^4x^4 + 6abc^2x^2 - 6ab\log\left(c^2x^2+1\right) - 3b\tan^{-1}(cx)\left(-4ac^5x^5 + b\left(c^4x^4 - 2c^2x^2 - 3\right) - 4b\log\left(1 \right.\right.}{30c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (9*a*b - 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 - 3*a*b*c^4*x^4 + 6*a^2*c^
5*x^5 + 6*b^2*(-I + c^5*x^5)*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^5*x^5
+ b*(-3 - 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 6*a*
b*Log[1 + c^2*x^2] - (6*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^4\arctan(cx)^2 + 2\,abx^4\arctan(cx) + a^2x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctan(c*x)^2 + 2*a*b*x^4*arctan(c*x) + a^2*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.10, size = 334, normalized size = 1.96

$$\frac{x^5a^2}{5} + \frac{x^5b^2\arctan(cx)^2}{5} - \frac{b^2\arctan(cx)\,x^4}{10c} + \frac{b^2\arctan(cx)\,x^2}{5c^3} - \frac{b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{5c^5} + \frac{b^2x^3}{30c^2} - \frac{3b^2x}{10c^4} + \frac{3b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/5*x^5*a^2+1/5*x^5*b^2*arctan(c*x)^2-1/10/c*b^2*arctan(c*x)*x^4+1/5/c^3*b^
2*arctan(c*x)*x^2-1/5/c^5*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/30*b^2*x^3/c^2-3/
10*b^2*x/c^4+3/10*b^2*arctan(c*x)/c^5+1/20*I/c^5*b^2*ln(c*x-I)^2-1/10*I/c^5
*b^2*dilog(1/2*I*(c*x-I))-1/20*I/c^5*b^2*ln(I+c*x)^2+1/10*I/c^5*b^2*dilog(-
1/2*I*(I+c*x))+1/10*I/c^5*b^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/10*I/c^5*b^2*l
n(c*x-I)*ln(c^2*x^2+1)+1/10*I/c^5*b^2*ln(I+c*x)*ln(c^2*x^2+1)-1/10*I/c^5*b^
2*ln(I+c*x)*ln(1/2*I*(c*x-I))+2/5*x^5*a*b*arctan(c*x)-1/10/c*x^4*a*b+1/5*a*
b*x^2/c^3-1/5/c^5*a*b*ln(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}a^2x^5 + \frac{1}{10}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)ab + \frac{1}{80}\left(4\,x^5\arctan(cx)^2 - x^5\log\left(c^2x^2+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^
2*x^2 + 1)/c^6))*a*b + 1/80*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 +
 80*integrate(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(
c^2*x^6 + x^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x
^2 + 1), x))*b^2
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,(a+b\,\mathrm{atan}\,(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x))^2,x)
```

```
[Out] int(x^4*(a + b*atan(c*x))^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,(a+b\,\mathrm{atan}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**4*(a + b*atan(c*x))**2, x)
```

3.15 $\int x^3 \left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=112

$$-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{abx}{2c^3}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{b^2x\tan^{-1}(cx)}{2c^3}+\frac{b^2x^2}{12c^2}-\frac{b^2\log\left(c^2x^2 +\right.}{3c^4}$$

```
[Out] 1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctan(c*x)/c^3-1/6*b*x^3*(a+b*arc
tan(c*x))/c-1/4*(a+b*arctan(c*x))^2/c^4+1/4*x^4*(a+b*arctan(c*x))^2-1/3*b^2
*ln(c^2*x^2+1)/c^4
```

Rubi [A] time = 0.21, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{abx}{2c^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{b^2x^2}{12c^2}-\frac{b^2\log\left(c^2x^2+1\right)}{3c^4}+\frac{b^2x\tan^{-1}(c}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTan[c*x])/(2*c^3) - (b*x^3
*(a + b*ArcTan[c*x]))/(6*c) - (a + b*ArcTan[c*x])^2/(4*c^4) + (x^4*(a + b*A
rcTan[c*x])^2)/4 - (b^2*Log[1 + c^2*x^2])/(3*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}(cx)\right)^2 dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(bc)\int\frac{x^4\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int x^2\left(a+b\tan^{-1}(cx)\right)dx}{2c}+\frac{b\int\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx}{2c}\\
&= -\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}b^2\int\frac{x^3}{1+c^2x^2}dx+\frac{b\int\left(a+b\right.}{}\\
&= \frac{abx}{2c^3}-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{12}b^2\\
&= \frac{abx}{2c^3}+\frac{b^2x\tan^{-1}(cx)}{2c^3}-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan\right.\\
&= \frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\tan^{-1}(cx)}{2c^3}-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a\right.
\end{aligned}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.99

$$\frac{cx\left(3a^2c^3x^3-2abc^2x^2+6ab+b^2cx\right)-2b\tan^{-1}(cx)\left(a\left(3-3c^4x^4\right)+bcx\left(c^2x^2-3\right)\right)+3b^2\left(c^4x^4-1\right)\tan^{-1}(cx)^2-}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(6*a*b + b^2*c*x - 2*a*b*c^2*x^2 + 3*a^2*c^3*x^3) - 2*b*(b*c*x*(-3 + c
^2*x^2) + a*(3 - 3*c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]
^2 - 4*b^2*Log[1 + c^2*x^2])/(12*c^4)
```

fricas [A] time = 0.43, size = 121, normalized size = 1.08

$$\frac{3\,a^2c^4x^4-2\,abc^3x^3+b^2c^2x^2+6\,abcx+3\left(b^2c^4x^4-b^2\right)\arctan(cx)^2-4\,b^2\log\left(c^2x^2+1\right)+2\left(3\,abc^4x^4-b^2c^3x^3-\right.}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*c^4*x^4 - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*a*b*c*x + 3*(b^2*c^4*
x^4 - b^2)*arctan(c*x)^2 - 4*b^2*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 - b^2*
c^3*x^3 + 3*b^2*c*x - 3*a*b)*arctan(c*x))/c^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 135, normalized size = 1.21

$$\frac{a^2x^4}{4}+\frac{b^2x^4\arctan(cx)^2}{4}-\frac{b^2\arctan(cx)x^3}{6c}+\frac{b^2x\arctan(cx)}{2c^3}-\frac{b^2\arctan(cx)^2}{4c^4}+\frac{b^2x^2}{12c^2}-\frac{b^2\ln\left(c^2x^2+1\right)}{3c^4}+\frac{x^4ab\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctan(c*x)^2-1/6/c*b^2*arctan(c*x)*x^3+1/2*b^2*x*a
rctan(c*x)/c^3-1/4/c^4*b^2*arctan(c*x)^2+1/12*b^2*x^2/c^2-1/3*b^2*ln(c^2*x^
2+1)/c^4+1/2*x^4*a*b*arctan(c*x)-1/6*a*b*x^3/c+1/2*a*b*x/c^3-1/2/c^4*a*b*ar
ctan(c*x)
```

maxima [A] time = 0.43, size = 136, normalized size = 1.21

$$\frac{1}{4}\,b^2x^4\arctan(cx)^2+\frac{1}{4}\,a^2x^4+\frac{1}{6}\left(3\,x^4\arctan(cx)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan(cx)}{c^5}\right)\right)ab-\frac{1}{12}\left(2\,c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\mathrm{a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 +
 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^
2 + 1))/c^4)*b^2
```

mupad [B] time = 0.32, size = 134, normalized size = 1.20

$$\frac{3\,a^2\,c^4\,x^4-4\,b^2\,\ln\left(c^2\,x^2+1\right)-3\,b^2\,\mathrm{atan}\left(c\,x\right)^2+b^2\,c^2\,x^2-6\,a\,b\,\mathrm{atan}\left(c\,x\right)-2\,b^2\,c^3\,x^3\,\mathrm{atan}\left(c\,x\right)+6\,b^2\,c\,x\,\mathrm{atan}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^2,x)
```

```
[Out] (3*a^2*c^4*x^4 - 4*b^2*log(c^2*x^2 + 1) - 3*b^2*atan(c*x)^2 + b^2*c^2*x^2 -
 6*a*b*atan(c*x) - 2*b^2*c^3*x^3*atan(c*x) + 6*b^2*c*x*atan(c*x) + 3*b^2*c^
4*x^4*atan(c*x)^2 - 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atan(c*x))/(1
2*c^4)
```

sympy [A] time = 1.44, size = 155, normalized size = 1.38

$$\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\,\mathrm{atan}(cx)}{2}-\frac{abx^3}{6c}+\frac{abx}{2c^3}-\frac{ab\,\mathrm{atan}(cx)}{2c^4}+\frac{b^2x^4\,\mathrm{atan}^2(cx)}{4}-\frac{b^2x^3\,\mathrm{atan}(cx)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2x\,\mathrm{atan}(cx)}{2c^3}-\frac{b^2\log\left(x^2+\frac{1}{c^2}\right)}{3c^4}-\frac{b^2}{} \\ \frac{a^2x^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x)/2 - a*b*x**3/(6*c) + a*b*x/(2*c
**3) - a*b*atan(c*x)/(2*c**4) + b**2*x**4*atan(c*x)**2/4 - b**2*x**3*atan(c
*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atan(c*x)/(2*c**3) - b**2*log(x**2
 + c**(-2))/(3*c**4) - b**2*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*x**4/4,
 True))
```

3.16 $\int x^2 \left(a + b \tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=138

$$-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{ib^2\text{Li}_2\left(1\right.}{3c^3}$$

```
[Out] 1/3*b^2*x/c^2-1/3*b^2*arctan(c*x)/c^3-1/3*b*x^2*(a+b*arctan(c*x))/c-1/3*I*(
a+b*arctan(c*x))^2/c^3+1/3*x^3*(a+b*arctan(c*x))^2-2/3*b*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/c^3-1/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^3
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4852, 4916, 321, 203, 4920, 4854, 2402, 2315}

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTan[c*x])/(3*c^3) - (b*x^2*(a + b*ArcTan[c*x]))/(
3*c) - ((I/3)*(a + b*ArcTan[c*x])^2)/c^3 + (x^3*(a + b*ArcTan[c*x])^2)/3 -
(2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) - ((I/3)*b^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2bc)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{(2b)\int x\left(a+b\tan^{-1}(cx)\right)dx}{3c}+\frac{(2b)\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{3c}\\
&= -\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}b^2\int\\
&= \frac{b^2x}{3c^2}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{2}{}\\
&= \frac{b^2x}{3c^2}-\frac{b^2\tan^{-1}(cx)}{3c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\right.\\
&= \frac{b^2x}{3c^2}-\frac{b^2\tan^{-1}(cx)}{3c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\right.
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 131, normalized size = 0.95

$$\frac{a^2c^3x^3-abc^2x^2+ab\log\left(c^2x^2+1\right)-b\tan^{-1}(cx)\left(-2ac^3x^3+bc^2x^2+2b\log\left(1+e^{2i\tan^{-1}(cx)}\right)+b\right)+b^2\left(c^3x^3+\right.}{3c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*c*x - a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(I + c^3*x^3)*ArcTan[c*x]^2 - b*
ArcTan[c*x]*(b + b*c^2*x^2 - 2*a*c^3*x^3 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x]
)]) + a*b*Log[1 + c^2*x^2] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c
^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\arctan(cx)^2 + 2\,abx^2\arctan(cx) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 298, normalized size = 2.16

$$\frac{x^3a^2}{3}+\frac{b^2x^3\arctan(cx)^2}{3}-\frac{b^2\arctan(cx)\,x^2}{3c}+\frac{b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{3c^3}+\frac{b^2x}{3c^2}-\frac{b^2\arctan(cx)}{3c^3}+\frac{ib^2\ln(cx-i)\ln\left(c\right.}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctan(c*x)^2-1/3/c*b^2*arctan(c*x)*x^2+1/3/c^3*b^2
*arctan(c*x)*ln(c^2*x^2+1)+1/3*b^2*x/c^2-1/3*b^2*arctan(c*x)/c^3+1/6*I/c^3*
b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/12*I/c^3*b^2*ln(I+c*x)^2+1/6*I/c^3*b^2*ln(I+c
*x)*ln(1/2*I*(c*x-I))-1/6*I/c^3*b^2*dilog(-1/2*I*(I+c*x))-1/6*I/c^3*b^2*ln(
c*x-I)*ln(-1/2*I*(I+c*x))-1/12*I/c^3*b^2*ln(c*x-I)^2+1/6*I/c^3*b^2*dilog(1/
2*I*(c*x-I))-1/6*I/c^3*b^2*ln(I+c*x)*ln(c^2*x^2+1)+2/3*a*b*x^3*arctan(c*x)-
1/3*a*b*x^2/c+1/3/c^3*a*b*ln(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2x^3+\frac{1}{3}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)ab+\frac{1}{48}\left(4x^3\arctan(cx)^2-x^3\log\left(c^2x^2+1\right)^2+48\int\frac{4c^2x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a*b + 1/48*(4*x^3*arctan(c*x)^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/4
8*(4*c^2*x^4*log(c^2*x^2 + 1) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*ar
ctan(c*x)^2 + 3*(c^2*x^4 + x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{atan}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2, x)
```

3.17 $\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=76

$$\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{abx}{c}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2}-\frac{b^2x\tan^{-1}(cx)}{c}$$

```
[Out] -a*b*x/c-b^2*x*arctan(c*x)/c+1/2*(a+b*arctan(c*x))^2/c^2+1/2*x^2*(a+b*arcta
n(c*x))^2+1/2*b^2*ln(c^2*x^2+1)/c^2
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4852, 4916, 4846, 260, 4884}

$$\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{abx}{c}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2}-\frac{b^2x\tan^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*x)/c) - (b^2*x*ArcTan[c*x])/c + (a + b*ArcTan[c*x])^2/(2*c^2) + (x^2
*(a + b*ArcTan[c*x])^2)/2 + (b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}(cx)\right)^2 dx = \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-(bc)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$
$$=\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int\left(a+b\tan^{-1}(cx)\right)dx}{c}+\frac{b\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}dx}{c}$$
$$=-\frac{abx}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{b^2\int\tan^{-1}(cx)\,dx}{c}$$
$$=-\frac{abx}{c}-\frac{b^2x\tan^{-1}(cx)}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2+b^2\int\frac{}{1+}$$
$$=-\frac{abx}{c}-\frac{b^2x\tan^{-1}(cx)}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{b^2\log\left(1\right.}{2c}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.99

$$\frac{2b\tan^{-1}(cx)\left(ac^2x^2+a-bcx\right)+acx(acx-2b)+b^2\log\left(c^2x^2+1\right)+b^2\left(c^2x^2+1\right)\tan^{-1}(cx)^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*c*x*(-2*b + a*c*x) + 2*b*(a - b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
c^2*x^2)*ArcTan[c*x]^2 + b^2*Log[1 + c^2*x^2])/(2*c^2)
```

fricas [A] time = 0.43, size = 83, normalized size = 1.09

$$\frac{a^2c^2x^2-2\,abcx+\left(b^2c^2x^2+b^2\right)\arctan\left(cx\right)^2+b^2\log\left(c^2x^2+1\right)+2\left(abc^2x^2-b^2cx+ab\right)\arctan\left(cx\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*c^2*x^2 - 2*a*b*c*x + (b^2*c^2*x^2 + b^2)*arctan(c*x)^2 + b^2*log(
c^2*x^2 + 1) + 2*(a*b*c^2*x^2 - b^2*c*x + a*b)*arctan(c*x))/c^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 97, normalized size = 1.28

$$\frac{a^2x^2}{2}+\frac{x^2b^2\arctan\left(cx\right)^2}{2}+\frac{b^2\arctan\left(cx\right)^2}{2c^2}-\frac{b^2x\arctan\left(cx\right)}{c}+\frac{b^2\ln\left(c^2x^2+1\right)}{2c^2}+ab\,x^2\arctan\left(cx\right)+\frac{ab\arctan\left(cx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*x^2*b^2*arctan(c*x)^2+1/2/c^2*b^2*arctan(c*x)^2-b^2*x*arcta
n(c*x)/c+1/2*b^2*ln(c^2*x^2+1)/c^2+a*b*x^2*arctan(c*x)+1/c^2*a*b*arctan(c*x
)-a*b*x/c
```

maxima [A] time = 0.47, size = 104, normalized size = 1.37

$$\frac{1}{2}\,b^2x^2\arctan\left(cx\right)^2+\frac{1}{2}\,a^2x^2+\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)ab-\frac{1}{2}\left(2\,c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\arctan\left(cx\right)+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c*x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arc
tan(c*x)/c^3))*a*b - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arct
an(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
```

mupad [B] time = 0.41, size = 88, normalized size = 1.16

$$\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{2}+\frac{b^2\,\ln\left(c^2\,x^2+1\right)}{2}-c\,\left(x\,\mathrm{atan}\left(c\,x\right)\,b^2+a\,x\,b\right)+a\,b\,\mathrm{atan}\left(c\,x\right)}{c^2}+\frac{a^2\,x^2}{2}+\frac{b^2\,x^2\,\mathrm{atan}\left(c\,x\right)^2}{2}+a\,b\,x^2\,\mathrm{atan}\left(c\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^2,x)
```

```
[Out] ((b^2*atan(c*x)^2)/2 + (b^2*log(c^2*x^2 + 1))/2 - c*(b^2*x*atan(c*x) + a*b*
x) + a*b*atan(c*x))/c^2 + (a^2*x^2)/2 + (b^2*x^2*atan(c*x)^2)/2 + a*b*x^2*a
tan(c*x)
```

sympy [A] time = 0.73, size = 107, normalized size = 1.41

$$\begin{cases} \frac{a^2x^2}{2}+abx^2\,\mathrm{atan}\left(cx\right)-\frac{abx}{c}+\frac{ab\,\mathrm{atan}\left(cx\right)}{c^2}+\frac{b^2x^2\,\mathrm{atan}^2\left(cx\right)}{2}-\frac{b^2x\,\mathrm{atan}\left(cx\right)}{c}+\frac{b^2\log\left(x^2+\frac{1}{c^2}\right)}{2c^2}+\frac{b^2\,\mathrm{atan}^2\left(cx\right)}{2c^2} & \text{for}\ c\neq 0 \\ \frac{a^2x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atan(c*x) - a*b*x/c + a*b*atan(c*x)/c**2
+ b**2*x**2*atan(c*x)**2/2 - b**2*x*atan(c*x)/c + b**2*log(x**2 + c**(-2))/
(2*c**2) + b**2*atan(c*x)**2/(2*c**2), Ne(c, 0)), (a**2*x**2/2, True))
```

3.18 $\int \left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=83

$$x\left(a+b\tan^{-1}(cx)\right)^2+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{ib^2\mathrm{Li}_2\left(1-\frac{2}{icx+1}\right)}{c}$$

```
[Out] I*(a+b*arctan(c*x))^2/c+x*(a+b*arctan(c*x))^2+2*b*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c+I*b^2*polylog(2,1-2/(1+I*c*x))/c
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4846, 4920, 4854, 2402, 2315}

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2)/c + x*(a + b*ArcTan[c*x])^2 + (2*b*(a + b*ArcTan[
c*x])*Log[2/(1 + I*c*x)])/c + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)^2 dx = x\left(a+b\tan^{-1}(cx)\right)^2 - (2bc)\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c} + x\left(a+b\tan^{-1}(cx)\right)^2 + (2b)\int \frac{a+b\tan^{-1}(cx)}{i-cx}dx$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c} + x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c} - \left(2b^2\right)$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c} + x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c} + \frac{\left(2ib^2\right)}{}$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c} + x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c} + \frac{ib^2\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 1.08

$$\frac{a\left(acx-b\log\left(c^2x^2+1\right)\right)+2b\tan^{-1}(cx)\left(acx+b\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right)-ib^2\mathrm{Li}_2\left(-e^{2i\tan^{-1}(cx)}\right)+b^2(cx-i)\tan^{-1}(c}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*(-I + c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(a*c*x + b*Log[1 + E^((2*I)
*ArcTan[c*x])]) + a*(a*c*x - b*Log[1 + c^2*x^2]) - I*b^2*PolyLog[2, -E^((2*
I)*ArcTan[c*x])])/c
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2\arctan(cx)^2 + 2\,ab\arctan(cx) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.19, size = 128, normalized size = 1.54

$$x\,b^2\arctan(cx)^2 - \frac{i\arctan(cx)^2\,b^2}{c} + 2xab\arctan(cx) + \frac{2\arctan(cx)\ln\left(\frac{(icx+1)^2}{c^2x^2+1}+1\right)b^2}{c} - \frac{i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2,x)
```

```
[Out] x*b^2*arctan(c*x)^2-I/c*arctan(c*x)^2*b^2+2*x*a*b*arctan(c*x)+2/c*arctan(c*
x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*b^2-I/c*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)
)*b^2+a^2*x-1/c*a*b*ln(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(4\,x\arctan\left(cx\right)^{2}+192\,c^{2}\int\frac{x^{2}\arctan\left(cx\right)^{2}}{16\left(c^{2}x^{2}+1\right)}\,dx+16\,c^{2}\int\frac{x^{2}\log\left(c^{2}x^{2}+1\right)^{2}}{16\left(c^{2}x^{2}+1\right)}\,dx+64\,c^{2}\int\frac{x^{2}\log\left(c^{2}x^{2}+1\right)}{16\left(c^{2}x^{2}+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x*arctan(c*x)^2 + 192*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2
 + 1), x) + 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
+ 64*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - x*log(c^2*
x^2 + 1)^2 + 4*arctan(c*x)^3/c - 128*c*integrate(1/16*x*arctan(c*x)/(c^2*x^
2 + 1), x) + 16*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x))*b^2 +
a^2*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{atan}\left(c\,x\right))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2,x)
```

```
[Out] int((a + b*atan(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{atan}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2, x)
```

3.19 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=132

$$-ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)\left(a+b\tan^{-1}(cx)\right)+ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)\left(a+b\tan^{-1}(cx)\right)+2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}\right.$$

```
[Out] -2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-I*b*(a+b*arctan(c*x))*polylo
g(2,1-2/(1+I*c*x))+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*
polylog(3,1-2/(1+I*c*x))+1/2*b^2*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A] time = 0.25, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4850, 4988, 4884, 4994, 6610}

$$-ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2
/(1 + I*c*x)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/2
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-(4bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh^{-1}\left(1-\right.}{1+c^2x^2}$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+(2bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,d$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-ib\left(a+b\tan^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2}{1+icx}\right)+$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-ib\left(a+b\tan^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2}{1+icx}\right)+$$

Mathematica [A] time = 0.08, size = 144, normalized size = 1.09

$$\frac{1}{2}b\left(2i\text{Li}_2\left(\frac{cx+i}{i-cx}\right)\left(a+b\tan^{-1}(cx)\right)-2i\text{Li}_2\left(\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)+b\left(\text{Li}_3\left(\frac{cx+i}{i-cx}\right)-\text{Li}_3\left(\frac{cx+i}{cx-i}\right)\right)\right)+2\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[(I + c*x)/(-I + c*x)] + (b*((2*I)*(a + b*Ar
cTan[c*x])*PolyLog[2, (I + c*x)/(I - c*x)] - (2*I)*(a + b*ArcTan[c*x])*Poly
Log[2, (I + c*x)/(-I + c*x)] + b*(PolyLog[3, (I + c*x)/(I - c*x)] - PolyLog
[3, (I + c*x)/(-I + c*x)])))/2
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.51, size = 1128, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x,x)
```

```
[Out] b^2*ln(c*x)*arctan(c*x)^2-b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b
^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*arctan(c*x)^2*ln(1-(
1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x
^2+1))-2*I*b^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2*a
rctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2*Pi*arctan(c*x)
^2+I*a*b*dilog(1+I*c*x)+2*a*b*ln(c*x)*arctan(c*x)-I*a*b*dilog(1-I*c*x)-1/2*
I*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*ar
ctan(c*x)^2+1/2*I*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+
2*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*polylog(3,(1+I*c*x)/(c^
2*x^2+1)^(1/2))+a^2*ln(c*x)-1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+
1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*ar
ctan(c*x)^2+1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^3*arctan(c*x)^2+I*a*b*ln(c*x)*ln(1+I*c*x)-I*a*b*ln(c*x)*ln(1-
I*c*x)+1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))*arctan(c*x)^2-1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \log(x) + \frac{1}{16} \int \frac{12\, b^2 \arctan\left(cx\right)^2 + b^2 \log\left(c^2x^2+1\right)^2 + 32\, ab \arctan\left(cx\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2 + 1)^2
+ 32*a*b*arctan(c*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x,x)
```

```
[Out] int((a + b*atan(c*x))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x, x)
```

3.20 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=82

$$-ic\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)$$

```
[Out] -I*c*(a+b*arctan(c*x))^2-(a+b*arctan(c*x))^2/x+2*b*c*(a+b*arctan(c*x))*ln(2
-2/(1-I*c*x))-I*b^2*c*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A] time = 0.15, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4852, 4924, 4868, 2447}

$$-ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-ic\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^2,x]
```

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/x + 2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^2 - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + (2ibc)\int \frac{a+b\tan^{-1}(cx)}{x(i+cx)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^2 - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1-icx}\right) -$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^2 - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1-icx}\right) -$$

Mathematica [A] time = 0.15, size = 102, normalized size = 1.24

$$\frac{-a\left(a+bcx\log\left(c^2x^2+1\right)-2bcx\log(cx)\right)+2b\tan^{-1}(cx)\left(-a+bcx\log\left(1-e^{2i\tan^{-1}(cx)}\right)\right)-ib^2cx\text{Li}_2\left(e^{2i\tan^{-1}(cx)}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^2,x]
```

```
[Out] (b^2*(-1 - I*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-a + b*c*x*Log[1 - E^((2
*I)*ArcTan[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 + c^2*x^2]) - I*
b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 323, normalized size = 3.94

$$-\frac{a^2}{x}-\frac{b^2\arctan(cx)^2}{x}+2c\,b^2\ln(cx)\arctan(cx)-c\,b^2\arctan(cx)\ln\left(c^2x^2+1\right)-ic\,b^2\ln(cx)\ln(-icx+1)-ic\,b^2\,\text{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2,x)
```

```
[Out] -a^2/x-b^2/x*arctan(c*x)^2+2*c*b^2*ln(c*x)*arctan(c*x)-c*b^2*arctan(c*x)*ln
(c^2*x^2+1)-I*c*b^2*ln(c*x)*ln(1-I*c*x)+1/2*I*c*b^2*ln(c*x-I)*ln(-1/2*I*(I+
c*x))+I*c*b^2*dilog(1+I*c*x)+I*c*b^2*ln(c*x)*ln(1+I*c*x)-1/2*I*c*b^2*ln(I+c
```

```
*x)*ln(1/2*I*(c*x-I))+1/2*I*c*b^2*ln(I+c*x)*ln(c^2*x^2+1)-I*c*b^2*dilog(1-I
*c*x)-1/4*I*c*b^2*ln(I+c*x)^2-1/2*I*c*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/2*I*c*b
^2*dilog(1/2*I*(c*x-I))+1/4*I*c*b^2*ln(c*x-I)^2+1/2*I*c*b^2*dilog(-1/2*I*(I
+c*x))-2*a*b/x*arctan(c*x)+2*c*a*b*ln(c*x)-c*a*b*ln(c^2*x^2+1)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}\,(c\,x))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^2,x)
[Out] int((a + b*atan(c*x))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2,x)
[Out] Integral((a + b*atan(c*x))**2/x**2, x)
```

3.21 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=79

$$-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(c^2x^2+1\right)+b^2c^2\log(x)$$

```
[Out] -b*c*(a+b*arctan(c*x))/x-1/2*c^2*(a+b*arctan(c*x))^2-1/2*(a+b*arctan(c*x))^
2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(c^2*x^2+1)
```

Rubi [A] time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4852, 4918, 266, 36, 29, 31, 4884}

$$-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(c^2x^2+1\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/x) - (c^2*(a + b*ArcTan[c*x])^2)/2 - (a + b*Arc
Tan[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\left(b^2c^2\right)\int \frac{}{x(}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\mathrm{Su}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\mathrm{Su}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+b^2c^2\log(x)$$

Mathematica [A] time = 0.07, size = 90, normalized size = 1.14

$$-\frac{a^2+2b\tan^{-1}(cx)\left(ac^2x^2+a+bcx\right)+2abcx-2b^2c^2x^2\log(x)+b^2c^2x^2\log\left(c^2x^2+1\right)+b^2\left(c^2x^2+1\right)\tan^{-1}(c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
c^2*x^2)*ArcTan[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b^2*c^2*x^2*Log[1 + c^2*x^2
])/x^2
```

fricas [A] time = 0.51, size = 94, normalized size = 1.19

$$-\frac{b^2c^2x^2\log\left(c^2x^2+1\right)-2\,b^2c^2x^2\log(x)+2\,abcx+\left(b^2c^2x^2+b^2\right)\arctan\left(cx\right)^2+a^2+2\left(abc^2x^2+b^2cx+ab\right)a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2*x^2*log(c^2*x^2 + 1) - 2*b^2*c^2*x^2*log(x) + 2*a*b*c*x + (b^
2*c^2*x^2 + b^2)*arctan(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + b^2*c*x + a*b)*arct
an(c*x))/x^2
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 110, normalized size = 1.39

$$-\frac{a^2}{2x^2}-\frac{b^2\arctan(cx)^2}{2x^2}-\frac{c\,b^2\arctan(cx)}{x}-\frac{c^2b^2\arctan(cx)^2}{2}+c^2b^2\ln(cx)-\frac{b^2c^2\ln\left(c^2x^2+1\right)}{2}-\frac{ab\arctan(cx)}{x^2}-\frac{abc}{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctan(c*x)^2-c*b^2*arctan(c*x)/x-1/2*c^2*b^2*arct
an(c*x)^2+c^2*b^2*ln(c*x)-1/2*b^2*c^2*ln(c^2*x^2+1)-a*b/x^2*arctan(c*x)-a*b
*c/x-c^2*a*b*arctan(c*x)
```

maxima [A] time = 0.44, size = 98, normalized size = 1.24

$$-\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)ab+\frac{1}{2}\left(\left(\arctan(cx)^2-\log\left(c^2x^2+1\right)+2\log(x)\right)c^2-2\left(c\arctan(cx)+\frac{1}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b + 1/2*((arctan(c*x)^2 - lo
g(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b^2
 - 1/2*b^2*arctan(c*x)^2/x^2 - 1/2*a^2/x^2
```

mupad [B] time = 2.31, size = 140, normalized size = 1.77

$$b^2c^2\ln(x)-\frac{a^2}{2x^2}-\frac{b^2c^2\operatorname{atan}(cx)^2}{2}-\frac{b^2c^2\ln(cx+1\mathrm{i})}{2}-\frac{b^2c^2\ln(1+cx1\mathrm{i})}{2}-\frac{b^2\operatorname{atan}(cx)^2}{2x^2}-\frac{abc}{x}-\frac{ab\operatorname{atan}(cx)}{x^2}-\frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^3,x)
```

```
[Out] b^2*c^2*log(x) - a^2/(2*x^2) - (b^2*c^2*atan(c*x)^2)/2 - (b^2*c^2*log(c*x +
 1i))/2 - (b^2*c^2*log(c*x*1i + 1))/2 - (b^2*atan(c*x)^2)/(2*x^2) - (a*b*c)
/x - (a*b*atan(c*x))/x^2 - (a*b*c^2*log(c*x + 1i)*1i)/2 + (a*b*c^2*log(c*x*
1i + 1)*1i)/2 - (b^2*c*atan(c*x))/x
```

sympy [A] time = 1.02, size = 119, normalized size = 1.51

$$\begin{cases}-\frac{a^2}{2x^2}-abc^2\operatorname{atan}(cx)-\frac{abc}{x}-\frac{ab\operatorname{atan}(cx)}{x^2}+b^2c^2\log(x)-\frac{b^2c^2\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b^2c^2\operatorname{atan}^2(cx)}{2}-\frac{b^2c\operatorname{atan}(cx)}{x}-\frac{b^2\operatorname{atan}^2(cx)}{2x^2} & \text{f} \\ -\frac{a^2}{2x^2} & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*c**2*atan(c*x) - a*b*c/x - a*b*atan(c*x)/x*
*2 + b**2*c**2*log(x) - b**2*c**2*log(x**2 + c**(-2))/2 - b**2*c**2*atan(c*
x)**2/2 - b**2*c*atan(c*x)/x - b**2*atan(c*x)**2/(2*x**2), Ne(c, 0)), (-a**
2/(2*x**2), True))
```

3.22 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4} dx$

Optimal. Leaf size=140

$$\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}$$

```
[Out] -1/3*b^2*c^2/x-1/3*b^2*c^3*arctan(c*x)-1/3*b*c*(a+b*arctan(c*x))/x^2+1/3*I*
c^3*(a+b*arctan(c*x))^2-1/3*(a+b*arctan(c*x))^2/x^3-2/3*b*c^3*(a+b*arctan(c
*x))*ln(2-2/(1-I*c*x))+1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A] time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4852, 4918, 325, 203, 4924, 4868, 2447}

$$\frac{1}{3}ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{bc\left(a+b\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^4,x]
```

```
[Out] -(b^2*c^2)/(3*x) - (b^2*c^3*ArcTan[c*x])/3 - (b*c*(a + b*ArcTan[c*x]))/(3*x
^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/(3*x^3) - (2*
b*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*PolyLog
[2, -1 + 2/(1 - I*c*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx &= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tan^{-1}(cx)}{x^3\left(1+c^2x^2\right)}\,dx\\
&= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx-\frac{1}{3}\left(2bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(b^2c^2\right)\int \frac{}{x^2}\\
&= -\frac{b^2c^2}{3x}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}-\frac{2}{3}bc^3\left(\right.\\
&= -\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\tan^{-1}(cx)-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan\right.}{3x}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 153, normalized size = 1.09

$$-\frac{a^2+2abc^3x^3\log(cx)+b\tan^{-1}(cx)\left(2a+bc^3x^3+2bc^3x^3\log\left(1-e^{2i\tan^{-1}(cx)}\right)+bcx\right)-abc^3x^3\log\left(c^2x^2+1\right)+a}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^4,x]
```

```
[Out] -1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - I*c^3*x^3)*ArcTan[c*x]^2 + b*A
rcTan[c*x]*(2*a + b*c*x + b*c^3*x^3 + 2*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan[c
*x])]) + 2*a*b*c^3*x^3*Log[c*x] - a*b*c^3*x^3*Log[1 + c^2*x^2] - I*b^2*c^3*
x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^3
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^4, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 399, normalized size = 2.85

$$-\frac{a^2}{3x^3}-\frac{b^2\arctan(cx)^2}{3x^3}-\frac{c\,b^2\arctan(cx)}{3x^2}-\frac{2c^3b^2\ln(cx)\arctan(cx)}{3}+\frac{c^3b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{ic^3b^2\ln(cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^4,x)
```

```
[Out] -1/3*a^2/x^3-1/3*b^2/x^3*arctan(c*x)^2-1/3*c*b^2*arctan(c*x)/x^2-2/3*c^3*b^
2*ln(c*x)*arctan(c*x)+1/3*c^3*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/6*I*c^3*b^2*l
n(I+c*x)*ln(1/2*I*(c*x-I))-1/12*I*c^3*b^2*ln(c*x-I)^2-1/6*I*c^3*b^2*ln(I+c*
x)*ln(c^2*x^2+1)-1/3*I*c^3*b^2*dilog(1+I*c*x)-1/3*I*c^3*b^2*ln(c*x)*ln(1+I*
c*x)+1/6*I*c^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/6*I*c^3*b^2*dilog(-1/2*I*(I+c*
x))+1/3*I*c^3*b^2*dilog(1-I*c*x)-1/3*b^2*c^2/x-1/3*b^2*c^3*arctan(c*x)+1/3*
I*c^3*b^2*ln(c*x)*ln(1-I*c*x)+1/6*I*c^3*b^2*dilog(1/2*I*(c*x-I))+1/12*I*c^3
*b^2*ln(I+c*x)^2-1/6*I*c^3*b^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))-2/3*a*b/x^3*arc
tan(c*x)-1/3*c*a*b/x^2-2/3*c^3*a*b*ln(c*x)+1/3*c^3*a*b*ln(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)ab+\frac{\frac{1}{4}\left(4x^3\int-\frac{12c^2x^2\log\left(c^2x^2+1\right)-56cx\arctan(cx)-108\left(c^2x\right.}{4\left(c^2x^6+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a
*b + 1/48*(48*x^3*integrate(-1/48*(4*c^2*x^2*log(c^2*x^2 + 1) - 8*c*x*arcta
n(c*x) - 36*(c^2*x^2 + 1)*arctan(c*x)^2 - 3*(c^2*x^2 + 1)*log(c^2*x^2 + 1)^
2)/(c^2*x^6 + x^4), x) - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x^3 - 1/
3*a^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^4,x)
```

```
[Out] int((a + b*atan(c*x))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atan}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x**4, x)
```

3.23 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=116

$$\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}-\frac{2}{3}b^2c^4\log(x)-\frac{b^2c^2}{12x^2}+$$

```
[Out] -1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctan(c*x))/x^3+1/2*b*c^3*(a+b*arctan(c*x))
/x+1/4*c^4*(a+b*arctan(c*x))^2-1/4*(a+b*arctan(c*x))^2/x^4-2/3*b^2*c^4*ln(x
)+1/3*b^2*c^4*ln(c^2*x^2+1)
```

Rubi [A] time = 0.22, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4852, 4918, 266, 44, 36, 29, 31, 4884}

$$\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}-\frac{b^2c^2}{12x^2}+\frac{1}{3}b^2c^4\log\left(c^2x\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] -(b^2*c^2)/(12*x^2) - (b*c*(a + b*ArcTan[c*x]))/(6*x^3) + (b*c^3*(a + b*Arc
Tan[c*x]))/(2*x) + (c^4*(a + b*ArcTan[c*x])^2)/4 - (a + b*ArcTan[c*x])^2/(4
*x^4) - (2*b^2*c^4*Log[x])/3 + (b^2*c^4*Log[1 + c^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tan^{-1}(cx)}{x^4\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx-\frac{1}{2}\left(bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}+\frac{1}{6}\left(b^2c^2\right)\int \frac{1}{x^3\left(1+c^2x^2\right)}\,dx-\frac{1}{2}\left(bc^3\right)\int$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}\right.}{4x^4}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}\right.}{4x^4}$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+\right.}{}$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+\right.}{}$$

Mathematica [A] time = 0.09, size = 128, normalized size = 1.10

$$\frac{-3a^2+6abc^3x^3+2b\tan^{-1}(cx)\left(3a\left(c^4x^4-1\right)+bcx\left(3c^2x^2-1\right)\right)-2abcx-8b^2c^4x^4\log(x)+3b^2\left(c^4x^4-1\right)\tan^{-1}(}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] (-3*a^2 - 2*a*b*c*x - b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(b*c*x*(-1 + 3*c^2*
x^2) + 3*a*(-1 + c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]^2
 - 8*b^2*c^4*x^4*Log[x] + 4*b^2*c^4*x^4*Log[1 + c^2*x^2])/(12*x^4)
```

fricas [A] time = 0.45, size = 135, normalized size = 1.16

$$\frac{4\,b^2c^4x^4\log\left(c^2x^2+1\right)-8\,b^2c^4x^4\log(x)+6\,abc^3x^3-b^2c^2x^2-2\,abcx+3\left(b^2c^4x^4-b^2\right)\arctan(cx)^2-3\,a^2+2\,(3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^2*c^4*x^4*log(c^2*x^2 + 1) - 8*b^2*c^4*x^4*log(x) + 6*a*b*c^3*x^3
 - b^2*c^2*x^2 - 2*a*b*c*x + 3*(b^2*c^4*x^4 - b^2)*arctan(c*x)^2 - 3*a^2 +
2*(3*a*b*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x - 3*a*b)*arctan(c*x))/x^4
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 147, normalized size = 1.27

$$-\frac{a^2}{4x^4}-\frac{b^2\arctan(cx)^2}{4x^4}-\frac{c\,b^2\arctan(cx)}{6x^3}+\frac{c^3b^2\arctan(cx)}{2x}+\frac{c^4b^2\arctan(cx)^2}{4}-\frac{b^2c^2}{12x^2}-\frac{2c^4b^2\ln(cx)}{3}+\frac{b^2c^4\ln\left(c^2\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arctan(c*x)^2-1/6*c*b^2*arctan(c*x)/x^3+1/2*c^3*b^
2*arctan(c*x)/x+1/4*c^4*b^2*arctan(c*x)^2-1/12*b^2*c^2/x^2-2/3*c^4*b^2*ln(c
*x)+1/3*b^2*c^4*ln(c^2*x^2+1)-1/2*a*b/x^4*arctan(c*x)-1/6*a*b*c/x^3+1/2*c^3
*a*b/x+1/2*c^4*a*b*arctan(c*x)
```

maxima [A] time = 0.42, size = 152, normalized size = 1.31

$$\frac{1}{6}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan(cx)}{x^4}\right)ab+\frac{1}{12}\left(2\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c\arctan(cx)-\frac{(3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b +
 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c*x) - (3*c^2*x
^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*log(x) + 1)*c^2/x
^2)*b^2 - 1/4*b^2*arctan(c*x)^2/x^4 - 1/4*a^2/x^4
```

mupad [B] time = 2.26, size = 171, normalized size = 1.47

$$\frac{b^2\,c^4\,\mathrm{atan}\left(c\,x\right)^2}{4}-\frac{2\,b^2\,c^4\,\ln(x)}{3}-\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{4}+\frac{a^2}{4}+x\,\left(\frac{c\,\mathrm{atan}(c\,x)\,b^2}{6}+\frac{a\,c\,b}{6}\right)-x^3\,\left(\frac{b^2\,c^3\,\mathrm{atan}(c\,x)}{2}+\frac{a\,b\,c^3}{2}\right)+\frac{b^2\,c^2\,x^2}{12}+\frac{a\,l}{}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^5,x)
```

```
[Out] (b^2*c^4*atan(c*x)^2)/4 - (2*b^2*c^4*log(x))/3 - ((b^2*atan(c*x)^2)/4 + a^2
/4 + x*((b^2*c*atan(c*x))/6 + (a*b*c)/6) - x^3*((b^2*c^3*atan(c*x))/2 + (a*
b*c^3)/2) + (b^2*c^2*x^2)/12 + (a*b*atan(c*x))/2)/x^4 + (b^2*c^4*log(c*x +
1i))/3 + (b^2*c^4*log(c*x*1i + 1))/3 + (a*b*c^4*log(c*x + 1i)*1i)/4 - (a*b*
c^4*log(c*x*1i + 1)*1i)/4
```

sympy [A] time = 1.78, size = 170, normalized size = 1.47

$$\begin{cases} -\frac{a^2}{4x^4} + \frac{abc^4\operatorname{atan}(cx)}{2} + \frac{abc^3}{2x} - \frac{abc}{6x^3} - \frac{ab\operatorname{atan}(cx)}{2x^4} - \frac{2b^2c^4\log(x)}{3} + \frac{b^2c^4\log\left(x^2+\frac{1}{c^2}\right)}{3} + \frac{b^2c^4\operatorname{atan}^2(cx)}{4} + \frac{b^2c^3\operatorname{atan}(cx)}{2x} - \frac{b^2c^2}{12x^2} - \frac{b^2c\,a}{} \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**4*atan(c*x)/2 + a*b*c**3/(2*x) - a*b*c/(
6*x**3) - a*b*atan(c*x)/(2*x**4) - 2*b**2*c**4*log(x)/3 + b**2*c**4*log(x**
2 + c**(-2))/3 + b**2*c**4*atan(c*x)**2/4 + b**2*c**3*atan(c*x)/(2*x) - b**
2*c**2/(12*x**2) - b**2*c*atan(c*x)/(6*x**3) - b**2*atan(c*x)**2/(4*x**4),
Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.24 $\int x^5 \left(a + b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=255

$$-\frac{23b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{15c^6}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{6c^6}-\frac{23i\cdots}{\cdots}$$

```
[Out] 19/60*b^3*x/c^5-1/60*b^3*x^3/c^3-19/60*b^3*arctan(c*x)/c^6-4/15*b^2*x^2*(a+
b*arctan(c*x))/c^4+1/20*b^2*x^4*(a+b*arctan(c*x))/c^2-23/30*I*b*(a+b*arctan
(c*x))^2/c^6-1/2*b*x*(a+b*arctan(c*x))^2/c^5+1/6*b*x^3*(a+b*arctan(c*x))^2/
c^3-1/10*b*x^5*(a+b*arctan(c*x))^2/c+1/6*(a+b*arctan(c*x))^3/c^6+1/6*x^6*(a
+b*arctan(c*x))^3-23/15*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^6-23/30*I*b
^3*polylog(2,1-2/(1+I*c*x))/c^6
```

Rubi [A] time = 0.95, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4852, 4916, 302, 203, 321, 4920, 4854, 2402, 2315, 4846, 4884}

$$-\frac{23ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{30c^6}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}-\frac{23b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(\cdots\right.}{15c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (19*b^3*x)/(60*c^5) - (b^3*x^3)/(60*c^3) - (19*b^3*ArcTan[c*x])/(60*c^6) -
(4*b^2*x^2*(a + b*ArcTan[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTan[c*x]))/(2
0*c^2) - (((23*I)/30)*b*(a + b*ArcTan[c*x])^2)/c^6 - (b*x*(a + b*ArcTan[c*x
])^2)/(2*c^5) + (b*x^3*(a + b*ArcTan[c*x])^2)/(6*c^3) - (b*x^5*(a + b*ArcTa
n[c*x])^2)/(10*c) + (a + b*ArcTan[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTan[c*x]
)^3)/6 - (23*b^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^6) - (((23*I
)/30)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}(cx)\right)^3 dx = \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3 - \frac{1}{2}(bc)\int \frac{x^6\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3 - \frac{b\int x^4\left(a+b\tan^{-1}(cx)\right)^2 dx}{2c} + \frac{b\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{2c}$$

$$= -\frac{bx^5\left(a+b\tan^{-1}(cx)\right)^2}{10c} + \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3 + \frac{1}{5}b^2\int \frac{x^5\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}$$

$$= \frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{6c^3} - \frac{bx^5\left(a+b\tan^{-1}(cx)\right)^2}{10c} + \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3 - \frac{b\int (}{}$$

$$= \frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2} - \frac{bx\left(a+b\tan^{-1}(cx)\right)^2}{2c^5} + \frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{6c^3} - \frac{bx^5(}{}$$

$$= -\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2} - \frac{23ib\left(a+b\tan^{-1}(cx)\right)^2}{30c^6} -$$

$$= \frac{19b^3x}{60c^5} - \frac{b^3x^3}{60c^3} - \frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2} - \frac{23ib(a+b}{30}$$

$$= \frac{19b^3x}{60c^5} - \frac{b^3x^3}{60c^3} - \frac{19b^3\tan^{-1}(cx)}{60c^6} - \frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tan^{-1}(c\right.}{20c^2}$$

$$= \frac{19b^3x}{60c^5} - \frac{b^3x^3}{60c^3} - \frac{19b^3\tan^{-1}(cx)}{60c^6} - \frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tan^{-1}(c\right.}{20c^2}$$

Mathematica [A] time = 0.83, size = 291, normalized size = 1.14

$$10a^3c^6x^6 + b\tan^{-1}(cx)\left(30a^2\left(c^6x^6+1\right) - 4abcx\left(3c^4x^4-5c^2x^2+15\right) + b^2\left(3c^4x^4-16c^2x^2-19\right) - 92b^2\log\left(1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-19*a*b^2 - 30*a^2*b*c*x + 19*b^3*c*x - 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x^
3 - b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 - 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*b^
2*(b*(23*I - 15*c*x + 5*c^3*x^3 - 3*c^5*x^5) + 15*a*(1 + c^6*x^6))*ArcTan[c
*x]^2 + 10*b^3*(1 + c^6*x^6)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*(-19 - 16*c
^2*x^2 + 3*c^4*x^4) - 4*a*b*c*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 30*a^2*(1 +
c^6*x^6) - 92*b^2*Log[1 + E^((2*I)*ArcTan[c*x])]) + 46*a*b^2*Log[1 + c^2*x^
2] + (46*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(60*c^6)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^5\arctan(cx)^3 + 3\,ab^2x^5\arctan(cx)^2 + 3\,a^2bx^5\arctan(cx) + a^3x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctan(c*x)^3 + 3*a*b^2*x^5*arctan(c*x)^2 + 3*a^2*b*x^5*ar
ctan(c*x) + a^3*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 528, normalized size = 2.07

$$-\frac{23ib^3\ln(cx-i)^2}{120c^6}+\frac{23ib^3\ln(cx+i)^2}{120c^6}-\frac{23ib^3\operatorname{dilog}\left(-\frac{i(cx+i)}{2}\right)}{60c^6}+\frac{23ib^3\operatorname{dilog}\left(\frac{i(cx-i)}{2}\right)}{60c^6}-\frac{b^3\arctan(cx)^2x^5}{10c}+\frac{b^3\arctan}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))^3,x)
```

```
[Out] -23/60*I/c^6*b^3*dilog(-1/2*I*(I+c*x))-23/120*I/c^6*b^3*ln(c*x-I)^2-1/10/c*
b^3*arctan(c*x)^2*x^5+1/6/c^3*b^3*arctan(c*x)^2*x^3+1/20/c^2*b^3*arctan(c*x
)*x^4-4/15/c^4*b^3*arctan(c*x)*x^2-1/2/c^5*b^3*arctan(c*x)^2*x+1/2*a*b^2*x^
6*arctan(c*x)^2+1/2*a^2*b*x^6*arctan(c*x)+23/60*I/c^6*b^3*dilog(1/2*I*(c*x-
I))+23/120*I/c^6*b^3*ln(I+c*x)^2+1/2/c^6*a^2*b*arctan(c*x)+1/2/c^6*a*b^2*ar
ctan(c*x)^2+23/30/c^6*a*b^2*ln(c^2*x^2+1)+23/30/c^6*b^3*arctan(c*x)*ln(c^2*
x^2+1)-1/2/c^5*x*a^2*b+1/20/c^2*a*b^2*x^4-1/10/c*x^5*a^2*b+1/6/c^3*a^2*b*x^
3-4/15/c^4*x^2*a*b^2+19/60*b^3*x/c^5-1/60*b^3*x^3/c^3-19/60*b^3*arctan(c*x)
/c^6+1/6*x^6*a^3+1/6*b^3*x^6*arctan(c*x)^3+1/6/c^6*b^3*arctan(c*x)^3+1/3/c^
3*a*b^2*x^3*arctan(c*x)-1/c^5*a*b^2*x*arctan(c*x)-23/60*I/c^6*b^3*ln(I+c*x)
*ln(c^2*x^2+1)-23/60*I/c^6*b^3*ln(c*x-I)*ln(-1/2*I*(I+c*x))+23/60*I/c^6*b^3
*ln(c*x-I)*ln(c^2*x^2+1)+23/60*I/c^6*b^3*ln(I+c*x)*ln(1/2*I*(c*x-I))-1/5/c*
a*b^2*x^5*arctan(c*x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^5*(a + b*atan(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \left(a + b \operatorname{atan}{\left(c x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**5*(a + b*atan(c*x))**3, x)
```

3.25 $\int x^4 \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=271

$$\frac{3ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}-\frac{9ab^2x}{10c^4}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{5c^5}+\frac{9b\left(a+b\tan^{-1}(cx\right.}{20c^5}$$

```
[Out] -9/10*a*b^2*x/c^4-1/20*b^3*x^2/c^3-9/10*b^3*x*arctan(c*x)/c^4+1/10*b^2*x^3*
(a+b*arctan(c*x))/c^2+9/20*b*(a+b*arctan(c*x))^2/c^5+3/10*b*x^2*(a+b*arctan
(c*x))^2/c^3-3/20*b*x^4*(a+b*arctan(c*x))^2/c+1/5*I*(a+b*arctan(c*x))^3/c^5
+1/5*x^5*(a+b*arctan(c*x))^3+3/5*b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^5+
1/2*b^3*ln(c^2*x^2+1)/c^5+3/5*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*
x))/c^5+3/10*b^3*polylog(3,1-2/(1+I*c*x))/c^5
```

Rubi [A] time = 0.76, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}-\frac{9ab^2x}{10c^4}+\frac{3bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-9*a*b^2*x)/(10*c^4) - (b^3*x^2)/(20*c^3) - (9*b^3*x*ArcTan[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTan[c*x]))/(10*c^2) + (9*b*(a + b*ArcTan[c*x])^2)/(20
*c^5) + (3*b*x^2*(a + b*ArcTan[c*x])^2)/(10*c^3) - (3*b*x^4*(a + b*ArcTan[c
*x])^2)/(20*c) + ((I/5)*(a + b*ArcTan[c*x])^3)/c^5 + (x^5*(a + b*ArcTan[c*x
])^3)/5 + (3*b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(5*c^5) + (b^3*Log
[1 + c^2*x^2])/(2*c^5) + (((3*I)/5)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 + I*c*x)])/c^5 + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(10*c^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^4 \left(a+b\tan^{-1}(cx)\right)^3 dx = \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3 - \frac{1}{5}(3bc)\int \frac{x^5\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3 - \frac{(3b)\int x^3\left(a+b\tan^{-1}(cx)\right)^2 dx}{5c} + \frac{(3b)\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}}{5c}$$

$$= -\frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c} + \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3 + \frac{1}{10}\left(3b^2\right)\int \frac{x^4\left(a+b\tan^{-1}\right.}{1+c^2x^2}$$

$$= \frac{3bx^2\left(a+b\tan^{-1}(cx)\right)^2}{10c^3} - \frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c} + \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{5c^5} + \frac{1}{5}x^5$$

$$= \frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2} + \frac{3bx^2\left(a+b\tan^{-1}(cx)\right)^2}{10c^3} - \frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c} + \frac{i\left(\right.}{}$$

$$= -\frac{9ab^2x}{10c^4} + \frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2} + \frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{20c^5} + \frac{3bx^2\left(a+b\tan^{-1}(c\right.}{10c^3}$$

$$= -\frac{9ab^2x}{10c^4} - \frac{9b^3x\tan^{-1}(cx)}{10c^4} + \frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2} + \frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{20c^5} + \frac{3b}{}$$

$$= -\frac{9ab^2x}{10c^4} - \frac{b^3x^2}{20c^3} - \frac{9b^3x\tan^{-1}(cx)}{10c^4} + \frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2} + \frac{9b\left(a+b\tan^{-1}(cx\right.}{20c^5}$$

Mathematica [A] time = 0.90, size = 396, normalized size = 1.46

$$\frac{4a^3c^5x^5 + 12a^2bc^5x^5\tan^{-1}(cx) - 3a^2bc^4x^4 + 6a^2bc^2x^2 - 6a^2b\log\left(c^2x^2+1\right) + 12ab^2c^5x^5\tan^{-1}(cx)^2 - 6ab^2c^4x}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-b^3 - 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 - b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 - 3*
a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 + 18*a*b^2*ArcTan[c*x] - 18*b^3*c*x*ArcTan[c*
x] + 12*a*b^2*c^2*x^2*ArcTan[c*x] + 2*b^3*c^3*x^3*ArcTan[c*x] - 6*a*b^2*c^4
*x^4*ArcTan[c*x] + 12*a^2*b*c^5*x^5*ArcTan[c*x] - (12*I)*a*b^2*ArcTan[c*x]^
2 + 9*b^3*ArcTan[c*x]^2 + 6*b^3*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^4*x^4*ArcTa
n[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTan[c*x]^2 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3
*c^5*x^5*ArcTan[c*x]^3 + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])
] + 12*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*a^2*b*Log[1 + c
^2*x^2] + 10*b^3*Log[1 + c^2*x^2] - (12*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[
2, -E^((2*I)*ArcTan[c*x])] + 6*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])])/(20*
c^5)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^4\arctan\left(cx\right)^3 + 3\,ab^2x^4\arctan\left(cx\right)^2 + 3\,a^2bx^4\arctan\left(cx\right) + a^3x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^4*arctan(c*x)^3 + 3*a*b^2*x^4*arctan(c*x)^2 + 3*a^2*b*x^4*ar
ctan(c*x) + a^3*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 5.52, size = 3053, normalized size = 11.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^3,x)
```

```
[Out] -3/160/c^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)+1)^2)*arctan(c*x)^2*x^3-9/80/c^4*b^3*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2*x+3/80/
c^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)+1)^2)^2*arctan(c*x)^2*x^3+9/80/c^4*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1
)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*arct
an(c*x)^2*x+9/160*I/c^3*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*
x)^2/(c^2*x^2+1)+I)^3*arctan(c*x)^2*x^2-1/20*b^3*x^2/c^3-3/80/c^2*b^3*Pi*cs
gn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1)+I)*arctan(c*x)^2*x^3+3/160/c^2*b^3*Pi*csgn(I*(1+I*c*x)^
4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)+I)^2*arctan(c*x)^2*x^3+3/10*I/c^5*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))
^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)^2+1/5*x^5*a^3-1/20/c^5*b
^3-3/10*I/c^5*a*b^2*ln(I+c*x)*ln(1/2*I*(c*x-I))-3/20*I/c^5*b^3*Pi*csgn(I*(1
+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2-3/20*I/c^5*b^3*Pi*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2+21/160*I/c^5*b^
3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*arct
an(c*x)^2+3/160*I/c^5*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan
(c*x)^2+3/160/c^2*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(
c^2*x^2+1)+I)^3*arctan(c*x)^2*x^3+9/160/c^4*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)+1)^2)^3*arctan(c*x)^2*x-3/160/c^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)+1)^2)^3*arctan(c*x)^2*x^3-9/160/c^4*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^
2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*arctan(c*x)^2*x-3/10*I/c^5*a*b^2*ln
(c*x-I)*ln(c^2*x^2+1)+3/10*I/c^5*a*b^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))+3/10*I/
c^5*a*b^2*ln(I+c*x)*ln(c^2*x^2+1)-9/160/c^4*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*
x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2
*arctan(c*x)^2*x-3/20*I/c^5*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))^2*arctan(c*x)^2+3/20*I/c^5*b^3*Pi*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1)^2)^2*arctan(c*x)^2+3/20*I/c^5*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c
*x)^2-9/80*I/c^3*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c
^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*arctan(c*x)^2*x^2+9/160*I/
c^3*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*arctan(c*x)^2*x^2-9/160*I/c^3*b^3*Pi*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*a
rctan(c*x)^2*x^2+9/80*I/c^3*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2*x^2-3/20*I/c^5*b^3*Pi*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-9/160*I
/c^3*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2*x^2-3/80*
I/c^5*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)+1)^2)^2*arctan(c*x)^2+21/160*I/c^5*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2
```

```
+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*ar
ctan(c*x)^2+9/20/c^5*b^3*arctan(c*x)^2-1/c^5*b^3*ln((1+I*c*x)^2/(c^2*x^2+1)
+1)+3/10/c^5*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/5*x^5*b^3*arctan(c*x
)^3-3/10/c*a*b^2*x^4*arctan(c*x)+3/5/c^5*b^3*ln(2)*arctan(c*x)^2+I/c^5*b^3*
arctan(c*x)-3/10/c^5*a^2*b*ln(c^2*x^2+1)+9/10/c^5*a*b^2*arctan(c*x)+1/10/c^
2*b^3*arctan(c*x)*x^3+3/10/c^3*b^3*arctan(c*x)^2*x^2-3/20/c*b^3*x^4*arctan(
c*x)^2-1/5*I/c^5*b^3*arctan(c*x)^3+9/160/c^4*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2*x-21/80*
I/c^5*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I
)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*arctan(c*x)^2+3/160*I/c^5*b^3*Pi*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arc
tan(c*x)^2+1/10*a*b^2*x^3/c^2-9/10*b^3*x*arctan(c*x)/c^4-9/10*a*b^2*x/c^4+3
/5/c^3*a*b^2*x^2*arctan(c*x)-3/5/c^5*a*b^2*arctan(c*x)*ln(c^2*x^2+1)+3/20*I
/c^5*a*b^2*ln(c*x-I)^2+3/10*I/c^5*a*b^2*dilog(-1/2*I*(I+c*x))-3/20*I/c^5*a*
b^2*ln(I+c*x)^2-3/10*I/c^5*a*b^2*dilog(1/2*I*(c*x-I))-3/5*I/c^5*b^3*arctan(
c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/20/c*a^2*b*x^4+3/10/c^3*b*x^2*a^
2+3/5*x^5*a^2*b*arctan(c*x)+3/5*x^5*a*b^2*arctan(c*x)^2+3/5/c^5*b^3*arctan(
c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3/10/c^5*b^3*arctan(c*x)^2*ln(c^2*x^
2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{40}\,b^3x^5\arctan\left(cx\right)^3-\frac{3}{160}\,b^3x^5\arctan\left(cx\right)\log\left(c^2x^2+1\right)^2+\frac{1}{5}\,a^3x^5+\frac{3}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/40*b^3*x^5*arctan(c*x)^3 - 3/160*b^3*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2 +
 1/5*a^3*x^5 + 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*a^2*b + integrate(1/160*(12*b^3*c^2*x^6*arctan(c*x)*log(c
^2*x^2 + 1) + 140*(b^3*c^2*x^6 + b^3*x^4)*arctan(c*x)^3 + 12*(40*a*b^2*c^2*
x^6 - b^3*c*x^5 + 40*a*b^2*x^4)*arctan(c*x)^2 + 3*(b^3*c*x^5 + 5*(b^3*c^2*x
^6 + b^3*x^4)*arctan(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^4*(a + b*atan(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \left(a + b \operatorname{atan}{\left(c x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**4*(a + b*atan(c*x))**3, x)
```

3.26 $\int x^3 \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=194

$$\frac{2b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4}+\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4c^4}+\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4}+\frac{3bx\left(a+b\,\mathrm{t}\right.}{4c}$$

```
[Out] -1/4*b^3*x/c^3+1/4*b^3*arctan(c*x)/c^4+1/4*b^2*x^2*(a+b*arctan(c*x))/c^2+I*
b*(a+b*arctan(c*x))^2/c^4+3/4*b*x*(a+b*arctan(c*x))^2/c^3-1/4*b*x^3*(a+b*ar
ctan(c*x))^2/c-1/4*(a+b*arctan(c*x))^3/c^4+1/4*x^4*(a+b*arctan(c*x))^3+2*b^
2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4+I*b^3*polylog(2,1-2/(1+I*c*x))/c^4
```

Rubi [A] time = 0.55, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 4846, 4884}

$$\frac{ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4}+\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{2b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4}+\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3}-\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -(b^3*x)/(4*c^3) + (b^3*ArcTan[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTan[c*x])
)/(4*c^2) + (I*b*(a + b*ArcTan[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTan[c*x])^2)
/(4*c^3) - (b*x^3*(a + b*ArcTan[c*x])^2)/(4*c) - (a + b*ArcTan[c*x])^3/(4*c
^4) + (x^4*(a + b*ArcTan[c*x])^3)/4 + (2*b^2*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/c^4 + (I*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}(cx)\right)^3 dx = \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3 - \frac{1}{4}(3bc)\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3 - \frac{(3b)\int x^2\left(a+b\tan^{-1}(cx)\right)^2 dx}{4c} + \frac{(3b)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}d}{4c}$$

$$= -\frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{4c} + \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3 + \frac{1}{2}b^2\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$

$$= \frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} - \frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{4c} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{4c^4} + \frac{1}{4}x^4\left(a+b\right.$$

$$= \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} - \frac{bx^3\left(a+\right.}{}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} -$$

$$= -\frac{b^3x}{4c^3} + \frac{b^3\tan^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\right.}{4}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^3\tan^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\right.}{4}$$

Mathematica [A] time = 0.56, size = 225, normalized size = 1.16

$$a^3c^4x^4 + b\tan^{-1}(cx)\left(3a^2\left(c^4x^4-1\right) - 2abcx\left(c^2x^2-3\right) + b^2\left(c^2x^2+1\right) + 8b^2\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right) - a^2bc^3x^3 + 3a$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2 + 3*a^2*b*c*x - b^3*c*x + a*b^2*c^2*x^2 - a^2*b*c^3*x^3 + a^3*c^4*x^
4 - b^2*(b*(4*I - 3*c*x + c^3*x^3) + a*(3 - 3*c^4*x^4))*ArcTan[c*x]^2 + b^3
*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(-2*a*b*c*x*(-3 + c^2*x^2) +
b^2*(1 + c^2*x^2) + 3*a^2*(-1 + c^4*x^4) + 8*b^2*Log[1 + E^((2*I)*ArcTan[c*
x])]) - 4*a*b^2*Log[1 + c^2*x^2] - (4*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*
x])])/(4*c^4)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\arctan(cx)^3 + 3\,ab^2x^3\arctan(cx)^2 + 3\,a^2bx^3\arctan(cx) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c*x)^3 + 3*a*b^2*x^3*arctan(c*x)^2 + 3*a^2*b*x^3*ar
ctan(c*x) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 445, normalized size = 2.29

$$\frac{ib^3\ln(cx-i)^2}{4c^4}-\frac{ib^3\,\mathrm{dilog}\left(\frac{i(cx-i)}{2}\right)}{2c^4}+\frac{ib^3\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2c^4}-\frac{ib^3\ln(cx+i)^2}{4c^4}-\frac{b^3\arctan(cx)\ln\left(c^2x^2+1\right)}{c^4}-\frac{b^3\,\mathrm{arctan}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^3,x)
```

```
[Out] -1/c^4*b^3*arctan(c*x)*ln(c^2*x^2+1)-1/4/c*b^3*arctan(c*x)^2*x^3+3/4/c^3*b^
3*arctan(c*x)^2*x+1/4/c^2*b^3*arctan(c*x)*x^2-1/4/c*a^2*b*x^3+1/4/c^2*x^2*a
*b^2+3/4/c^3*x*a^2*b-1/2*I/c^4*b^3*dilog(1/2*I*(c*x-I))-1/4*I/c^4*b^3*ln(I+
c*x)^2+1/4*I/c^4*b^3*ln(c*x-I)^2-1/c^4*a*b^2*ln(c^2*x^2+1)-3/4/c^4*a^2*b*ar
ctan(c*x)+3/4*x^4*a^2*b*arctan(c*x)+3/4*a*b^2*x^4*arctan(c*x)^2+1/2*I/c^4*b
^3*dilog(-1/2*I*(I+c*x))-3/4/c^4*a*b^2*arctan(c*x)^2-1/4*b^3*x/c^3+1/4*b^3*
arctan(c*x)/c^4+1/4*x^4*a^3+1/4*b^3*x^4*arctan(c*x)^3-1/4/c^4*b^3*arctan(c*
x)^3+1/2*I/c^4*b^3*ln(I+c*x)*ln(c^2*x^2+1)-1/2*I/c^4*b^3*ln(I+c*x)*ln(1/2*I
*(c*x-I))+1/2*I/c^4*b^3*ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/2/c*a*b^2*x^3*arctan
(c*x)+3/2/c^3*a*b^2*x*arctan(c*x)-1/2*I/c^4*b^3*ln(c*x-I)*ln(c^2*x^2+1)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^3*(a + b*atan(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\,\mathrm{atan}\left(cx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))**3, x)
```

3.27 $\int x^2 \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=206

$$-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}-\frac{b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}\right.}{c^3}$$

```
[Out] a*b^2*x/c^2+b^3*x*arctan(c*x)/c^2-1/2*b*(a+b*arctan(c*x))^2/c^3-1/2*b*x^2*(
a+b*arctan(c*x))^2/c-1/3*I*(a+b*arctan(c*x))^3/c^3+1/3*x^3*(a+b*arctan(c*x)
)^3-b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3-1/2*b^3*ln(c^2*x^2+1)/c^3-I*b
^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3-1/2*b^3*polylog(3,1-2/(1+
I*c*x))/c^3
```

Rubi [A] time = 0.43, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3}-\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{i\left(a+b\tan\right.}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTan[c*x])/c^2 - (b*(a + b*ArcTan[c*x])^2)/(2*c^3)
 - (b*x^2*(a + b*ArcTan[c*x])^2)/(2*c) - ((I/3)*(a + b*ArcTan[c*x])^3)/c^3
+ (x^3*(a + b*ArcTan[c*x])^3)/3 - (b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x
)])/c^3 - (b^3*Log[1 + c^2*x^2])/(2*c^3) - (I*b^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/c^3 - (b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}(cx)\right)^3dx=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-(bc)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\int x\left(a+b\tan^{-1}(cx)\right)^2dx}{c}+\frac{b\int\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{c}$$

$$=-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3+b^2\int$$

$$=-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\left(a+\right.}{}$$

$$=\frac{ab^2x}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tan^{-1}(cx)}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\right.}{}$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tan^{-1}(cx)}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\right.}{}$$

Mathematica [A] time = 0.62, size = 269, normalized size = 1.31

$$2a^3c^3x^3+6a^2bc^3x^3\tan^{-1}(cx)-3a^2bc^2x^2+3a^2b\log\left(c^2x^2+1\right)+6ab^2\left(\left(c^3x^3+i\right)\tan^{-1}(cx)^2-\tan^{-1}(cx)\left(c^2x^2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTan[c*x] + 3*a^2*b*L
og[1 + c^2*x^2] + 6*a*b^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*
(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*A
rcTan[c*x])]) + b^3*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*ArcTan
[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x]*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(6*
c^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\arctan\left(cx\right)^3 + 3\,ab^2x^2\arctan\left(cx\right)^2 + 3\,a^2bx^2\arctan\left(cx\right) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c*x)^3 + 3*a*b^2*x^2*arctan(c*x)^2 + 3*a^2*b*x^2*ar
ctan(c*x) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 2.53, size = 2020, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^3,x)
```

```
[Out] b^3*x*arctan(c*x)/c^2+a*b^2*x/c^2+1/4*I/c^3*b^3*csgn(I*(1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I/(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2*Pi-1/2/c^3*b^3*polylog(3,-(1+I*
c*x)^2/(c^2*x^2+1))-1/2/c^3*b^3*arctan(c*x)^2+1/3*x^3*b^3*arctan(c*x)^3+1/c
^3*b^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1/8/c^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)^3*arctan(c*x)^2*x+1/8/c^2*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2
+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*arctan(c*x)^2*x+1/4*I/c^3*b^3*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2*Pi+
1/4*I/c^3*b^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2*Pi-1/8*I/c^3*
b^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2*Pi-1/8*I/c^3*b^3*
csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*arctan(c*
x)^2*Pi+1/2*I/c^3*a*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/2*I/c^3*a*b^2*ln(c*x-I)*l
n(-1/2*I*(I+c*x))-1/2*I/c^3*a*b^2*ln(I+c*x)*ln(c^2*x^2+1)+1/2*I/c^3*a*b^2*l
n(I+c*x)*ln(1/2*I*(c*x-I))-1/c*a*b^2*x^2*arctan(c*x)+I/c^3*b^3*arctan(c*x)*
polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/c^3*a*b^2*arctan(c*x)*ln(c^2*x^2+1)-1
/4*I/c^3*a*b^2*ln(c*x-I)^2-1/2*I/c^3*a*b^2*dilog(-1/2*I*(I+c*x))+1/4*I/c^3*
a*b^2*ln(I+c*x)^2+1/2*I/c^3*a*b^2*dilog(1/2*I*(c*x-I))-1/8/c^2*b^3*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arct
an(c*x)^2*x+1/4/c^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2*x-1/4/c^2*b^3*Pi*csgn(I*(1+I*c*x)
^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^2/(c^2*x
```

```
^2+1)+I)*arctan(c*x)^2*x+1/8/c^2*b^3*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*
I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*arctan(c*x
)^2*x-1/4*I/c^3*b^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2*Pi-1/4*I/c^3*b^3*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c
*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2*Pi+1/4*I/c^3*b^3*csgn(I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2*Pi-1/2*I/c^
3*b^3*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2
*arctan(c*x)^2*Pi+1/4*I/c^3*b^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2*Pi-1/8*I/c^3*b^3*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x
)^2*Pi+1/4*I/c^3*b^3*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*
x^2+1)+I)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*arctan(c*x)^2*Pi-1/8*I/c^3*b^
3*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1
+I*c*x)^2/(c^2*x^2+1)+I)^2*arctan(c*x)^2*Pi-I/c^3*b^3*arctan(c*x)+1/3*I/c^3
*b^3*arctan(c*x)^3-1/2/c*b*x^2*a^2+b^2*x^3*a*arctan(c*x)^2+a^2*b*x^3*arctan
(c*x)-1/c^3*b^3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2/c^3*b^3*a
rctan(c*x)^2*ln(c^2*x^2+1)-1/c^3*a*b^2*arctan(c*x)+1/2/c^3*a^2*b*ln(c^2*x^2
+1)-1/c^3*b^3*ln(2)*arctan(c*x)^2-1/2/c*b^3*arctan(c*x)^2*x^2+1/3*a^3*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}\,b^3x^3\arctan\left(cx\right)^3-\frac{1}{32}\,b^3x^3\arctan\left(cx\right)\log\left(c^2x^2+1\right)^2+\frac{1}{3}\,a^3x^3+\frac{1}{2}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan(c*x)^3 - 1/32*b^3*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2 +
1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a^2*b + integrate(1/32*(4*b^3*c^2*x^4*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^
3*c^2*x^4 + b^3*x^2)*arctan(c*x)^3 + 4*(24*a*b^2*c^2*x^4 - b^3*c*x^3 + 24*a
*b^2*x^2)*arctan(c*x)^2 + (b^3*c*x^3 + 3*(b^3*c^2*x^4 + b^3*x^2)*arctan(c*x
))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\operatorname{atan}{\left(cx\right)}\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**3, x)
```

3.28 $\int x\left(a+b\tan^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=131

$$-\frac{3b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2}-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{3bx\left(a+\right.}{}$$

```
[Out] -3/2*I*b*(a+b*arctan(c*x))^2/c^2-3/2*b*x*(a+b*arctan(c*x))^2/c+1/2*(a+b*arc
tan(c*x))^3/c^2+1/2*x^2*(a+b*arctan(c*x))^3-3*b^2*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c^2-3/2*I*b^3*polylog(2,1-2/(1+I*c*x))/c^2
```

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4852, 4916, 4846, 4920, 4854, 2402, 2315, 4884}

$$-\frac{3ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}-\frac{3b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2}-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*x*(a + b*ArcTan[c*x])^2)/(2
*c) + (a + b*ArcTan[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b^
2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 - (((3*I)/2)*b^3*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c^2
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}(cx)\right)^3\,dx=\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{2}(3bc)\int\frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{(3b)\int\left(a+b\tan^{-1}(cx)\right)^2\,dx}{2c}+\frac{(3b)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{2c}$$

$$=-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3+\left(3b^2\right)\int$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

Mathematica [A] time = 0.30, size = 152, normalized size = 1.16

$$\frac{a\left(acx(acx-3b)+3b^2\log\left(c^2x^2+1\right)\right)+3b^2\tan^{-1}(cx)^2\left(ac^2x^2+a+b(-cx+i)\right)+3b\tan^{-1}(cx)\left(a\left(ac^2x^2+a\right.\right.}{2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^2 + b*(I - c*x))*ArcTan[c*x]^2 + b^3*(1 + c^2*x^2)*ArcT
an[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a - 2*b*c*x + a*c^2*x^2) - 2*b^2*Log[1 + E^
((2*I)*ArcTan[c*x])]) + a*(a*c*x*(-3*b + a*c*x) + 3*b^2*Log[1 + c^2*x^2]) +
 (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\arctan(cx)^3+3\,ab^2x\arctan(cx)^2+3\,a^2bx\arctan(cx)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c*x)^3 + 3*a*b^2*x*arctan(c*x)^2 + 3*a^2*b*x*arctan(c
*x) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 352, normalized size = 2.69

$$\frac{x^2a^3}{2}+\frac{x^2b^3\arctan(cx)^3}{2}+\frac{b^3\arctan(cx)^3}{2c^2}-\frac{3b^3\arctan(cx)^2x}{2c}+\frac{3b^3\arctan(cx)\ln\left(c^2x^2+1\right)}{2c^2}-\frac{3ib^3\ln(cx-i)\ln\left(-\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^3,x)
```

```
[Out] 1/2*x^2*a^3+1/2*x^2*b^3*arctan(c*x)^3+1/2/c^2*b^3*arctan(c*x)^3-3/2/c*b^3*a
rctan(c*x)^2*x+3/2/c^2*b^3*arctan(c*x)*ln(c^2*x^2+1)-3/4*I/c^2*b^3*ln(c*x-I
)*ln(-1/2*I*(I+c*x))+3/4*I/c^2*b^3*dilog(1/2*I*(c*x-I))+3/4*I/c^2*b^3*ln(c*
x-I)*ln(c^2*x^2+1)-3/4*I/c^2*b^3*ln(I+c*x)*ln(c^2*x^2+1)-3/8*I/c^2*b^3*ln(c
*x-I)^2+3/8*I/c^2*b^3*ln(I+c*x)^2-3/4*I/c^2*b^3*dilog(-1/2*I*(I+c*x))+3/4*I
/c^2*b^3*ln(I+c*x)*ln(1/2*I*(c*x-I))+3/2*x^2*a*b^2*arctan(c*x)^2+3/2/c^2*a*
b^2*arctan(c*x)^2-3/c*a*b^2*x*arctan(c*x)+3/2/c^2*a*b^2*ln(c^2*x^2+1)+3/2*b
*x^2*a^2*arctan(c*x)-3/2/c*x*a^2*b+3/2/c^2*a^2*b*arctan(c*x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x*(a + b*atan(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**3, x)
```

3.29 $\int \left(a+b\tan^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=119

$$\frac{3ib^2\mathrm{Li}_2\left(1-\frac{2}{icx+1}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+\frac{3b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)^2}{c}$$

```
[Out] I*(a+b*arctan(c*x))^3/c+x*(a+b*arctan(c*x))^3+3*b*(a+b*arctan(c*x))^2*ln(2/
(1+I*c*x))/c+3*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3*p
olylog(3,1-2/(1+I*c*x))/c
```

Rubi [A] time = 0.21, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4846, 4920, 4854, 4884, 4994, 6610}

$$\frac{3ib^2\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{3b^3\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^3)/c + x*(a + b*ArcTan[c*x])^3 + (3*b*(a + b*ArcTan[
c*x])^2*Log[2/(1 + I*c*x)])/c + ((3*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
```

```
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)^3\,dx = x\left(a+b\tan^{-1}(cx)\right)^3-(3bc)\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+(3b)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{i-cx}\,dx$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}-(6$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}+\frac{3i}{}$$
$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}+\frac{3i}{}$$

Mathematica [A] time = 0.10, size = 192, normalized size = 1.61

$$a^3x-\frac{3a^2b\log\left(c^2x^2+1\right)}{2c}+3a^2bx\tan^{-1}(cx)+\frac{3ab^2\left(-i\text{Li}_2\left(-e^{2i\tan^{-1}(cx)}\right)+cx\tan^{-1}(cx)^2-i\tan^{-1}(cx)^2+2\tan^{-1}\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c*x] - (3*a^2*b*Log[1 + c^2*x^2])/(2*c) + (3*a*b^2
*((-I)*ArcTan[c*x]^2 + c*x*ArcTan[c*x]^2 + 2*ArcTan[c*x]*Log[1 + E^((2*I)*A
rcTan[c*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b^3*((-I)*ArcTan
[c*x]^3 + c*x*ArcTan[c*x]^3 + 3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])
] - (3*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*PolyLog[3, -E
^((2*I)*ArcTan[c*x])])/2))/c
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\arctan(cx)^3+3\,ab^2\arctan(cx)^2+3\,a^2b\arctan(cx)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.23, size = 270, normalized size = 2.27

$$x\,a^{3}-\frac{i b^{3} \arctan \left(c x\right)^{3}}{c}+b^{3} x \arctan \left(c x\right)^{3}+\frac{3 b^{3} \arctan \left(c x\right)^{2} \ln \left(\frac{\left(i c x+1\right)^{2}}{c^{2} x^{2}+1}+1\right)}{c}-\frac{3 i b^{3} \arctan \left(c x\right) \operatorname{polylog}\left(2,-\frac{\left(i c x+1\right)^{2}}{c^{2} x^{2}+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3,x)
```

```
[Out] x*a^3-I/c*b^3*arctan(c*x)^3+b^3*x*arctan(c*x)^3+3/c*b^3*arctan(c*x)^2*ln((1
+I*c*x)^2/(c^2*x^2+1)+1)-3*I/c*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*
x^2+1))+3/2/c*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3*I/c*arctan(c*x)^2*a
*b^2+3*x*a*b^2*arctan(c*x)^2+6/c*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*arctan(c*x)*
a*b^2-3*I/c*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*a*b^2+3*x*a^2*b*arctan(c*x)
-3/2/c*a^2*b*ln(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,b^{3} x \arctan \left(c x\right)^{3}-\frac{3}{32}\,b^{3} x \arctan \left(c x\right) \log \left(c^{2} x^{2}+1\right)^{2}+\frac{7\,b^{3} \arctan \left(c x\right)^{4}}{32\,c}+28\,b^{3} c^{2} \int \frac{x^{2} \arctan \left(c x\right)^{3}}{32\,\left(c^{2} x^{2}+1\right)}\,dx+3\,b^{3} c^{2} \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x*arctan(c*x)^3 - 3/32*b^3*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 7/32*
b^3*arctan(c*x)^4/c + 28*b^3*c^2*integrate(1/32*x^2*arctan(c*x)^3/(c^2*x^2
+ 1), x) + 3*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2
*x^2 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1)
, x) + 12*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2
+ 1), x) + a*b^2*arctan(c*x)^3/c - 12*b^3*c*integrate(1/32*x*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 3*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 +
1), x) + a^3*x + 3*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x
^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3,x)
```

```
[Out] int((a + b*atan(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atan}{\left(c x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3, x)
```

3.30 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx$

Optimal. Leaf size=206

$$-\frac{3}{2}b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{3}{2}b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)\left(a+b\,\text{t}\right.$$

```
[Out] -2*(a+b*arctan(c*x))^3*arctanh(-1+2/(1+I*c*x))-3/2*I*b*(a+b*arctan(c*x))^2*
polylog(2,1-2/(1+I*c*x))+3/2*I*b*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1+I*c*
x))-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))+3/2*b^2*(a+b*arctan(
c*x))*polylog(3,-1+2/(1+I*c*x))+3/4*I*b^3*polylog(4,1-2/(1+I*c*x))-3/4*I*b^
3*polylog(4,-1+2/(1+I*c*x))
```

Rubi [A] time = 0.43, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{3}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ib\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[1 - 2/(1 + I*c*x)] - ((3*I)/2)*b*(a + b*Arc
Tan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + ((3*I)/2)*b*(a + b*ArcTan[c*x])
^2*PolyLog[2, -1 + 2/(1 + I*c*x)] - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1
 - 2/(1 + I*c*x)])/2 + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*
c*x)])/2 + ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)] - ((3*I)/4)*b^3*Poly
Log[4, -1 + 2/(1 + I*c*x)]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx = 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-(6bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\dots}\right.}{1+c^2x^2}$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)+$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)+$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx}\right)+$$

Mathematica [A] time = 0.15, size = 212, normalized size = 1.03

$$\frac{3}{4}ib\left(2\text{Li}_2\left(\frac{cx+i}{i-cx}\right)\left(a+b\tan^{-1}(cx)\right)^2-2\text{Li}_2\left(\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)^2+b\left(-2i\text{Li}_3\left(\frac{cx+i}{i-cx}\right)\left(a+b\tan^{-1}(cx)\right)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[(I + c*x)/(-I + c*x)] + ((3*I)/4)*b*(2*(a +
 b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(I - c*x)] - 2*(a + b*ArcTan[c*x])^2
*PolyLog[2, (I + c*x)/(-I + c*x)] + b*((-2*I)*(a + b*ArcTan[c*x])*PolyLog[3
, (I + c*x)/(I - c*x)] + (2*I)*(a + b*ArcTan[c*x])*PolyLog[3, (I + c*x)/(-I
 + c*x)] + b*(-PolyLog[4, (I + c*x)/(I - c*x)] + PolyLog[4, (I + c*x)/(-I +
 c*x)])))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(cx\right)^3+3\,ab^2\arctan\left(cx\right)^2+3\,a^2b\arctan\left(cx\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.24, size = 2309, normalized size = 11.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x,x)
[Out] 3/2*I*a*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^3*arctan(c*x)^2+3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^3-1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3-1/2*I*b^3*Pi*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*arctan(c*x)^3-1/2*I*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x
)^3-3/2*I*a*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))^2*arctan(c*x)^2+3/2*I*a^2*b*ln(c*x)*ln(1+I*c*x)-3/2*I*a^2*b*ln(c*x)*l
n(1-I*c*x)-1/2*I*b^3*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*arctan(c*x)^3+1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^3+a^3*ln(c*x)+1/2*I*b^3*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^3-3/2*I
*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*a*b^2*Pi*csgn(I/((
1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3*a*b^2*ln(c*x)*arctan(c*x)^2-3
*a*b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+3*a*b^2*arctan(c*x)^2*ln
(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x
^2+1)^(1/2))+3*a^2*b*ln(c*x)*arctan(c*x)+3/2*I*a^2*b*dilog(1+I*c*x)-3/2*I*a
^2*b*dilog(1-I*c*x)+1/2*I*b^3*Pi*arctan(c*x)^3-3*I*b^3*arctan(c*x)^2*polylo
g(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^3*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^3-6*I*a*b^2*arctan(c*x)*pol
ylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*a*b^2*arctan(c*x)*polylog(2,-(1+I*c
*x)/(c^2*x^2+1)^(1/2))+3/2*I*a*b^2*Pi*arctan(c*x)^2+3*I*a*b^2*arctan(c*x)*p
olylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+6*I*b^3*polylog(4,(1+I*c*x)/(c^2*x^2+1)^
(1/2))-3/2*a*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*b^3*arctan(c*x)^
2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3*I*b^3*arctan(c*x)^2*polylog(2,(1+I*
c*x)/(c^2*x^2+1)^(1/2))+3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*
csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+6*a*b^2*polylog(3,-(1+I*c*x)/(c^2*x^
2+1)^(1/2))+6*a*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^3*ln(c*x)*arct
an(c*x)^3-b^3*arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^3*arctan(c*x)^3
*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3*arctan(c*x)*polylog(3,-(1+I*c*x)/(
c^2*x^2+1)^(1/2))+b^3*arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3
```

```
*arctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*b^3*arctan(c*x)*pol
ylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+6*I*b^3*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(
1/2))-3/4*I*b^3*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\log(x)+\frac{1}{32}\int\frac{28\,b^3\arctan(cx)^3+3\,b^3\arctan(cx)\log\left(c^2x^2+1\right)^2+96\,ab^2\arctan(cx)^2+96\,a^2b\arctan(cx)}{x}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*x)*log(c
^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x,x)
```

```
[Out] int((a + b*atan(c*x))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{atan}{\left(cx\right)}\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x, x)
```

3.31 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal. Leaf size=116

$$-3ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)\left(a+b\tan^{-1}(cx)\right)-ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+3bc\log\left(2-\frac{2}{1-icx}\right)\left(a+\right.$$

```
[Out] -I*c*(a+b*arctan(c*x))^3-(a+b*arctan(c*x))^3/x+3*b*c*(a+b*arctan(c*x))^2*ln
(2-2/(1-I*c*x))-3*I*b^2*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))+3/2*b
^3*c*polylog(3,-1+2/(1-I*c*x))
```

Rubi [A] time = 0.27, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4852, 4924, 4868, 4884, 4992, 6610}

$$-3ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{3}{2}b^3c\text{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)-ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^3 - (a + b*ArcTan[c*x])^3/x + 3*b*c*(a + b*ArcTa
n[c*x])^2*Log[2 - 2/(1 - I*c*x)] - (3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[
2, -1 + 2/(1 - I*c*x)] + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
```

```
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2}\,dx &= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(1+c^2x^2\right)}\,dx\\ &= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+(3ibc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(i+cx)}\,dx\\ &= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right)\\ &= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right)\\ &= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right)\end{aligned}$$

Mathematica [A] time = 0.40, size = 214, normalized size = 1.84

$$-\frac{a^3}{x}-\frac{3}{2}a^2bc\log\left(c^2x^2+1\right)+3a^2bc\log(x)-\frac{3a^2b\tan^{-1}(cx)}{x}+3ab^2c\left(-i\left(\tan^{-1}(cx)^2+\text{Li}_2\left(e^{2i\tan^{-1}(cx)}\right)\right)-\frac{\tan^{-1}(cx)^2}{cx}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTan[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 +
c^2*x^2])/2 + 3*a*b^2*c*(-(ArcTan[c*x]^2/(c*x)) + 2*ArcTan[c*x]*Log[1 - E^(
(2*I)*ArcTan[c*x])] - I*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])
) + b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x]^3 - ArcTan[c*x]^3/(c*x) + 3*ArcTan
[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (3*I)*ArcTan[c*x]*PolyLog[2, E^((
-2*I)*ArcTan[c*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan(cx)^3+3\,ab^2\arctan(cx)^2+3\,a^2b\arctan(cx)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.29, size = 2159, normalized size = 18.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^2,x)
```

```
[Out] 3/2*I*c*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2
*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
*arctan(c*x)^2-3/4*I*c*b^3*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1)^2)-3/2*I*c*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/4*I*c*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-3/2*I*c*b^3*
Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*c*b^3*Pi*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*arctan(c*x)^2+3/2*I*c*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2-3/2*I*c*b^3*arctan(c*x)^2*
Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2
)^2+3/4*I*c*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+3/4*I*c*b^3*arctan(c*
x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-3*c*b^3*arctan(c*x)^2*ln((1+I*c*x)^2/(c^
2*x^2+1)-1)+3*c*b^3*ln(c*x)*arctan(c*x)^2-3*a*b^2/x*arctan(c*x)^2-3*a^2*b/x
*arctan(c*x)-I*c*b^3*arctan(c*x)^3+3*c*a^2*b*ln(c*x)+3*c*b^3*ln(2)*arctan(c
*x)^2-3/2*c*a^2*b*ln(c^2*x^2+1)+3*c*b^3*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x
^2+1)^(1/2))+3*c*b^3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*c*b^
3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*c*b^3*arctan(c*x)^2*ln(
c^2*x^2+1)-b^3/x*arctan(c*x)^3+6*c*b^3*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2
))+6*c*b^3*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/4*I*c*b^3*arctan(c*x)^
2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-3/4*I*
c*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3+3/2*I*c*b^3*Pi*csg
n(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-
3/2*I*c*b^3*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^2*arctan(c*x)^2+3/2*I*c*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3/4*I*c*b^3*arctan(c*x)^2*Pi*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-3/2*I*c*a*b^2*ln(c*x-I)*ln(c^2*x^2+1)+3/2*I
*c*a*b^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))+3/2*I*c*a*b^2*ln(I+c*x)*ln(c^2*x^2+1)
-3/2*I*c*a*b^2*ln(I+c*x)*ln(1/2*I*(c*x-I))+3*I*c*a*b^2*ln(c*x)*ln(1+I*c*x)-
3*I*c*a*b^2*ln(c*x)*ln(1-I*c*x)-3*c*a*b^2*arctan(c*x)*ln(c^2*x^2+1)+6*c*a*b
^2*ln(c*x)*arctan(c*x)+3/2*I*c*a*b^2*dilog(-1/2*I*(I+c*x))-3/4*I*c*a*b^2*ln
(I+c*x)^2-3/2*I*c*a*b^2*dilog(1/2*I*(c*x-I))+3*I*c*a*b^2*dilog(1+I*c*x)-3*I
*c*a*b^2*dilog(1-I*c*x)-6*I*c*b^3*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+
1)^(1/2))-6*I*c*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2
*I*c*b^3*Pi*arctan(c*x)^2+3/4*I*c*a*b^2*ln(c*x-I)^2-a^3/x+3/2*I*c*b^3*Pi*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/4*I*c*b^3*
arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^2,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**2, x)
```

3.32 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=133

$$3b^2c^2\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}$$

```
[Out] -3/2*I*b*c^2*(a+b*arctan(c*x))^2-3/2*b*c*(a+b*arctan(c*x))^2/x-1/2*c^2*(a+b
*arctan(c*x))^3-1/2*(a+b*arctan(c*x))^3/x^2+3*b^2*c^2*(a+b*arctan(c*x))*ln(
2-2/(1-I*c*x))-3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A] time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4852, 4918, 4924, 4868, 2447, 4884}

$$-\frac{3}{2}ib^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+3b^2c^2\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] ((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/(2*x
) - (c^2*(a + b*ArcTan[c*x])^3)/2 - (a + b*ArcTan[c*x])^3/(2*x^2) + 3*b^2*c
^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - ((3*I)/2)*b^3*c^2*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx-\frac{1}{2}\left(3bc^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right.}{1+c^2x^2}$$
$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\left(3b^2c^2\right)\int \frac{a}{\ }$$
$$= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{(a+b\,\mathrm{t}}{\ }$$
$$= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{(a+b\,\mathrm{t}}{\ }$$
$$= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{(a+b\,\mathrm{t}}{\ }$$

Mathematica [A] time = 0.34, size = 176, normalized size = 1.32

$$-\frac{a\left(a(a+3bcx)-6b^2c^2x^2\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)\right)+3b^2\tan^{-1}(cx)^2\left(ac^2x^2+a+bcx(1+icx)\right)+3b\tan^{-1}(cx)\left(a\left(ac^2x^2+\right.\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] -1/2*(3*b^2*(a + a*c^2*x^2 + b*c*x*(1 + I*c*x))*ArcTan[c*x]^2 + b^3*(1 + c^
2*x^2)*ArcTan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a + 2*b*c*x + a*c^2*x^2) - 2*b^2
*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]) + a*(a*(a + 3*b*c*x) - 6*b^2*c^2*x
^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (3*I)*b^3*c^2*x^2*PolyLog[2, E^((2*I)*Ar
cTan[c*x])])/x^2
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\arctan(cx)^3+3\,ab^2\arctan(cx)^2+3\,a^2b\arctan(cx)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 457, normalized size = 3.44

$$-\frac{3ic^2b^3\ln(cx+i)^2}{8}+\frac{3ic^2b^3\ln(cx-i)^2}{8}-\frac{3ic^2b^3\operatorname{dilog}(-icx+1)}{2}+\frac{3ic^2b^3\operatorname{dilog}\left(-\frac{i(cx+i)}{2}\right)}{4}-\frac{3ic^2b^3\operatorname{dilog}\left(\frac{i(cx-i)}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^3,x)
```

```
[Out] 3*c^2*a*b^2*ln(c*x)-3/2*c^2*a*b^2*arctan(c*x)^2-3/2*c*a^2*b/x+3/2*I*c^2*b^3
*dilog(1+I*c*x)+3/4*I*c^2*b^3*dilog(-1/2*I*(I+c*x))-3/8*I*c^2*b^3*ln(I+c*x)
^2-3/4*I*c^2*b^3*dilog(1/2*I*(c*x-I))-3/2*c^2*b^3*arctan(c*x)*ln(c^2*x^2+1)
+3*c^2*b^3*arctan(c*x)*ln(c*x)-3/2*c^2*a^2*b*arctan(c*x)-3/2*c^2*a*b^2*ln(c
^2*x^2+1)-3/2*a*b^2/x^2*arctan(c*x)^2-3/2*a^2*b/x^2*arctan(c*x)-3/2*c*b^3*a
rctan(c*x)^2/x+3/8*I*c^2*b^3*ln(c*x-I)^2-3/2*I*c^2*b^3*dilog(1-I*c*x)-1/2*a
^3/x^2-1/2*b^3/x^2*arctan(c*x)^3-1/2*c^2*b^3*arctan(c*x)^3-3/2*I*c^2*b^3*ln
(c*x)*ln(1-I*c*x)-3/4*I*c^2*b^3*ln(c*x-I)*ln(c^2*x^2+1)-3/4*I*c^2*b^3*ln(I+
c*x)*ln(1/2*I*(c*x-I))+3/4*I*c^2*b^3*ln(I+c*x)*ln(c^2*x^2+1)+3/2*I*c^2*b^3*
ln(c*x)*ln(1+I*c*x)+3/4*I*c^2*b^3*ln(c*x-I)*ln(-1/2*I*(I+c*x))-3*c*a*b^2/x*
arctan(c*x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^3,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atan}(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**3, x)
```

3.33 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=213

$$ib^2c^3\text{Li}_2\left(\frac{2}{1-icx}-1\right)\left(a+b\tan^{-1}(cx)\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2$$

```
[Out] -b^2*c^2*(a+b*arctan(c*x))/x-1/2*b*c^3*(a+b*arctan(c*x))^2-1/2*b*c*(a+b*arc
tan(c*x))^2/x^2+1/3*I*c^3*(a+b*arctan(c*x))^3-1/3*(a+b*arctan(c*x))^3/x^3+b
^3*c^3*ln(x)-1/2*b^3*c^3*ln(c^2*x^2+1)-b*c^3*(a+b*arctan(c*x))^2*ln(2-2/(1-
I*c*x))+I*b^2*c^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))-1/2*b^3*c^3*p
olylog(3,-1+2/(1-I*c*x))
```

Rubi [A] time = 0.48, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^3c^3\text{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{3}ic^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^4,x]
```

```
[Out] -((b^2*c^2*(a + b*ArcTan[c*x]))/x) - (b*c^3*(a + b*ArcTan[c*x])^2)/2 - (b*c
*(a + b*ArcTan[c*x])^2)/(2*x^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^3 - (a + b*
ArcTan[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 + c^2*x^2])/2 - b*
c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + I*b^2*c^3*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 - I*c*
x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^4}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} + (bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} + (bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx - \left(bc^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} + \left(b^2c^2\right)\int \frac{a+}{x^2}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} - bc^3\left(a+b\tan\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+\right.$$

Mathematica [A] time = 0.88, size = 321, normalized size = 1.51

$$-\frac{a^3}{3x^3} - a^2bc^3\log(x) + \frac{1}{2}a^2bc^3\log\left(c^2x^2+1\right) - \frac{a^2b\tan^{-1}(cx)}{x^3} - \frac{a^2bc}{2x^2} + \frac{iab^2\left(c^3x^3\mathrm{Li}_2\left(e^{2i\tan^{-1}(cx)}\right) + \left(c^3x^3+i\right)\tan^{-1}(cx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (a^2*b*c)/(2*x^2) - (a^2*b*ArcTan[c*x])/x^3 - a^2*b*c^3*Log[
x] + (a^2*b*c^3*Log[1 + c^2*x^2])/2 + (I*a*b^2*(I*c^2*x^2 + (I + c^3*x^3)*A
rcTan[c*x]^2 + I*c*x*ArcTan[c*x]*(1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^((2*I)*
ArcTan[c*x])]) + c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^3 + (b^3*c^3
*(I*Pi^3 - (24*ArcTan[c*x])/(c*x) - 12*ArcTan[c*x]^2 - (12*ArcTan[c*x]^2)/(
c^2*x^2) - (8*I)*ArcTan[c*x]^3 - (8*ArcTan[c*x]^3)/(c^3*x^3) - 24*ArcTan[c*
x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 24*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (2
4*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, E^((-2*
I)*ArcTan[c*x])]))/24
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\arctan(cx)^3 + 3\,ab^2\arctan(cx)^2 + 3\,a^2b\arctan(cx) + a^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^4, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 3.19, size = 5974, normalized size = 28.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^4,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^4,x)
[Out] int((a + b*atan(c*x))^3/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**4,x)
[Out] Integral((a + b*atan(c*x))**3/x**4, x)
```

3.34 $\int \frac{(a+b\tan^{-1}(cx))^3}{x^5}\,dx$

Optimal. Leaf size=198

$$-2b^2c^4\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^3+ibc^4\left(a+b\tan^{-1}(cx)\right)$$

```
[Out] -1/4*b^3*c^3/x-1/4*b^3*c^4*arctan(c*x)-1/4*b^2*c^2*(a+b*arctan(c*x))/x^2+I*
b*c^4*(a+b*arctan(c*x))^2-1/4*b*c*(a+b*arctan(c*x))^2/x^3+3/4*b*c^3*(a+b*ar
ctan(c*x))^2/x+1/4*c^4*(a+b*arctan(c*x))^3-1/4*(a+b*arctan(c*x))^3/x^4-2*b^
2*c^4*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+I*b^3*c^4*polylog(2,-1+2/(1-I*c*x
))
```

Rubi [A] time = 0.60, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4852, 4918, 325, 203, 4924, 4868, 2447, 4884}

$$ib^3c^4\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}-2b^2c^4\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}c^4\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -(b^3*c^3)/(4*x) - (b^3*c^4*ArcTan[c*x])/4 - (b^2*c^2*(a + b*ArcTan[c*x]))/
(4*x^2) + I*b*c^4*(a + b*ArcTan[c*x])^2 - (b*c*(a + b*ArcTan[c*x])^2)/(4*x^
3) + (3*b*c^3*(a + b*ArcTan[c*x])^2)/(4*x) + (c^4*(a + b*ArcTan[c*x])^3)/4
- (a + b*ArcTan[c*x])^3/(4*x^4) - 2*b^2*c^4*(a + b*ArcTan[c*x])*Log[2 - 2/(
1 - I*c*x)] + I*b^3*c^4*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^5}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx-\frac{1}{4}\left(3bc^3\right)\int \frac{\left(a+b\tan^{-1}(\cdots}{x^2\left(1+c^2x\cdots\right.}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4x^4}+\frac{1}{2}\left(b^2c^2\right)\int \frac{a+b\tan^{-1}(cx)}{x^3\left(1+c^2x^2\right)}\,dx-\frac{1}{4}\left(\cdots\right.$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}+\frac{3bc^3\left(a+b\tan^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\cdots\right.}{\cdots}$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}+\frac{3bc^3\left(a\cdots\right.}{\cdots}$$

$$= -\frac{b^3c^3}{4x}-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}+\cdots$$

$$= -\frac{b^3c^3}{4x}-\frac{1}{4}b^3c^4\tan^{-1}(cx)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc\left(a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.72, size = 265, normalized size = 1.34

$$-\frac{a^3+b\tan^{-1}(cx)\left(a^2\left(3-3c^4x^4\right)+ab\left(2cx-6c^3x^3\right)+8b^2c^4x^4\log\left(1-e^{2i\tan^{-1}(cx)}\right)+b^2c^2x^2\left(c^2x^2+1\right)\right)-3a^2\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -1/4*(a^3 + a^2*b*c*x + a*b^2*c^2*x^2 - 3*a^2*b*c^3*x^3 + b^3*c^3*x^3 + a*b
^2*c^4*x^4 + b^2*(b*c*x*(1 - 3*c^2*x^2 - (4*I)*c^3*x^3) + a*(3 - 3*c^4*x^4)
)*ArcTan[c*x]^2 - b^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*c^2
*x^2*(1 + c^2*x^2) + a*b*(2*c*x - 6*c^3*x^3) + a^2*(3 - 3*c^4*x^4) + 8*b^2*
c^4*x^4*Log[1 - E^((2*I)*ArcTan[c*x])]) + 8*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1
+ c^2*x^2]] - (4*I)*b^3*c^4*x^4*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan(cx)^3 + 3\,ab^2\arctan(cx)^2 + 3\,a^2b\arctan(cx) + a^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^5, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 550, normalized size = 2.78

$$ic^4b^3\,\mathrm{dilog}(-icx+1) - \frac{ic^4b^3\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2} - ic^4b^3\,\mathrm{dilog}(icx+1) + \frac{ic^4b^3\,\mathrm{dilog}\left(\frac{i(cx-i)}{2}\right)}{2} + \frac{ic^4b^3\ln(cx+i)^2}{4} - \frac{ic^4b^3\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^5,x)
```

```
[Out] -1/4*c*a^2*b/x^3-1/4*c^2*a*b^2/x^2+3/4*c^3*a^2*b/x+c^4*a*b^2*ln(c^2*x^2+1)+
3/4*c^4*a^2*b*arctan(c*x)+3/4*c^4*a*b^2*arctan(c*x)^2+I*c^4*b^3*dilog(1-I*c
*x)-3/4*a^2*b/x^4*arctan(c*x)-3/4*a*b^2/x^4*arctan(c*x)^2-1/2*I*c^4*b^3*dil
og(-1/2*I*(I+c*x))+1/4*I*c^4*b^3*ln(I+c*x)^2-I*c^4*b^3*dilog(1+I*c*x)-1/4*I
*c^4*b^3*ln(c*x-I)^2+1/2*I*c^4*b^3*dilog(1/2*I*(c*x-I))-1/4*c^2*b^3*arctan(
c*x)/x^2-1/4*c*b^3*arctan(c*x)^2/x^3+3/4*c^3*b^3*arctan(c*x)^2/x+c^4*b^3*ar
ctan(c*x)*ln(c^2*x^2+1)-2*c^4*b^3*arctan(c*x)*ln(c*x)-2*c^4*a*b^2*ln(c*x)-1
/4*b^3*c^3/x-1/4*b^3*c^4*arctan(c*x)-1/2*c*a*b^2/x^3*arctan(c*x)+3/2*c^3*a*
b^2/x*arctan(c*x)+1/2*I*c^4*b^3*ln(c*x-I)*ln(c^2*x^2+1)-1/2*I*c^4*b^3*ln(I+
c*x)*ln(c^2*x^2+1)-1/2*I*c^4*b^3*ln(c*x-I)*ln(-1/2*I*(I+c*x))-I*c^4*b^3*ln(
c*x)*ln(1+I*c*x)+1/2*I*c^4*b^3*ln(I+c*x)*ln(1/2*I*(c*x-I))+I*c^4*b^3*ln(c*x
)*ln(1-I*c*x)-1/4*a^3/x^4-1/4*b^3/x^4*arctan(c*x)^3+1/4*c^4*b^3*arctan(c*x)
^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}\,(c\,x))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^5,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**5, x)
```

3.35 $\int \frac{x}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=11

$$\mathrm{Int}\left(\frac{x}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcTan[a*x],x]
[Out] Defer[Int][x/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)}\,dx = \int \frac{x}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x],x]
[Out] Integrate[x/ArcTan[a*x], x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="fricas")
[Out] integral(x/arctan(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x),x)
```

```
[Out] int(x/arctan(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/arctan(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x),x)
```

```
[Out] int(x/atan(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x),x)
```

```
[Out] Integral(x/atan(a*x), x)
```

3.36 $\int \frac{1}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\text{Int}\left(\frac{1}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x),x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-1),x]
[Out] Defer[Int][ArcTan[a*x]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)}\,dx = \int \frac{1}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-1),x]
[Out] Integrate[ArcTan[a*x]^(-1), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="fricas")
[Out] integral(1/arctan(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x),x)
```

```
[Out] int(1/arctan(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/arctan(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x),x)
```

```
[Out] int(1/atan(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x),x)
```

```
[Out] Integral(1/atan(a*x), x)
```

3.37 $\int \frac{1}{x\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\tan^{-1}(ax)}\,dx = \int \frac{1}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]),x]
[Out] Integrate[1/(x*ArcTan[a*x]), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\arctan(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="fricas")
[Out] integral(1/(x*arctan(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x),x)
```

```
[Out] int(1/x/arctan(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arctan (ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctan(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)),x)
```

```
[Out] int(1/(x*atan(a*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x),x)
```

```
[Out] Integral(1/(x*atan(a*x)), x)
```

3.38 $\int \frac{x}{\tan^{-1}(ax)^2} \, dx$

Optimal. Leaf size=11

$$\text{Int}\left(\frac{x}{\tan^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcTan[a*x]^2,x]
[Out] Defer[Int][x/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)^2} \, dx = \int \frac{x}{\tan^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x}{\tan^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x]^2,x]
[Out] Integrate[x/ArcTan[a*x]^2, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\arctan(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(x/arctan(a*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan(ax)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^2,x)
```

```
[Out] int(x/arctan(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2x^3 - sage_0 x \arctan(ax) + x}{a\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^3 - arctan(a*x)*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + x)/(a*a
rctan(a*x))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x)^2,x)
```

```
[Out] int(x/atan(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**2,x)
```

```
[Out] Integral(x/atan(a*x)**2, x)
```

3.39 $\int \frac{1}{\tan^{-1}(ax)^2} dx$

Optimal. Leaf size=9

$$\mathrm{Int}\left(\frac{1}{\tan^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^2,x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-2),x]
[Out] Defer[Int][ArcTan[a*x]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)^2} dx = \int \frac{1}{\tan^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-2),x]
[Out] Integrate[ArcTan[a*x]^(-2), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\arctan(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(arctan(a*x)^(-2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^2,x)
```

```
[Out] int(1/arctan(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{-2\,a^2 sage_0 x \arctan\left(ax\right) + a^2x^2 + 1}{a\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - 2*a^2*arctan(a*x)*integrate(x/arctan(a*x), x) + 1)/(a*arctan(a*
x))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x)^2,x)
```

```
[Out] int(1/atan(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**2,x)
```

```
[Out] Integral(atan(a*x)**(-2), x)
```

3.40 $\int \frac{1}{x\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x\tan^{-1}(ax)^2}\,dx = \int \frac{1}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*ArcTan[a*x]^2), x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\arctan(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arctan(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^2,x)
```

```
[Out] int(1/x/arctan(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2x^2 - sage_0x^2 \arctan(ax) + 1}{ax \arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - x*arctan(a*x)*integrate((a^2*x^2 - 1)/(x^2*arctan(a*x)), x) + 1
)/(a*x*arctan(a*x))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^2),x)
```

```
[Out] int(1/(x*atan(a*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{atan}^{2}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x*atan(a*x)**2), x)
```

3.41 $\int x\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(x\sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x*arctan(a*x)^(1/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x\sqrt{\tan^{-1}(ax)}\,dx=\int x\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.11, size = 0, normalized size = 0.00

$$\int x\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*Sqrt[ArcTan[a*x]], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.97, size = 0, normalized size = 0.00

$$\int x\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2),x)
```

```
[Out] int(x*atan(a*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{atan}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(atan(a*x)), x)
```

3.42 $\int \sqrt{\tan^{-1}(ax)}\, dx$

Optimal. Leaf size=11

$$\text{Int}\left(\sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2),x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{\tan^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \sqrt{\tan^{-1}(ax)}\, dx = \int \sqrt{\tan^{-1}(ax)}\, dx$$

Mathematica [A] time = 1.93, size = 0, normalized size = 0.00

$$\int \sqrt{\tan^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]],x]
[Out] Integrate[Sqrt[ArcTan[a*x]], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \sqrt{\arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int \sqrt{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2),x)
```

```
[Out] int(atan(a*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{atan}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x)), x)
```

3.43 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/x,x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$$

Mathematica [A] time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/x,x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/x, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan(ax)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x,x)
```

```
[Out] int(arctan(a*x)^(1/2)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/x,x)
```

```
[Out] int(atan(a*x)^(1/2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(atan(a*x))/x, x)
```

3.44 $\int x \tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(x\tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x*arctan(a*x)^(3/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x\tan^{-1}(ax)^{3/2}\,dx=\int x\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A] time = 1.02, size = 0, normalized size = 0.00

$$\int x\tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x*ArcTan[a*x]^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.95, size = 0, normalized size = 0.00

$$\int x\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2),x)
```

```
[Out] int(x*atan(a*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**(3/2), x)
```

3.45 $\int \tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=11

$$\mathrm{Int}\left(\tan^{-1}(ax)^{3/2},x\right)$$

[Out] Unintegrable(arctan(a*x)^(3/2),x)

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

[In] Int[ArcTan[a*x]^(3/2),x]

[Out] Defer[Int][ArcTan[a*x]^(3/2), x]

Rubi steps

$$\int \tan^{-1}(ax)^{3/2}\,dx=\int \tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A] time = 2.37, size = 0, normalized size = 0.00

$$\int \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcTan[a*x]^(3/2),x]

[Out] Integrate[ArcTan[a*x]^(3/2), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int \operatorname{atan}(a\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2),x)
```

```
[Out] int(atan(a*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2), x)
```

3.46 $\int \frac{\tan^{-1}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\tan^{-1}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x}\,dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/x,x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/x, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x,x)
```

```
[Out] int(arctan(a*x)^(3/2)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/x,x)
```

```
[Out] int(atan(a*x)^(3/2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/x, x)
```

3.47 $\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(1/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x/Sqrt[ArcTan[a*x]], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\arctan(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x}{\sqrt{\operatorname{atan}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x)^(1/2),x)
```

```
[Out] int(x/atan(a*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(atan(a*x)), x)
```

3.48 $\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^(1/2),x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][1/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[1/Sqrt[ArcTan[a*x]], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\arctan \left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x)^(1/2),x)
```

```
[Out] int(1/atan(a*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{atan}{\left(a x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(atan(a*x)), x)
```

3.49 $\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{1}{x\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^(1/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*Sqrt[ArcTan[a*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{atan}{\left(ax \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(atan(a*x))), x)
```

3.50 $\int \frac{x}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(3/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{x}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x/ArcTan[a*x]^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/arctan(a*x)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x}{\operatorname{atan}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x)^(3/2),x)
```

```
[Out] int(x/atan(a*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/atan(a*x)**(3/2), x)
```

3.51 $\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\frac{1}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^(3/2),x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-3/2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(-3/2), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-3/2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(-3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/arctan(a*x)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x)^(3/2),x)
```

```
[Out] int(1/atan(a*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(-3/2), x)
```

3.52 $\int \frac{1}{x \tan^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \tan^{-1}(ax)^{3/2}}, x\right)$$

[Out] Unintegrable(1/x/arctan(a*x)^(3/2),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*ArcTan[a*x]^(3/2)),x]

[Out] Defer[Int][1/(x*ArcTan[a*x]^(3/2)), x]

Rubi steps

$$\int \frac{1}{x \tan^{-1}(ax)^{3/2}} \, dx = \int \frac{1}{x \tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A] time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{1}{x \tan^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*ArcTan[a*x]^(3/2)),x]

[Out] Integrate[1/(x*ArcTan[a*x]^(3/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arctan(ax)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*atan(a*x)**(3/2)), x)
```

3.53 $\int \sqrt{x}\,\tan^{-1}(x)\,dx$

Optimal. Leaf size=117

$$\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{4\sqrt{x}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\right.$$

```
[Out] 2/3*x^(3/2)*arctan(x)-1/6*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/6*ln(1+x+2^(1/2
)*x^(1/2))*2^(1/2)+1/3*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/3*arctan(1+2^(1
/2)*x^(1/2))*2^(1/2)-4/3*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {4852, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{4\sqrt{x}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-4*Sqrt[x])/3 - (Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]])/3 + (Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[x]])/3 + (2*x^(3/2)*ArcTan[x])/3 - Log[1 - Sqrt[2]*Sqrt[x]
 + x]/(3*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]/(3*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\tan^{-1}(x)\,dx &= \frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{2}{3}\int \frac{x^{3/2}}{1+x^2}\,dx\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{2}{3}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{2}{3}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right.\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x\cdots}\right.\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}}+\frac{1}{3}\sqrt{2}\,\text{Subst}\left(\cdots\right.\\
&= -\frac{4\sqrt{x}}{3}-\frac{1}{3}\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)+\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{\log\left(1\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 108, normalized size = 0.92

$$\frac{1}{6}\left(4x^{3/2}\tan^{-1}(x)-8\sqrt{x}-\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-8*Sqrt[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[x]] + 4*x^(3/2)*ArcTan[x] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] +
x] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/6
```

fricas [A] time = 0.48, size = 118, normalized size = 1.01

$$\frac{2}{3}\left(x\arctan(x)-2\right)\sqrt{x}-\frac{2}{3}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x*arctan(x) - 2)*sqrt(x) - 2/3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqr
t(x) + x + 1) - sqrt(2)*sqrt(x) - 1) - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(
-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 1/6*sqrt(2)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) - 1/6*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 1.00, size = 86, normalized size = 0.74

$$\frac{2}{3}x^{\frac{3}{2}}\arctan(x)+\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{6}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

maple [A] time = 0.02, size = 74, normalized size = 0.63

$$\frac{2x^{\frac{3}{2}}\arctan(x)}{3}-\frac{4\sqrt{x}}{3}+\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{3}+\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{3}+\frac{\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arctan(x)-4/3*x^(1/2)+1/3*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+1/3
*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/6*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1
+x-2^(1/2)*x^(1/2)))
```

maxima [A] time = 0.41, size = 86, normalized size = 0.74

$$\frac{2}{3}x^{\frac{3}{2}}\arctan(x)+\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{6}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

mupad [B] time = 0.32, size = 49, normalized size = 0.42

$$\frac{2\,x^{3/2}\,\mathrm{atan}(x)}{3}-\frac{4\,\sqrt{x}}{3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atan(x),x)
```

```
[Out] (2*x^(3/2)*atan(x))/3 + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/3 + 1
i/3) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/3 - 1i/3) - (4*x^(1/2)
)/3
```

sympy [A] time = 3.22, size = 104, normalized size = 0.89

$$\frac{2x^{\frac{3}{2}}\operatorname{atan}(x)}{3}-\frac{4\sqrt{x}}{3}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)}{6}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{3}+\frac{\sqrt{2}\operatorname{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atan(x)/3 - 4*sqrt(x)/3 - sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)/
6 + sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1)/6 + sqrt(2)*atan(sqrt(2)*sqrt(x) -
 1)/3 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/3
```

3.54 $\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\tan^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$$

Mathematica [A] time = 4.12, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \arctan(cx)^3 + 3\,ab^2 \arctan(cx)^2 + 3\,a^2b \arctan(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 3.37, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan(cx)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx)^{m+1}a^3}{d(m+1)}+\frac{\frac{15}{2}b^3d^mxx^m\arctan(cx)^3-\frac{21}{8}b^3d^mxx^m\arctan(cx)\log\left(c^2x^2+1\right)^2+(m+1)\int\frac{84b^3c^2d^mx^2x^m\arctan(c}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x)^3 - 3*b^3
*d^m*x*x^m*arctan(c*x)*log(c^2*x^2 + 1)^2 + 32*(m + 1)*integrate(1/32*(12*b
^3*c^2*d^m*x^2*x^m*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^3*d^m*m + b^3*d^m +
 (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x)^3 - 12*(b^3*c*d^m*x - 8
*a*b^2*d^m*m - 8*a*b^2*d^m - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*a
rctan(c*x)^2 + 96*(a^2*b*d^m*m + a^2*b*d^m + (a^2*b*c^2*d^m*m + a^2*b*c^2*d
^m)*x^2)*x^m*arctan(c*x) + 3*(b^3*c*d^m*x*x^m + (b^3*d^m*m + b^3*d^m + (b^3
*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x))*log(c^2*x^2 + 1)^2)/((c^2*m
 + c^2)*x^2 + m + 1), x))/(m + 1)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (a+b\,\mathrm{atan}\left(c\,x\right))^3\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))^3*(d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\mathrm{atan}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x))**3, x)
```

3.55 $\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\tan^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 2.65, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\arctan(cx)^2 + 2ab\arctan(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 2.93, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx)^{m+1}\,a^2}{d(m+1)}+\frac{7\,b^2d^mxx^m\arctan(cx)^2-\frac{3}{4}\,b^2d^mxx^m\log\left(c^2x^2+1\right)^2+(m+1)\int\frac{12\,b^2c^2d^mx^2x^m\log\left(c^2x^2+1\right)+36\left(b^2d^mm+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x)^2 - b^2*d
^m*x*x^m*log(c^2*x^2 + 1)^2 + 16*(m + 1)*integrate(1/16*(4*b^2*c^2*d^m*x^2*
x^m*log(c^2*x^2 + 1) + 12*(b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^2*d
^m)*x^2)*x^m*arctan(c*x)^2 + (b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^
2*d^m)*x^2)*x^m*log(c^2*x^2 + 1)^2 - 8*(b^2*c*d^m*x - 4*a*b*d^m*m - 4*a*b*d
^m - 4*(a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(c*x))/((c^2*m + c^2)*x
^2 + m + 1), x))/(m + 1)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))^2*(d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\mathrm{atan}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x))**2, x)
```

3.56 $\int (dx)^m \left(a + b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=73

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{d^2(m+1)(m+2)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m
],[2+1/2*m],-c^2*x^2)/d^2/(1+m)/(2+m)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4852, 364}

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyperg
eometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d^2*(1 + m)*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tan^{-1}(cx)\right)dx&=\frac{(dx)^{1+m}\left(a+b\tan^{-1}(cx)\right)}{d(1+m)}-\frac{(bc)\int\frac{(dx)^{1+m}}{1+c^2x^2}\,dx}{d(1+m)}\\&=\frac{(dx)^{1+m}\left(a+b\tan^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{d^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.82

$$-\frac{x(dx)^m\left(bcx\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;-c^2x^2\right)-(m+2)\left(a+b\tan^{-1}(cx)\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((x*(d*x)^m*(-((2 + m)*(a + b*ArcTan[c*x])) + b*c*x*Hypergeometric2F1[1, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((b\arctan(cx)+a)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 2.77, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(d^m x x^m \arctan(cx)-(cd^m m+cd^m)\int \frac{xx^m}{(c^2m+c^2)x^2+m+1}\,dx\right)b}{m+1}+\frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] (d^m*x*x^m*arctan(c*x) - (c*d^m*m + c*d^m)*integrate(x*x^m/((c^2*m + c^2)*x
^2 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{atan}(c\,x))\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))*(d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\,\mathrm{atan}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x)), x)
```

3.57 $\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{a+b\tan^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x)),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x]), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\arctan(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctan(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\arctan(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{atan}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x)),x)
[Out] int((d*x)^m/(a + b*atan(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{atan}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x)),x)
[Out] Integral((d*x)**m/(a + b*atan(c*x)), x)
```

3.58 $\int \left(a+b\tan^{-1}(cx)\right)^p \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\left(a+b\tan^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*arctan(c*x))^p,x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\tan^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])^p,x]
[Out] Defer[Int][(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)^p \, dx=\int \left(a+b\tan^{-1}(cx)\right)^p \, dx$$

Mathematica [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^p,x]
[Out] Integrate[(a + b*ArcTan[c*x])^p, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((b\arctan(cx)+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.70, size = 0, normalized size = 0.00

$$\int (a+b\arctan(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^p,x)
[Out] int((a+b*arctan(c*x))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\arctan(cx) + a)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int (a + b\,\mathrm{atan}(c\,x))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^p,x)
[Out] int((a + b*atan(c*x))^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atan}(cx))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**p,x)
[Out] Integral((a + b*atan(c*x))**p, x)
```

3.59 $\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^p \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\tan^{-1}(cx)\right)^p, x\right)$$

[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^p,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^p,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^p, x]

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^p \, dx = \int (dx)^m \left(a+b\tan^{-1}(cx)\right)^p \, dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p, x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m (b\arctan(cx)+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="fricas")

[Out] integral((d*x)^m*(b*arctan(c*x) + a)^p, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="giac")

[Out] sage0*x

maple [A] time = 1.36, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\arctan(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^p,x)
[Out] int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b\arctan(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="maxima")
[Out] integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b\,\mathrm{atan}(c\,x))^p \, (d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^p*(d*x)^m,x)
[Out] int((a + b*atan(c*x))^p*(d*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**p,x)
[Out] Timed out
```

3.60 $\int x^7 \left(a+b \tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=54

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}+\frac{bx^2}{8c^3}-\frac{bx^6}{24c}$$

```
[Out] 1/8*b*x^2/c^3-1/24*b*x^6/c-1/8*b*arctan(c*x^2)/c^4+1/8*x^8*(a+b*arctan(c*x^
2))
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5033, 275, 302, 203}

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{bx^2}{8c^3}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}-\frac{bx^6}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*ArcTan[c*x^2])/(8*c^4) + (x^8*(a + b*
ArcTan[c*x^2]))/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\int\frac{x^9}{1+c^2x^4}\,dx\\
&= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\text{Subst}\left(\int\frac{x^4}{1+c^2x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)dx,x,x^2\right)\\
&= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{8c^3}\\
&= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.09

$$\frac{ax^8}{8}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}+\frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{1}{8}bx^8\tan^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) + (a*x^8)/8 - (b*ArcTan[c*x^2])/(8*c^4) +
(b*x^8*ArcTan[c*x^2])/8
```

fricas [A] time = 0.43, size = 51, normalized size = 0.94

$$\frac{3\,ac^4x^8-bc^3x^6+3\,bcx^2+3\left(bc^4x^8-b\right)\arctan\left(cx^2\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*a*c^4*x^8 - b*c^3*x^6 + 3*b*c*x^2 + 3*(b*c^4*x^8 - b)*arctan(c*x^2)
)/c^4
```

giac [A] time = 0.16, size = 60, normalized size = 1.11

$$\frac{3\,acx^8+\left(3\,cx^8\arctan\left(cx^2\right)-\frac{3\,\arctan\left(cx^2\right)}{c^3}-\frac{c^9x^6-3\,c^7x^2}{c^9}\right)b}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*c*x^8 + (3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x^6 -
 3*c^7*x^2)/c^9)*b)/c
```

maple [A] time = 0.03, size = 50, normalized size = 0.93

$$\frac{x^8a}{8}+\frac{b\,x^8\arctan\left(c\,x^2\right)}{8}-\frac{b\,x^6}{24c}+\frac{b\,x^2}{8c^3}-\frac{b\arctan\left(c\,x^2\right)}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctan(c*x^2)-1/24*b*x^6/c+1/8*b*x^2/c^3-1/8*b*arctan(c
*x^2)/c^4
```

maxima [A] time = 0.41, size = 54, normalized size = 1.00

$$\frac{1}{8}ax^8 + \frac{1}{24}\left(3x^8 \arctan\left(cx^2\right) - c\left(\frac{c^2x^6 - 3x^2}{c^4} + \frac{3\arctan\left(cx^2\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/24*(3*x^8*arctan(c*x^2) - c*((c^2*x^6 - 3*x^2)/c^4 + 3*arctan
(c*x^2)/c^5))*b
```

mupad [B] time = 0.36, size = 49, normalized size = 0.91

$$\frac{a\,x^8}{8} + \frac{b\,x^2}{8\,c^3} - \frac{b\,x^6}{24\,c} - \frac{b\,\mathrm{atan}\left(c\,x^2\right)}{8\,c^4} + \frac{b\,x^8\,\mathrm{atan}\left(c\,x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^8)/8 + (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*atan(c*x^2))/(8*c^4) + (b
*x^8*atan(c*x^2))/8
```

sympy [A] time = 65.10, size = 58, normalized size = 1.07

$$\begin{cases} \frac{ax^8}{8} + \frac{bx^8\,\mathrm{atan}\left(cx^2\right)}{8} - \frac{bx^6}{24c} + \frac{bx^2}{8c^3} - \frac{b\,\mathrm{atan}\left(cx^2\right)}{8c^4} & \text{for } c \neq 0 \\ \frac{ax^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atan(c*x**2)/8 - b*x**6/(24*c) + b*x**2/(8*c**
3) - b*atan(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

3.61 $\int x^5 \left(a+b \tan^{-1}\left(c x^2\right)\right) \, dx$

Optimal. Leaf size=47

$$\frac{1}{6} x^6 \left(a+b \tan^{-1}\left(c x^2\right)\right)+\frac{b \log \left(c^2 x^4+1\right)}{12 c^3}-\frac{b x^4}{12 c}$$

```
[Out] -1/12*b*x^4/c+1/6*x^6*(a+b*arctan(c*x^2))+1/12*b*ln(c^2*x^4+1)/c^3
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 43}

$$\frac{1}{6} x^6 \left(a+b \tan^{-1}\left(c x^2\right)\right)+\frac{b \log \left(c^2 x^4+1\right)}{12 c^3}-\frac{b x^4}{12 c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -(b*x^4)/(12*c) + (x^6*(a + b*ArcTan[c*x^2]))/6 + (b*Log[1 + c^2*x^4])/(12*
c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+b \tan^{-1}\left(c x^2\right)\right) \, dx &= \frac{1}{6} x^6 \left(a+b \tan^{-1}\left(c x^2\right)\right)-\frac{1}{3} (b c) \int \frac{x^7}{1+c^2 x^4} \, dx\\
&= \frac{1}{6} x^6 \left(a+b \tan^{-1}\left(c x^2\right)\right)-\frac{1}{12} (b c) \text{Subst}\left(\int \frac{x}{1+c^2 x} \, dx,x,x^4\right)\\
&= \frac{1}{6} x^6 \left(a+b \tan^{-1}\left(c x^2\right)\right)-\frac{1}{12} (b c) \text{Subst}\left(\int \left(\frac{1}{c^2}-\frac{1}{c^2 \left(1+c^2 x\right)}\right) \, dx,x,x^4\right)\\
&= -\frac{b x^4}{12 c}+\frac{1}{6} x^6 \left(a+b \tan^{-1}\left(c x^2\right)\right)+\frac{b \log \left(1+c^2 x^4\right)}{12 c^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.11

$$\frac{a x^6}{6}+\frac{b \log \left(c^2 x^4+1\right)}{12 c^3}-\frac{b x^4}{12 c}+\frac{1}{6} b x^6 \tan^{-1}\left(c x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/12*(b*x^4)/c + (a*x^6)/6 + (b*x^6*ArcTan[c*x^2])/6 + (b*Log[1 + c^2*x^4]
)/(12*c^3)
```

fricas [A] time = 0.43, size = 51, normalized size = 1.09

$$\frac{2\,bc^3x^6\arctan\left(cx^2\right)+2\,ac^3x^6-bc^2x^4+b\log\left(c^2x^4+1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b*c^3*x^6*arctan(c*x^2) + 2*a*c^3*x^6 - b*c^2*x^4 + b*log(c^2*x^4 +
 1))/c^3
```

giac [A] time = 0.17, size = 47, normalized size = 1.00

$$\frac{2\,acx^6+\left(2\,cx^6\arctan\left(cx^2\right)-x^4+\frac{\log\left(c^2x^4+1\right)}{c^2}\right)b}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*(2*a*c*x^6 + (2*c*x^6*arctan(c*x^2) - x^4 + log(c^2*x^4 + 1)/c^2)*b)/c
```

maple [A] time = 0.04, size = 45, normalized size = 0.96

$$\frac{x^6a}{6}+\frac{b\,x^6\arctan\left(c\,x^2\right)}{6}-\frac{b\,x^4}{12c}+\frac{b\ln\left(c^2x^4+1\right)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctan(c*x^2)-1/12*b*x^4/c+1/12*b*ln(c^2*x^4+1)/c^3
```

maxima [A] time = 0.31, size = 48, normalized size = 1.02

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\arctan\left(cx^2\right)-\left(\frac{x^4}{c^2}-\frac{\log\left(c^2x^4+1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c)
*b
```

mupad [B] time = 0.35, size = 44, normalized size = 0.94

$$\frac{a\,x^6}{6}+\frac{b\,\ln\left(c^2\,x^4+1\right)}{12\,c^3}-\frac{b\,x^4}{12\,c}+\frac{b\,x^6\,\mathrm{atan}\left(c\,x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*log(c^2*x^4 + 1))/(12*c^3) - (b*x^4)/(12*c) + (b*x^6*atan(c*
x^2))/6
```

sympy [A] time = 46.37, size = 80, normalized size = 1.70

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atan}\left(cx^2\right)}{6} - \frac{bx^4}{12c} + \frac{ib\sqrt{\frac{1}{c^2}} \operatorname{atan}\left(cx^2\right)}{6c^2} + \frac{b \log\left(x^2 + i\sqrt{\frac{1}{c^2}}\right)}{6c^3} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**2)/6 - b*x**4/(12*c) + I*b*sqrt(c**(
-2))*atan(c*x**2)/(6*c**2) + b*log(x**2 + I*sqrt(c**(-2)))/(6*c**3), Ne(c,
0)), (a*x**6/6, True))
```

3.62 $\int x^3 \left(a+b\tan^{-1}\left(cx^2\right)\right) \, dx$

Optimal. Leaf size=43

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\tan^{-1}\left(cx^2\right)}{4c^2}-\frac{bx^2}{4c}$$

```
[Out] -1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2+1/4*x^4*(a+b*arctan(c*x^2))
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 321, 203}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\tan^{-1}\left(cx^2\right)}{4c^2}-\frac{bx^2}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -(b*x^2)/(4*c) + (b*ArcTan[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2]))/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \left(a+b\tan^{-1}\left(cx^2\right)\right)dx = \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{2}(bc)\int\frac{x^5}{1+c^2x^4}\,dx$$
$$=\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\,\text{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^2\right)$$
$$=-\frac{bx^2}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\,\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{4c}$$
$$=-\frac{bx^2}{4c}+\frac{b\tan^{-1}\left(cx^2\right)}{4c^2}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.12

$$\frac{ax^4}{4}+\frac{b\tan^{-1}\left(cx^2\right)}{4c^2}-\frac{bx^2}{4c}+\frac{1}{4}bx^4\tan^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/4*(b*x^2)/c + (a*x^4)/4 + (b*ArcTan[c*x^2])/(4*c^2) + (b*x^4*ArcTan[c*x^
2])/4
```

fricas [A] time = 0.43, size = 38, normalized size = 0.88

$$\frac{ac^2x^4-bcx^2+\left(bc^2x^4+b\right)\arctan\left(cx^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*c^2*x^4 - b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2))/c^2
```

giac [A] time = 0.20, size = 43, normalized size = 1.00

$$\frac{acx^4+\frac{\left(c^2x^4\arctan\left(cx^2\right)-cx^2+\arctan\left(cx^2\right)\right)b}{c}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/4*(a*c*x^4 + (c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*b/c)/c
```

maple [A] time = 0.03, size = 41, normalized size = 0.95

$$\frac{x^4a}{4}+\frac{b\,x^4\arctan\left(c\,x^2\right)}{4}-\frac{b\,x^2}{4c}+\frac{b\arctan\left(c\,x^2\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctan(c*x^2)-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2
```

maxima [A] time = 0.41, size = 43, normalized size = 1.00

$$\frac{1}{4}ax^4+\frac{1}{4}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/4*(x^4*arctan(c*x^2) - c*(x^2/c^2 - arctan(c*x^2)/c^3))*b
```

mupad [B] time = 0.33, size = 40, normalized size = 0.93

$$\frac{a\,x^4}{4}-\frac{b\,x^2}{4\,c}+\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{4\,c^2}+\frac{b\,x^4\,\mathrm{atan}\left(c\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^4)/4 - (b*x^2)/(4*c) + (b*atan(c*x^2))/(4*c^2) + (b*x^4*atan(c*x^2))/4
```

sympy [A] time = 23.24, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4}+\frac{bx^4\,\mathrm{atan}\left(cx^2\right)}{4}-\frac{bx^2}{4c}+\frac{b\,\mathrm{atan}\left(cx^2\right)}{4c^2} & \text{for}\ c\neq 0\\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x**2)/4 - b*x**2/(4*c) + b*atan(c*x**2)
/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

3.63 $\int x \left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(c^2x^4+1\right)}{4c}$$

```
[Out] 1/2*x^2*(a+b*arctan(c*x^2))-1/4*b*ln(c^2*x^4+1)/c
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 260}

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(c^2x^4+1\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTan[c*x^2]))/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)-(bc)\int\frac{x^3}{1+c^2x^4}\,dx\\&=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(1+c^2x^4\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.14

$$\frac{ax^2}{2}-\frac{b\log\left(c^2x^4+1\right)}{4c}+\frac{1}{2}bx^2\tan^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTan[c*x^2])/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

fricas [A] time = 0.42, size = 39, normalized size = 1.08

$$\frac{2\,bcx^2\arctan\left(cx^2\right)+2\,acx^2-b\log\left(c^2x^4+1\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] 1/4*(2*b*c*x^2*arctan(c*x^2) + 2*a*c*x^2 - b*log(c^2*x^4 + 1))/c
```

giac [A] time = 0.15, size = 40, normalized size = 1.11

$$\frac{2\,acx^2 + \left(2\,cx^2 \arctan\left(cx^2\right) - \log\left(c^2x^4 + 1\right)\right)b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] 1/4*(2*a*c*x^2 + (2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b)/c
```

maple [A] time = 0.02, size = 36, normalized size = 1.00

$$\frac{a\,x^2}{2} + \frac{b\,x^2 \arctan\left(c\,x^2\right)}{2} - \frac{b\ln\left(c^2x^4 + 1\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2)),x)
[Out] 1/2*a*x^2+1/2*b*x^2*arctan(c*x^2)-1/4*b*ln(c^2*x^4+1)/c
```

maxima [A] time = 0.31, size = 38, normalized size = 1.06

$$\frac{1}{2}\,ax^2 + \frac{\left(2\,cx^2 \arctan\left(cx^2\right) - \log\left(c^2x^4 + 1\right)\right)b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b/c
```

mupad [B] time = 0.31, size = 35, normalized size = 0.97

$$\frac{a\,x^2}{2} - \frac{b\,\ln\left(c^2\,x^4 + 1\right)}{4\,c} + \frac{b\,x^2\,\mathrm{atan}\left(c\,x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^2)),x)
[Out] (a*x^2)/2 - (b*log(c^2*x^4 + 1))/(4*c) + (b*x^2*atan(c*x^2))/2
```

sympy [A] time = 13.50, size = 66, normalized size = 1.83

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2\,\mathrm{atan}\left(cx^2\right)}{2} - \frac{ib\sqrt{\frac{1}{c^2}}\,\mathrm{atan}\left(cx^2\right)}{2} - \frac{b\log\left(x^2 + i\sqrt{\frac{1}{c^2}}\right)}{2c} & \text{for}\ c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2)),x)
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x**2)/2 - I*b*sqrt(c**(-2))*atan(c*x**2
)/2 - b*log(x**2 + I*sqrt(c**(-2)))/(2*c), Ne(c, 0)), (a*x**2/2, True))
```

3.64 $\int \frac{a+b\tan^{-1}(cx^2)}{x}\,dx$

Optimal. Leaf size=39

$$a\log(x)+\frac{1}{4}ib\text{Li}_2\left(-icx^2\right)-\frac{1}{4}ib\text{Li}_2\left(icx^2\right)$$

```
[Out] a*ln(x)+1/4*I*b*polylog(2,-I*c*x^2)-1/4*I*b*polylog(2,I*c*x^2)
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5031, 4848, 2391}

$$\frac{1}{4}ib\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\text{PolyLog}\left(2,icx^2\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,x^2\right)\\ &= a\log(x)+\frac{1}{4}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,x^2\right)-\frac{1}{4}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,\right.\\ &= a\log(x)+\frac{1}{4}ib\text{Li}_2\left(-icx^2\right)-\frac{1}{4}ib\text{Li}_2\left(icx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$a\log(x)+\frac{1}{4}ib\text{Li}_2\left(-icx^2\right)-\frac{1}{4}ib\text{Li}_2\left(icx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\arctan\left(cx^2\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="fricas")
[Out] integral((b*arctan(c*x^2) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\arctan\left(cx^2\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)/x, x)
```

maple [C] time = 0.11, size = 63, normalized size = 1.62

$$a\ln(x)+b\ln(x)\arctan\left(c\,x^2\right)-\frac{b\left(\sum_{\_R1=\text{RootOf}\left(c^2\_Z^4+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x,x)
[Out] a*ln(x)+b*ln(x)*arctan(c*x^2)-1/2*b/c*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\arctan\left(cx^2\right)}{x}\,dx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="maxima")
[Out] b*integrate(arctan(c*x^2)/x, x) + a*log(x)
```

mupad [B] time = 0.33, size = 32, normalized size = 0.82

$$a\,\ln(x)-\frac{b\,\left(\mathrm{Li}_2\left(1-c\,x^2\,1\mathrm{i}\right)-\mathrm{Li}_2\left(1\mathrm{i}\,c\,x^2+1\right)\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x,x)
[Out] a*log(x) - (b*(dilog(1 - c*x^2*1i) - dilog(c*x^2*1i + 1))*1i)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{atan}\left(cx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))/x, x)
```

3.65 $\int \frac{a+b\tan^{-1}(cx^2)}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(c^2x^4+1\right)+bc\log(x)$$

```
[Out] 1/2*(-a-b*arctan(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(c^2*x^4+1)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5033, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(c^2x^4+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^3,x]
```

```
[Out] -(a + b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^3}\,dx = -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2} + (bc)\int \frac{1}{x\left(1+c^2x^4\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) - \frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x,\right.$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2} + bc\log(x) - \frac{1}{4}bc\log\left(1+c^2x^4\right)$$

Mathematica [**A**] time = 0.01, size = 44, normalized size = 1.13

$$-\frac{a}{2x^2} - \frac{1}{4}bc\log\left(c^2x^4+1\right) - \frac{b\tan^{-1}\left(cx^2\right)}{2x^2} + bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4]
)/4
```

fricas [**A**] time = 0.44, size = 43, normalized size = 1.10

$$-\frac{bcx^2\log\left(c^2x^4+1\right) - 4\,bcx^2\log(x) + 2\,b\arctan\left(cx^2\right) + 2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2*log(c^2*x^4 + 1) - 4*b*c*x^2*log(x) + 2*b*arctan(c*x^2) + 2*a
)/x^2
```

giac [**A**] time = 0.17, size = 60, normalized size = 1.54

$$-\frac{bc^3x^2\log\left(c^2x^4+1\right) - 2\,bc^3x^2\log\left(cx^2\right) + 2\,bc^2\arctan\left(cx^2\right) + 2\,ac^2}{4\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*c^3*x^2*log(c^2*x^4 + 1) - 2*b*c^3*x^2*log(c*x^2) + 2*b*c^2*arctan(
c*x^2) + 2*a*c^2)/(c^2*x^2)
```

maple [**A**] time = 0.03, size = 39, normalized size = 1.00

$$-\frac{a}{2x^2} - \frac{b\arctan\left(c\,x^2\right)}{2x^2} + bc\ln(x) - \frac{bc\ln\left(c^2x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x^2)+b*c*ln(x)-1/4*b*c*ln(c^2*x^4+1)
```

maxima [A] time = 0.32, size = 41, normalized size = 1.05

$$-\frac{1}{4}\left(c\left(\log\left(c^2x^4+1\right)-\log\left(x^4\right)\right)+\frac{2\arctan\left(cx^2\right)}{x^2}\right)b-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="maxima")
[Out] -1/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*b - 1/2*a/x^2
```

mupad [B] time = 0.34, size = 38, normalized size = 0.97

$$b\,c\,\ln(x)-\frac{a}{2\,x^2}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{2\,x^2}-\frac{b\,c\,\ln\left(c^2\,x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^3,x)
[Out] b*c*log(x) - a/(2*x^2) - (b*atan(c*x^2))/(2*x^2) - (b*c*log(c^2*x^4 + 1))/4
```

sympy [A] time = 30.28, size = 75, normalized size = 1.92

$$\begin{cases} -\frac{a}{2x^2}+bc\log\left(x\right)-\frac{bc\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{2}-\frac{ib\,\mathrm{atan}\left(cx^2\right)}{2\sqrt{\frac{1}{c^2}}}-\frac{b\,\mathrm{atan}\left(cx^2\right)}{2x^2} & \text{for } c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**3,x)
[Out] Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x**2 + I*sqrt(c**(-2)))/2 - I
*b*atan(c*x**2)/(2*sqrt(c**(-2))) - b*atan(c*x**2)/(2*x**2), Ne(c, 0)), (-a
/(2*x**2), True))
```

3.66 $\int \frac{a+b\tan^{-1}(cx^2)}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}-\frac{1}{4}bc^2\tan^{-1}\left(cx^2\right)-\frac{bc}{4x^2}$$

```
[Out] -1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)+1/4*(-a-b*arctan(c*x^2))/x^4
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 325, 203}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}-\frac{1}{4}bc^2\tan^{-1}\left(cx^2\right)-\frac{bc}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^5,x]
```

```
[Out] -(b*c)/(4*x^2) - (b*c^2*ArcTan[c*x^2])/4 - (a + b*ArcTan[c*x^2])/(4*x^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^5}\,dx = -\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{2}(bc)\int \frac{1}{x^3\left(1+c^2x^4\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx,x,x^2\right)$$
$$= -\frac{bc}{4x^2}-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}-\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^2\right)$$
$$= -\frac{bc}{4x^2}-\frac{1}{4}bc^2\tan^{-1}\left(cx^2\right)-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 1.17

$$-\frac{a}{4x^4}-\frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^4\right)}{4x^2}-\frac{b\tan^{-1}\left(cx^2\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*ArcTan[c*x^2])/(4*x^4) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^4)])/(4*x^2)
```

fricas [A] time = 0.40, size = 30, normalized size = 0.73

$$-\frac{bcx^2+\left(bc^2x^4+b\right)\arctan\left(cx^2\right)+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2) + a)/x^4
```

giac [B] time = 2.81, size = 74, normalized size = 1.80

$$\frac{bc^5ix^4\log\left(cix^2+1\right)-bc^5ix^4\log\left(-cix^2+1\right)-2\,bc^4x^2-2\,bc^3\arctan\left(cx^2\right)-2\,ac^3}{8\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(b*c^5*i*x^4*log(c*i*x^2 + 1) - b*c^5*i*x^4*log(-c*i*x^2 + 1) - 2*b*c^4
*x^2 - 2*b*c^3*arctan(c*x^2) - 2*a*c^3)/(c^3*x^4)
```

maple [A] time = 0.03, size = 39, normalized size = 0.95

$$-\frac{a}{4x^4}-\frac{b\arctan\left(c\,x^2\right)}{4x^4}-\frac{b\,c^2\arctan\left(c\,x^2\right)}{4}-\frac{bc}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x^2)-1/4*b*c^2*arctan(c*x^2)-1/4*b*c/x^2
```

maxima [A] time = 0.42, size = 35, normalized size = 0.85

$$-\frac{1}{4}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*b - 1/4*a/x^4
```

mupad [B] time = 0.36, size = 41, normalized size = 1.00

$$-\frac{\frac{b\,c\,x^2}{2}+\frac{a}{2}}{2\,x^4}-\frac{b\,c^2\operatorname{atan}\left(c\,x^2\right)}{4}-\frac{b\operatorname{atan}\left(c\,x^2\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^5,x)
```

```
[Out] - (a/2 + (b*c*x^2)/2)/(2*x^4) - (b*c^2*atan(c*x^2))/4 - (b*atan(c*x^2))/(4*
x^4)
```

sympy [A] time = 24.12, size = 42, normalized size = 1.02

$$-\frac{a}{4x^4}-\frac{bc^2\operatorname{atan}\left(cx^2\right)}{4}-\frac{bc}{4x^2}-\frac{b\operatorname{atan}\left(cx^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**5,x)
```

```
[Out] -a/(4*x**4) - b*c**2*atan(c*x**2)/4 - b*c/(4*x**2) - b*atan(c*x**2)/(4*x**4
)
```

3.67 $\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^7}\,dx$

Optimal. Leaf size=55

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(c^2x^4+1\right)-\frac{bc}{12x^4}$$

```
[Out] -1/12*b*c/x^4+1/6*(-a-b*arctan(c*x^2))/x^6-1/3*b*c^3*ln(x)+1/12*b*c^3*ln(c^
2*x^4+1)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 44}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{12}bc^3\log\left(c^2x^4+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^7,x]
```

```
[Out] -(b*c)/(12*x^4) - (a + b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^4])/12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^7}\,dx &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{3}(bc)\int \frac{1}{x^5\left(1+c^2x^4\right)}\,dx\\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{12}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^4\right)\\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{12}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^4\right)\\
&= -\frac{bc}{12x^4}-\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(1+c^2x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.09

$$-\frac{a}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(c^2x^4+1\right)-\frac{bc}{12x^4}-\frac{b\tan^{-1}\left(cx^2\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^7,x]
```

```
[Out] -1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3
+ (b*c^3*Log[1 + c^2*x^4])/12
```

fricas [A] time = 0.44, size = 54, normalized size = 0.98

$$\frac{bc^3x^6\log\left(c^2x^4+1\right)-4\,bc^3x^6\log(x)-bcx^2-2\,b\arctan\left(cx^2\right)-2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^6*log(c^2*x^4 + 1) - 4*b*c^3*x^6*log(x) - b*c*x^2 - 2*b*arcta
n(c*x^2) - 2*a)/x^6
```

giac [A] time = 0.14, size = 69, normalized size = 1.25

$$\frac{bc^7x^6\log\left(c^2x^4+1\right)-2\,bc^7x^6\log\left(cx^2\right)-bc^5x^2-2\,bc^4\arctan\left(cx^2\right)-2\,ac^4}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(b*c^7*x^6*log(c^2*x^4 + 1) - 2*b*c^7*x^6*log(c*x^2) - b*c^5*x^2 - 2*b
*c^4*arctan(c*x^2) - 2*a*c^4)/(c^4*x^6)
```

maple [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{a}{6x^6}-\frac{b\arctan\left(c\,x^2\right)}{6x^6}-\frac{bc}{12x^4}-\frac{b\,c^3\ln(x)}{3}+\frac{b\,c^3\ln\left(c^2x^4+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctan(c*x^2)-1/12*b*c/x^4-1/3*b*c^3*ln(x)+1/12*b*c^3*
ln(c^2*x^4+1)
```

maxima [A] time = 0.32, size = 53, normalized size = 0.96

$$\frac{1}{12}\left(\left(c^2\log\left(c^2x^4+1\right)-c^2\log\left(x^4\right)-\frac{1}{x^4}\right)c-\frac{2\arctan\left(cx^2\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*((c^2*log(c^2*x^4 + 1) - c^2*log(x^4) - 1/x^4)*c - 2*arctan(c*x^2)/x^6
)*b - 1/6*a/x^6
```

mupad [B] time = 0.37, size = 50, normalized size = 0.91

$$\frac{b\,c^3\,\ln\left(c^2\,x^4+1\right)}{12}-\frac{a}{6\,x^6}-\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{6\,x^6}-\frac{b\,c}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^7,x)
```

```
[Out] (b*c^3*log(c^2*x^4 + 1))/12 - a/(6*x^6) - (b*c^3*log(x))/3 - (b*atan(c*x^2)
)/(6*x^6) - (b*c)/(12*x^4)
```

sympy [A] time = 81.39, size = 92, normalized size = 1.67

$$\begin{cases} -\frac{a}{6x^6} - \frac{bc^3 \log(x)}{3} + \frac{bc^3 \log\left(x^2 + i\sqrt{\frac{1}{c^2}}\right)}{6} + \frac{ibc^2 \operatorname{atan}(cx^2)}{6\sqrt{\frac{1}{c^2}}} - \frac{bc}{12x^4} - \frac{b \operatorname{atan}(cx^2)}{6x^6} & \text{for } c \neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) - b*c**3*log(x)/3 + b*c**3*log(x**2 + I*sqrt(c**(-2)
))/6 + I*b*c**2*atan(c*x**2)/(6*sqrt(c**(-2))) - b*c/(12*x**4) - b*atan(c*x
**2)/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

3.68 $\int x^4 \left(a + b\tan^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=161

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\tan^{-}}{}$$

```
[Out] -2/15*b*x^3/c+1/5*x^5*(a+b*arctan(c*x^2))+1/10*b*arctan(-1+x*2^(1/2)*c^(1/2
))/c^(5/2)*2^(1/2)+1/10*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)+1/20*
b*ln(1+c*x^2-x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)-1/20*b*ln(1+c*x^2+x*2^(1/2)
*c^(1/2))/c^(5/2)*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (x^5*(a + b*ArcTan[c*x^2]))/5 - (b*ArcTan[1 - Sqrt[2]*S
qrt[c]*x])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(5*Sqrt[
2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2)) -
 (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{5}(2bc)\int\frac{x^6}{1+c^2x^4}\,dx\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{(2b)\int\frac{x^2}{1+c^2x^4}\,dx}{5c}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{5c^2}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{5c^2}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3}+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3}+\frac{b\int\frac{\sqrt{}}{-\frac{1}{c}-}}{10\sqrt{}}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\log\left(1+\sqrt{2}\sqrt{c}\,x\right.}{10\sqrt{2}\,c^{5/2}}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\tan^{-1}\left(1+\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 179, normalized size = 1.11

$$\frac{ax^5}{5}+\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}\,x-\sqrt{2}}{\sqrt{2}}\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}\,x+\sqrt{2}}{\sqrt{2}}\right)}{5\sqrt{2}\,c^{5/2}}-\frac{2bx^3}{15c}+\frac{1}{5}bx$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^2])/5 + (b*ArcTan[(-Sqrt[
2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[(Sqrt[2] + 2*Sq
```

```
rt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x
^2])/(10*Sqrt[2]*c^(5/2)) - (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt
[2]*c^(5/2))
```

fricas [B] time = 0.46, size = 372, normalized size = 2.31

$$\frac{12\,bcx^5\arctan\left(cx^2\right)+12\,acx^5-8\,bx^3-12\,\sqrt{2}\,c\left(\frac{b^4}{c^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,b^3c^3x\left(\frac{b^4}{c^{10}}\right)^{\frac{1}{4}}-\sqrt{2}\,\sqrt{\sqrt{2}\,b^3c^7x\left(\frac{b^4}{c^{10}}\right)^{\frac{3}{4}}+b^4c^4\sqrt{\frac{b^4}{c^{10}}}+b^6x^2}}{b^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b*c*x^5*arctan(c*x^2) + 12*a*c*x^5 - 8*b*x^3 - 12*sqrt(2)*c*(b^4/c
^10)^(1/4)*arctan(-(sqrt(2)*b^3*c^3*x*(b^4/c^10)^(1/4) - sqrt(2)*sqrt(sqrt(
2)*b^3*c^7*x*(b^4/c^10)^(3/4) + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2)*c^3*(b^4/
c^10)^(1/4) + b^4)/b^4) - 12*sqrt(2)*c*(b^4/c^10)^(1/4)*arctan(-(sqrt(2)*b^
3*c^3*x*(b^4/c^10)^(1/4) - sqrt(2)*sqrt(-sqrt(2)*b^3*c^7*x*(b^4/c^10)^(3/4)
 + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2)*c^3*(b^4/c^10)^(1/4) - b^4)/b^4) - 3*s
qrt(2)*c*(b^4/c^10)^(1/4)*log(sqrt(2)*b^3*c^7*x*(b^4/c^10)^(3/4) + b^4*c^4*
sqrt(b^4/c^10) + b^6*x^2) + 3*sqrt(2)*c*(b^4/c^10)^(1/4)*log(-sqrt(2)*b^3*c
^7*x*(b^4/c^10)^(3/4) + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2))/c
```

giac [A] time = 2.85, size = 169, normalized size = 1.05

$$\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{2}\,\sqrt{|c|}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}}+\frac{2\,\sqrt{2}\,\sqrt{|c|}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}}-\frac{\sqrt{2}\,\log\left(x^2+\right.}{c^{10}|c|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/c^12 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^12 - sqrt(2)*log(x^2 + sqrt(2)*x/s
qrt(abs(c)) + 1/abs(c))/(c^10*abs(c)^(3/2)) + sqrt(2)*sqrt(abs(c))*log(x^2
- sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^12) + 1/15*(3*b*c*x^5*arctan(c*x^2)
+ 3*a*c*x^5 - 2*b*x^3)/c
```

maple [A] time = 0.04, size = 140, normalized size = 0.87

$$\frac{a\,x^5}{5}+\frac{b\,x^5\arctan\left(c\,x^2\right)}{5}-\frac{2b\,x^3}{15c}+\frac{b\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\right.}{10c^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctan(c*x^2)-2/15*b*x^3/c+1/20*b/c^3/(1/c^2)^(1/4)*2^(
1/2)*ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^
(1/2)+(1/c^2)^(1/2)))+1/10*b/c^3/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^
2)^(1/4)*x+1)+1/10*b/c^3/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)
*x-1)
```

maxima [A] time = 0.45, size = 147, normalized size = 0.91

$$\frac{1}{5}ax^5+\frac{1}{60}\left(12x^5\arctan\left(cx^2\right)-c\left(\frac{8x^3}{c^2}-\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2cx+\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2cx-\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x\right.}{c^{\frac{3}{2}}}\right.}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/60*(12*x^5*arctan(c*x^2) - c*(8*x^3/c^2 - 3*(2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) + 2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) - sqrt(2)*log(c*x^2
+ sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x +
1)/c^(3/2))/c^2))*b
```

mupad [B] time = 0.36, size = 64, normalized size = 0.40

$$\frac{a\,x^5}{5}-\frac{2\,b\,x^3}{15\,c}+\frac{b\,x^5\,\mathrm{atan}\left(c\,x^2\right)}{5}+\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{5\,c^{5/2}}+\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^5)/5 - (2*b*x^3)/(15*c) + (b*x^5*atan(c*x^2))/5 + ((-1)^(1/4)*b*atan((
-1)^(1/4)*c^(1/2)*x))/(5*c^(5/2)) + ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x
*1i)*1i)/(5*c^(5/2))
```

sympy [A] time = 33.03, size = 184, normalized size = 1.14

$$\begin{cases}\frac{ax^5}{5}+\frac{bx^5\,\mathrm{atan}\left(cx^2\right)}{5}-\frac{2bx^3}{15c}-\frac{(-1)^{\frac{3}{4}}b\log\left(x-\sqrt[4]{-1}\sqrt[4]{\frac{1}{c^2}}\right)}{5c^3\sqrt[4]{\frac{1}{c^2}}}+\frac{(-1)^{\frac{3}{4}}b\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{10c^3\sqrt[4]{\frac{1}{c^2}}}+\frac{(-1)^{\frac{3}{4}}b\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}x}{\sqrt[4]{\frac{1}{c^2}}}\right)}{5c^3\sqrt[4]{\frac{1}{c^2}}}-\frac{\sqrt[4]{-1}\,b\,\mathrm{atan}\left(cx^2\right)}{5c^6\left(\frac{1}{c^2}\right)^{\frac{7}{4}}} & \text{for } c \\ \frac{ax^5}{5} & \text{other}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x**2)/5 - 2*b*x**3/(15*c) - (-1)**(3/4)
*b*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(5*c**3*(c**(-2))**(1/4)) + (-1)**
(3/4)*b*log(x**2 + I*sqrt(c**(-2)))/(10*c**3*(c**(-2))**(1/4)) + (-1)**(3/4
)*b*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(5*c**3*(c**(-2))**(1/4)) - (-1)**
(1/4)*b*atan(c*x**2)/(5*c**6*(c**(-2))**(7/4)), Ne(c, 0)), (a*x**5/5, True)
)
```

3.69 $\int x^2 \left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=159

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}\,c^{3/2}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}\,c^{3/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}\,c^{3/2}}+\frac{b\tan^{-1}}{}$$

```
[Out] -2/3*b*x/c+1/3*x^3*(a+b*arctan(c*x^2))+1/6*b*arctan(-1+x*2^(1/2)*c^(1/2))/c
^(3/2)*2^(1/2)+1/6*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)-1/12*b*ln(
1+c*x^2-x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)+1/12*b*ln(1+c*x^2+x*2^(1/2)*c^(1
/2))/c^(3/2)*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}\,c^{3/2}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}\,c^{3/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}\,c^{3/2}}+\frac{b\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (x^3*(a + b*ArcTan[c*x^2]))/3 - (b*ArcTan[1 - Sqrt[2]*Sqrt
[c]*x])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[2]*
c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2)) + (b*
Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)dx=\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{3}(2bc)\int\frac{x^4}{1+c^2x^4}\,dx$$

$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{(2b)\int\frac{1}{1+c^2x^4}\,dx}{3c}$$

$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{3c}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{3c}$$

$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}-\frac{b\int\frac{\frac{\sqrt{2}}{\sqrt{c}}}{-\frac{1}{c}-\cdots}}{6\sqrt{2}\cdots}$$

$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x\cdots\right)}{6\sqrt{2}c^{3/2}}$$

$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}\cdots$$

Mathematica [A] time = 0.04, size = 177, normalized size = 1.11

$$\frac{ax^3}{3}-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}x-\sqrt{2}}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}x+\sqrt{2}}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}+\frac{1}{3}bx^3\tan^{-1}\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (a*x^3)/3 + (b*x^3*ArcTan[c*x^2])/3 + (b*ArcTan[(-Sqrt[2]
+ 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[(Sqrt[2] + 2*Sqrt[
```

```
c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(6*Sqrt[2]*c^(3/2)) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c
^(3/2))
```

fricas [B] time = 0.44, size = 337, normalized size = 2.12

$$4\,bcx^{3}\arctan\left(cx^{2}\right)+4\,acx^{3}-4\,\sqrt{2}\,c\left(\frac{b^{4}}{c^{6}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,bc^{5}x\left(\frac{b^{4}}{c^{6}}\right)^{\frac{3}{4}}-\sqrt{2}\,\sqrt{b^{2}x^{2}+\sqrt{2}\,bcx\left(\frac{b^{4}}{c^{6}}\right)^{\frac{1}{4}}+c^{2}\sqrt{\frac{b^{4}}{c^{6}}}}\,c^{5}\left(\frac{b^{4}}{c^{6}}\right)^{\frac{3}{4}}+b^{4}}{b^{4}}\right)-4\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*b*c*x^3*arctan(c*x^2) + 4*a*c*x^3 - 4*sqrt(2)*c*(b^4/c^6)^(1/4)*arc
tan(-(sqrt(2)*b*c^5*x*(b^4/c^6)^(3/4) - sqrt(2)*sqrt(b^2*x^2 + sqrt(2)*b*c*
x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6))*c^5*(b^4/c^6)^(3/4) + b^4)/b^4) - 4*
sqrt(2)*c*(b^4/c^6)^(1/4)*arctan(-(sqrt(2)*b*c^5*x*(b^4/c^6)^(3/4) - sqrt(2
)*sqrt(b^2*x^2 - sqrt(2)*b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6))*c^5*(b^
4/c^6)^(3/4) - b^4)/b^4) + sqrt(2)*c*(b^4/c^6)^(1/4)*log(b^2*x^2 + sqrt(2)*
b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6)) - sqrt(2)*c*(b^4/c^6)^(1/4)*log(
b^2*x^2 - sqrt(2)*b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6)) - 8*b*x)/c
```

giac [A] time = 3.07, size = 165, normalized size = 1.04

$$\frac{1}{12}\,bc^{5}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{6}\sqrt{|c|}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{6}\sqrt{|c|}}+\frac{\sqrt{2}\,\log\left(x^{2}+\frac{\sqrt{2}\,x}{\sqrt{|c|}}+\frac{1}{|c|}\right.}{c^{6}\sqrt{|c|}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*c^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(
abs(c)))/(c^6*sqrt(abs(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/s
qrt(abs(c)))*sqrt(abs(c)))/(c^6*sqrt(abs(c))) + sqrt(2)*log(x^2 + sqrt(2)*x
/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c))) - sqrt(2)*log(x^2 - sqrt(2)*x/
sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c)))) + 1/3*(b*c*x^3*arctan(c*x^2) +
 a*c*x^3 - 2*b*x)/c
```

maple [A] time = 0.03, size = 138, normalized size = 0.87

$$\frac{x^{3}a}{3}+\frac{b\,x^{3}\arctan\left(c\,x^{2}\right)}{3}-\frac{2bx}{3c}+\frac{b\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)}{12c}+\frac{b\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)}{6c}+\frac{b\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c*x^2)-2/3*b*x/c+1/12*b/c*(1/c^2)^(1/4)*2^(1/2)*
ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)
+(1/c^2)^(1/2)))+1/6*b/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)
*x+1)+1/6*b/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)
```

maxima [A] time = 0.43, size = 145, normalized size = 0.91

$$\frac{1}{3}ax^3+\frac{1}{12}\left(4x^3\arctan\left(cx^2\right)-c\left(\frac{8x}{c^2}-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}}-\cdots}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2*
sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2*s
qrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 + sqr
t(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/sq
rt(c))/c^2))*b
```

mupad [B] time = 0.39, size = 62, normalized size = 0.39

$$\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}\left(c\,x^2\right)}{3}-\frac{2\,b\,x}{3\,c}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{3\,c^{3/2}}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{3\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^3)/3 + (b*x^3*atan(c*x^2))/3 - (2*b*x)/(3*c) - ((-1)^(1/4)*b*atan((-1)
^(1/4)*c^(1/2)*x)*1i)/(3*c^(3/2)) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x
*1i))/(3*c^(3/2))
```

sympy [A] time = 17.68, size = 173, normalized size = 1.09

$$\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{atan}\left(cx^2\right)}{3}+\frac{(-1)^{\frac{3}{4}}b\left(\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(cx^2\right)}{3}-\frac{2bx}{3c}-\frac{\sqrt[4]{-1}\,b\sqrt[4]{\frac{1}{c^2}}\log\left(x-\sqrt[4]{-1}\sqrt[4]{\frac{1}{c^2}}\right)}{3c}+\frac{\sqrt[4]{-1}\,b\sqrt[4]{\frac{1}{c^2}}\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{6c}-\frac{\sqrt[4]{-1}\,b\sqrt[4]{\frac{1}{c^2}}\operatorname{atan}\cdots}{3c} \\ \frac{ax^3}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x**2)/3 + (-1)**(3/4)*b*(c**(-2))**(3/4
)*atan(c*x**2)/3 - 2*b*x/(3*c) - (-1)**(1/4)*b*(c**(-2))**(1/4)*log(x - (-1
)**(1/4)*(c**(-2))**(1/4))/(3*c) + (-1)**(1/4)*b*(c**(-2))**(1/4)*log(x**2
+ I*sqrt(c**(-2)))/(6*c) - (-1)**(1/4)*b*(c**(-2))**(1/4)*atan((-1)**(3/4)*
x/(c**(-2))**(1/4))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

3.70 $\int \left(a + b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=140

$$ax - \frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}\sqrt{c}} + \frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}\sqrt{c}} + bx\tan^{-1}\left(cx^2\right) + \frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}\sqrt{c}} - \frac{b\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}}$$

```
[Out] a*x+b*x*arctan(c*x^2)-1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/
2*b*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/2)*
c^(1/2))*2^(1/2)/c^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2
)
```

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5027, 297, 1162, 617, 204, 1165, 628}

$$ax - \frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}\sqrt{c}} + \frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}\sqrt{c}} + bx\tan^{-1}\left(cx^2\right) + \frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}\sqrt{c}} - \frac{b\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^2] + (b*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c
]) - (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*Log[1 - Sqrt[
2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*Sqrt[c]) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x +
 c*x^2])/(2*Sqrt[2]*Sqrt[c])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(cx^2\right)\right)dx = ax+b\int \tan^{-1}\left(cx^2\right)dx$$
$$= ax+bx\tan^{-1}\left(cx^2\right)-(2bc)\int \frac{x^2}{1+c^2x^4}dx$$
$$= ax+bx\tan^{-1}\left(cx^2\right)+b\int \frac{1-cx^2}{1+c^2x^4}dx-b\int \frac{1+cx^2}{1+c^2x^4}dx$$
$$= ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}dx}{2c}-\frac{b\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}dx}{2c}-\frac{b\int \frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}dx}{2\sqrt{2}\sqrt{c}}-\frac{b\int}{}$$
$$= ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}-\frac{b\,\mathrm{Sub}}{}$$
$$= ax+bx\tan^{-1}\left(cx^2\right)+\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\log\left(1-\sqrt{2}\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.76

$$ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\left(\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)-\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)+2\tan^{-1}\left(\sqrt{2}\right.\right.}{2\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^2] - (b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[1
 + Sqrt[2]*Sqrt[c]*x] + Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt[2
]*Sqrt[c]*x + c*x^2]))/(2*Sqrt[2]*Sqrt[c])
```

fricas [B] time = 0.44, size = 319, normalized size = 2.28

$$bx\arctan\left(cx^2\right)+ax+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}b^3cx+b^4-\sqrt{2}\sqrt{b^6x^2+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{3}{4}}b^3cx+\sqrt{\frac{b^4}{c^2}}b^4}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}c}{b^4}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="fricas")
```

```
[Out] b*x*arctan(c*x^2) + a*x + sqrt(2)*(b^4/c^2)^(1/4)*arctan(-(sqrt(2)*(b^4/c^2
)^(1/4)*b^3*c*x + b^4 - sqrt(2)*sqrt(b^6*x^2 + sqrt(2)*(b^4/c^2)^(3/4)*b^3*
c*x + sqrt(b^4/c^2)*b^4)*(b^4/c^2)^(1/4)*c)/b^4) + sqrt(2)*(b^4/c^2)^(1/4)*
arctan(-(sqrt(2)*(b^4/c^2)^(1/4)*b^3*c*x - b^4 - sqrt(2)*sqrt(b^6*x^2 - sqr
t(2)*(b^4/c^2)^(3/4)*b^3*c*x + sqrt(b^4/c^2)*b^4)*(b^4/c^2)^(1/4)*c)/b^4) +
 1/4*sqrt(2)*(b^4/c^2)^(1/4)*log(b^6*x^2 + sqrt(2)*(b^4/c^2)^(3/4)*b^3*c*x
+ sqrt(b^4/c^2)*b^4) - 1/4*sqrt(2)*(b^4/c^2)^(1/4)*log(b^6*x^2 - sqrt(2)*(b
^4/c^2)^(3/4)*b^3*c*x + sqrt(b^4/c^2)*b^4)
```

giac [A] time = 0.18, size = 149, normalized size = 1.06

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{|c|}\log\left(x^2\right.}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(
c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 + sqr
t(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 4*x*arctan(c*x^2))*b + a*x
```

maple [A] time = 0.03, size = 125, normalized size = 0.89

$$ax+bx\arctan\left(c\,x^2\right)-\frac{b\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x^2),x)
```

```
[Out] a*x+b*x*arctan(c*x^2)-1/4*b/c/(1/c^2)^(1/4)*2^(1/2)*ln((x^2-(1/c^2)^(1/4)*x
*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))-1/2*b/
c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/2*b/c/(1/c^2)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)
```

maxima [A] time = 0.43, size = 127, normalized size = 0.91

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\log\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^
(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(
3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x
^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*b + a*x
```

mupad [**B**] time = 0.39, size = 49, normalized size = 0.35

$$a\,x+b\,x\,\mathrm{atan}\left(c\,x^2\right)-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c*x^2),x)
```

```
[Out] a*x + b*x*atan(c*x^2) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x))/c^(1/2) -
 ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x*1i)*1i)/c^(1/2)
```

sympy [**A**] time = 10.23, size = 930, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((-x*atan((-sqrt(2)/2 - sqrt(2)*I/2)**(-2)), Eq(c, -1/(x**
2*(-sqrt(2)/2 - sqrt(2)*I/2)**2))), (-x*atan((-sqrt(2)/2 + sqrt(2)*I/2)**(-
2)), Eq(c, -1/(x**2*(-sqrt(2)/2 + sqrt(2)*I/2)**2))), (-x*atan((sqrt(2)/2 -
 sqrt(2)*I/2)**(-2)), Eq(c, -1/(x**2*(sqrt(2)/2 - sqrt(2)*I/2)**2))), (-x*a
tan((sqrt(2)/2 + sqrt(2)*I/2)**(-2)), Eq(c, -1/(x**2*(sqrt(2)/2 + sqrt(2)*I
/2)**2))), (0, Eq(c, 0)), (-2*(-1)**(3/4)*c**5*x**5*(c**(-2))**(7/4)*atan(c
*x**2)/(-2*(-1)**(3/4)*c**5*x**4*(c**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**
(-2))**(7/4)) + 2*I*c**4*x**4*(c**(-2))**(3/2)*log(x - (-1)**(1/4)*(c**(-2)
)**(1/4))/(-2*(-1)**(3/4)*c**5*x**4*(c**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(
c**(-2))**(7/4)) - I*c**4*x**4*(c**(-2))**(3/2)*log(x**2 + I*sqrt(c**(-2)))
/(-2*(-1)**(3/4)*c**5*x**4*(c**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**(-2))*
*(7/4)) - 2*I*c**4*x**4*(c**(-2))**(3/2)*atan((-1)**(3/4)*x/(c**(-2))**(1/4
))/(-2*(-1)**(3/4)*c**5*x**4*(c**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**(-2)
)**(7/4)) - 2*(-1)**(3/4)*c**3*x*(c**(-2))**(7/4)*atan(c*x**2)/(-2*(-1)**(3
/4)*c**5*x**4*(c**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**(-2))**(7/4)) + 2*I
*c**2*(c**(-2))**(3/2)*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-2*(-1)**(3/4
)*c**5*x**4*(c**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**(-2))**(7/4)) - I*c**
2*(c**(-2))**(3/2)*log(x**2 + I*sqrt(c**(-2)))/(-2*(-1)**(3/4)*c**5*x**4*(c
**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**(-2))**(7/4)) - 2*I*c**2*(c**(-2))*
*(3/2)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-2*(-1)**(3/4)*c**5*x**4*(c**(
-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**(-2))**(7/4)) + 2*c*x**4*atan(c*x**2)/
(-2*(-1)**(3/4)*c**5*x**4*(c**(-2))**(7/4) - 2*(-1)**(3/4)*c**3*(c**(-2))**
(7/4)) + 2*atan(c*x**2)/(-2*(-1)**(3/4)*c**6*x**4*(c**(-2))**(7/4) - 2*(-1)
**(3/4)*c**4*(c**(-2))**(7/4)), True))
```

3.71 $\int \frac{a+b\tan^{-1}(cx^2)}{x^2}\,dx$

Optimal. Leaf size=143

$$-\frac{a+b\tan^{-1}(cx^2)}{x}-\frac{b\sqrt{c}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}}-\frac{b\sqrt{c}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}}+$$

```
[Out] (-a-b*arctan(c*x^2))/x+1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)+1
/2*b*arctan(1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/2)
*c^(1/2))*c^(1/2)*2^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/
2)
```

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5033, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a+b\tan^{-1}(cx^2)}{x}-\frac{b\sqrt{c}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}}-\frac{b\sqrt{c}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x^2])/x) - (b*Sqrt[c]*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/Sqrt
[2] + (b*Sqrt[c]*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/Sqrt[2] - (b*Sqrt[c]*Log[1
- Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[c]*Log[1 + Sqrt[2]*Sqrt
[c]*x + c*x^2])/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^2}\,dx &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}+(2bc)\int \frac{1}{1+c^2x^4}\,dx\\ &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}+(bc)\int \frac{1-cx^2}{1+c^2x^4}\,dx+(bc)\int \frac{1+cx^2}{1+c^2x^4}\,dx\\ &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}+\frac{1}{2}b\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx+\frac{1}{2}b\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{\left(b\sqrt{c}\right)\int \cdots}{2\sqrt{2}\cdots}\\ &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}-\frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}+\cdots\\ &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}-\frac{b\sqrt{c}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}-\frac{b\sqrt{c}\log\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.05, size = 158, normalized size = 1.10

$$-\frac{a}{x}-\frac{b\sqrt{c}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}-\frac{b\tan^{-1}\left(cx^2\right)}{x}+\frac{b\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}x-\sqrt{2}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{b\sqrt{c}\tan\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x^2])/x + (b*Sqrt[c]*ArcTan[(-Sqrt[2] + 2*Sqrt[c]*x)/S
qrt[2]])/Sqrt[2] + (b*Sqrt[c]*ArcTan[(Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/Sqrt
[2] - (b*Sqrt[c]*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[
c]*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2])
```

fricas [B] time = 0.45, size = 322, normalized size = 2.25

$$-\frac{4\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}x\arctan\left(-\frac{b^4c^2+\sqrt{2}\left(b^4c^2\right)^{\frac{3}{4}}bcx-\sqrt{2}\left(b^4c^2\right)^{\frac{3}{4}}\sqrt{b^2c^2x^2+\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}bcx+\sqrt{b^4c^2}}}{b^4c^2}\right)+4\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}x\arctan\left(\frac{b^4c^2-\sqrt{2}\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(b^4*c^2)^(1/4)*x*arctan(-(b^4*c^2 + sqrt(2)*(b^4*c^2)^(3/4
)*b*c*x - sqrt(2)*(b^4*c^2)^(3/4)*sqrt(b^2*c^2*x^2 + sqrt(2)*(b^4*c^2)^(1/4
)*b*c*x + sqrt(b^4*c^2)))/(b^4*c^2)) + 4*sqrt(2)*(b^4*c^2)^(1/4)*x*arctan((
b^4*c^2 - sqrt(2)*(b^4*c^2)^(3/4)*b*c*x + sqrt(2)*(b^4*c^2)^(3/4)*sqrt(b^2*
c^2*x^2 - sqrt(2)*(b^4*c^2)^(1/4)*b*c*x + sqrt(b^4*c^2)))/(b^4*c^2)) - sqrt
(2)*(b^4*c^2)^(1/4)*x*log(b^2*c^2*x^2 + sqrt(2)*(b^4*c^2)^(1/4)*b*c*x + sqr
t(b^4*c^2)) + sqrt(2)*(b^4*c^2)^(1/4)*x*log(b^2*c^2*x^2 - sqrt(2)*(b^4*c^2)
^(1/4)*b*c*x + sqrt(b^4*c^2)) + 4*b*arctan(c*x^2) + 4*a)/x
```

giac [A] time = 0.21, size = 138, normalized size = 0.97

$$\frac{1}{4}bc\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{\sqrt{|c|}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs
(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(c
)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c)) +
 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(
c))/sqrt(abs(c))) - (b*arctan(c*x^2) + a)/x
```

maple [A] time = 0.03, size = 125, normalized size = 0.87

$$-\frac{a}{x}-\frac{b\arctan\left(c x^2\right)}{x}+\frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)}{2}+\frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\;\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{4}+\frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\;\text{arcta}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^2,x)
```

```
[Out] -a/x-b/x*arctan(c*x^2)+1/2*b*c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)
^(1/4)*x-1)+1/4*b*c*(1/c^2)^(1/4)*2^(1/2)*ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(
1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+1/2*b*c*(1/c^2)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)
```

maxima [A] time = 0.42, size = 132, normalized size = 0.92

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}}-\frac{\sqrt{2}\log\left(\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqr
t(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt
(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^
2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*b - a/x
```

mupad [B] time = 0.20, size = 55, normalized size = 0.38

$$-\frac{a}{x}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{x}-(-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}-(-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^2,x)
```

```
[Out] - a/x - (b*atan(c*x^2))/x - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x)
*1i - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x*1i)
```

sympy [A] time = 20.44, size = 151, normalized size = 1.06

$$\begin{cases} -\frac{a}{x}-\sqrt[4]{-1}\,bc\sqrt[4]{\frac{1}{c^2}}\log\left(x-\sqrt[4]{-1}\sqrt[4]{\frac{1}{c^2}}\right)+\frac{\sqrt[4]{-1}\,bc\sqrt[4]{\frac{1}{c^2}}\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{2}-\sqrt[4]{-1}\,bc\sqrt[4]{\frac{1}{c^2}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}x}{\sqrt[4]{\frac{1}{c^2}}}\right)+\frac{(-1)^{\frac{3}{4}}b\,\mathrm{atan}\left(cx^2\right)}{\sqrt[4]{\frac{1}{c^2}}}-\frac{b \cdots}{} \\ -\frac{a}{x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x - (-1)**(1/4)*b*c*(c**(-2))**(1/4)*log(x - (-1)**(1/4)*(c**
(-2))**(1/4)) + (-1)**(1/4)*b*c*(c**(-2))**(1/4)*log(x**2 + I*sqrt(c**(-2))
)/2 - (-1)**(1/4)*b*c*(c**(-2))**(1/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))
 + (-1)**(3/4)*b*atan(c*x**2)/(c**(-2))**(1/4) - b*atan(c*x**2)/x, Ne(c, 0)
), (-a/x, True))
```

3.72 $\int \frac{a+b\tan^{-1}(cx^2)}{x^4}\,dx$

Optimal. Leaf size=159

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}}-$$

```
[Out] -2/3*b*c/x+1/3*(-a-b*arctan(c*x^2))/x^3-1/6*b*c^(3/2)*arctan(-1+x*2^(1/2)*c
^(1/2))*2^(1/2)-1/6*b*c^(3/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)-1/12*b*c^
(3/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)+1/12*b*c^(3/2)*ln(1+c*x^2+x*2^(
1/2)*c^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 325, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^4,x]
```

```
[Out] (-2*b*c)/(3*x) - (a + b*ArcTan[c*x^2])/(3*x^3) + (b*c^(3/2)*ArcTan[1 - Sqrt
[2]*Sqrt[c]*x])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*
Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]) + (b*
c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^4}\,dx = -\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}(2bc)\int \frac{1}{x^2\left(1+c^2x^4\right)}\,dx$$

$$= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{3}\left(2bc^3\right)\int \frac{x^2}{1+c^2x^4}\,dx$$

$$= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}\left(bc^2\right)\int \frac{1-cx^2}{1+c^2x^4}\,dx-\frac{1}{3}\left(bc^2\right)\int \frac{1+cx^2}{1+c^2x^4}\,dx$$

$$= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{6}(bc)\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{1}{6}(bc)\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx - \cdots$$

$$= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}\,x+c\cdots\right)}{6\sqrt{2}}$$

$$= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}+\frac{bc^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\tan^{-1}\left(1+\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}}-\cdots$$

Mathematica [**A**] time = 0.06, size = 177, normalized size = 1.11

$$-\frac{a}{3x^3}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}-\frac{bc^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\,x-\sqrt{2}}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{bc^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\,x+\cdots}{\sqrt{2}}\right)}{3\sqrt{2}}\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*c)/(3*x) - (b*ArcTan[c*x^2])/(3*x^3) - (b*c^(3/2)*ArcTan[
(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[(Sqrt[2]
 + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*
x + c*x^2])/(6*Sqrt[2]) + (b*c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6
*Sqrt[2])
```

fricas [B] time = 0.45, size = 389, normalized size = 2.45

$$\frac{4\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{b^4c^6+\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}b^3c^5x-\sqrt{2}\sqrt{b^6c^{10}x^2+\sqrt{b^4c^6}\,b^4c^6+\sqrt{2}\left(b^4c^6\right)^{\frac{3}{4}}b^3c^5x}\left(b^4c^6\right)^{\frac{1}{4}}}{b^4c^6}\right)+4\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(2)*(b^4*c^6)^(1/4)*x^3*arctan(-(b^4*c^6 + sqrt(2)*(b^4*c^6)^(1
/4)*b^3*c^5*x - sqrt(2)*sqrt(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 + sqrt(2)
*(b^4*c^6)^(3/4)*b^3*c^5*x)*(b^4*c^6)^(1/4))/(b^4*c^6)) + 4*sqrt(2)*(b^4*c^
6)^(1/4)*x^3*arctan((b^4*c^6 - sqrt(2)*(b^4*c^6)^(1/4)*b^3*c^5*x + sqrt(2)*
sqrt(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 - sqrt(2)*(b^4*c^6)^(3/4)*b^3*c^5
*x)*(b^4*c^6)^(1/4))/(b^4*c^6)) + sqrt(2)*(b^4*c^6)^(1/4)*x^3*log(b^6*c^10*
x^2 + sqrt(b^4*c^6)*b^4*c^6 + sqrt(2)*(b^4*c^6)^(3/4)*b^3*c^5*x) - sqrt(2)*
(b^4*c^6)^(1/4)*x^3*log(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 - sqrt(2)*(b^4
*c^6)^(3/4)*b^3*c^5*x) - 8*b*c*x^2 - 4*b*arctan(c*x^2) - 4*a)/x^3
```

giac [A] time = 5.36, size = 159, normalized size = 1.00

$$-\frac{1}{12}bc^3\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{|c|}\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(
abs(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 +
 sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sq
rt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 1/3*(2*b*c*x^2 + b*arctan(c*x^2) +
a)/x^3
```

maple [A] time = 0.03, size = 132, normalized size = 0.83

$$-\frac{a}{3x^3}-\frac{b\arctan\left(cx^2\right)}{3x^3}-\frac{2bc}{3x}-\frac{bc\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{12\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)}{6\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\right.}{6\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x^2)-2/3*b*c/x-1/12*b*c/(1/c^2)^(1/4)*2^(1/2)
*ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2
)+(1/c^2)^(1/2)))-1/6*b*c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4
)*x+1)-1/6*b*c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)
```

maxima [A] time = 0.41, size = 142, normalized size = 0.89

$$-\frac{1}{12}\left(\left(\left(c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/12*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c)
)/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))
/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log
(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)*b
- 1/3*a/x^3
```

mupad [B] time = 0.43, size = 63, normalized size = 0.40

$$-\frac{2\,b\,c\,x^2+a}{3\,x^3}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{3\,x^3}-\frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{3}-\frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^4,x)
```

```
[Out] - (a + 2*b*c*x^2)/(3*x^3) - (b*atan(c*x^2))/(3*x^3) - ((-1)^(1/4)*b*c^(3/2)
*atan((-1)^(1/4)*c^(1/2)*x))/3 - ((-1)^(1/4)*b*c^(3/2)*atan((-1)^(1/4)*c^(1
/2)*x*1i)*1i)/3
```

sympy [A] time = 38.52, size = 590, normalized size = 3.71

$$\begin{cases} -\frac{a}{3x^3} \\ -\frac{a-\infty ib}{3x^3} \\ -\frac{a+\infty ib}{3x^3} \\ -\frac{2ax^4}{6x^7+\frac{6x^3}{c^2}}-\frac{2a}{6c^2x^7+6x^3}+\frac{2(-1)^{\frac{3}{4}}bc^3x^7\left(\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-1}\sqrt[4]{\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}}-\frac{(-1)^{\frac{3}{4}}bc^3x^7\left(\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}}-\frac{2(-1)^{\frac{3}{4}}bc^3x^7\left(\frac{1}{c^2}\right)^{\frac{3}{4}}\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}x}{\sqrt[4]{\frac{1}{c^2}}}\right)}{6x^7+\frac{6x^3}{c^2}}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*I*b)/(3*x**3), Eq(c, -I/x**2))
, (-(a + oo*I*b)/(3*x**3), Eq(c, I/x**2)), (-2*a*x**4/(6*x**7 + 6*x**3/c**2
) - 2*a/(6*c**2*x**7 + 6*x**3) + 2*(-1)**(3/4)*b*c**3*x**7*(c**(-2))**(3/4)
*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(6*x**7 + 6*x**3/c**2) - (-1)**(3/4)
*b*c**3*x**7*(c**(-2))**(3/4)*log(x**2 + I*sqrt(c**(-2)))/(6*x**7 + 6*x**3/
c**2) - 2*(-1)**(3/4)*b*c**3*x**7*(c**(-2))**(3/4)*atan((-1)**(3/4)*x/(c**(
-2))**(1/4))/(6*x**7 + 6*x**3/c**2) + 2*(-1)**(1/4)*b*c**2*x**7*(c**(-2))**
(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*c*x**6/(6*x**7 + 6*x**3/c**
2) + 2*(-1)**(3/4)*b*c*x**3*(c**(-2))**(3/4)*log(x - (-1)**(1/4)*(c**(-2))*
*(1/4))/(6*x**7 + 6*x**3/c**2) - (-1)**(3/4)*b*c*x**3*(c**(-2))**(3/4)*log(
x**2 + I*sqrt(c**(-2)))/(6*x**7 + 6*x**3/c**2) - 2*(-1)**(3/4)*b*c*x**3*(c*
*(-2))**(3/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(6*x**7 + 6*x**3/c**2) -
 2*b*x**4*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) + 2*(-1)**(1/4)*b*x**3*(c**(-
2))**(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*x**2/(6*c*x**7 + 6*x**
3/c) - 2*b*atan(c*x**2)/(6*c**2*x**7 + 6*x**3), True))
```

3.73 $\int \frac{a+b\tan^{-1}(cx^2)}{x^6}\,dx$

Optimal. Leaf size=159

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}+\frac{bc^{5/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}-$$

```
[Out] -2/15*b*c/x^3+1/5*(-a-b*arctan(c*x^2))/x^5-1/10*b*c^(5/2)*arctan(-1+x*2^(1/
2)*c^(1/2))*2^(1/2)-1/10*b*c^(5/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)+1/20
*b*c^(5/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)-1/20*b*c^(5/2)*ln(1+c*x^2+
x*2^(1/2)*c^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 325, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}+\frac{bc^{5/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] (-2*b*c)/(15*x^3) - (a + b*ArcTan[c*x^2])/(5*x^5) + (b*c^(5/2)*ArcTan[1 - S
qrt[2]*Sqrt[c]*x])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/
(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]) -
 (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^6}\,dx &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}(2bc)\int \frac{1}{x^4\left(1+c^2x^4\right)}\,dx\\
&= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}-\frac{1}{5}\left(2bc^3\right)\int \frac{1}{1+c^2x^4}\,dx\\
&= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}-\frac{1}{5}\left(bc^3\right)\int \frac{1-cx^2}{1+c^2x^4}\,dx-\frac{1}{5}\left(bc^3\right)\int \frac{1+cx^2}{1+c^2x^4}\,dx\\
&= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}-\frac{1}{10}\left(bc^2\right)\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{1}{10}\left(bc^2\right)\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}
\\
&= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}\,x+\right.}{10\sqrt{2}}\\
&= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{bc^{5/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\tan^{-1}\left(1+\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}+
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 177, normalized size = 1.11

$$
-\frac{a}{5x^5}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\tan^{-1}\left(\frac{2\sqrt{c}\,x-\sqrt{2}}{\sqrt{2}}\right)}{5\sqrt{2}}-\frac{bc^{5/2}\tan^{-1}\left(\frac{2\sqrt{c}\,x+\sqrt{}}{\sqrt{2}}\right.}{5\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (2*b*c)/(15*x^3) - (b*ArcTan[c*x^2])/(5*x^5) - (b*c^(5/2)*ArcT
an[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[(Sqrt
[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[
c]*x + c*x^2])/(10*Sqrt[2]) - (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(10*Sqrt[2])
```

fricas [B] time = 0.47, size = 350, normalized size = 2.20

$$\frac{12\sqrt{2}\left(b^4c^{10}\right)^{\frac{1}{4}}x^5\arctan\left(-\frac{b^4c^{10}+\sqrt{2}\left(b^4c^{10}\right)^{\frac{3}{4}}bc^3x-\sqrt{2}\left(b^4c^{10}\right)^{\frac{3}{4}}\sqrt{b^2c^6x^2+\sqrt{2}\left(b^4c^{10}\right)^{\frac{1}{4}}bc^3x+\sqrt{b^4c^{10}}}}{b^4c^{10}}\right)+12\sqrt{2}\left(b^4c^{10}\right)^{\frac{1}{4}}x^5\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*sqrt(2)*(b^4*c^10)^(1/4)*x^5*arctan(-(b^4*c^10 + sqrt(2)*(b^4*c^10
)^(3/4)*b*c^3*x - sqrt(2)*(b^4*c^10)^(3/4)*sqrt(b^2*c^6*x^2 + sqrt(2)*(b^4*
c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10)))/(b^4*c^10)) + 12*sqrt(2)*(b^4*c^10)^
(1/4)*x^5*arctan((b^4*c^10 - sqrt(2)*(b^4*c^10)^(3/4)*b*c^3*x + sqrt(2)*(b^
4*c^10)^(3/4)*sqrt(b^2*c^6*x^2 - sqrt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^
4*c^10)))/(b^4*c^10)) - 3*sqrt(2)*(b^4*c^10)^(1/4)*x^5*log(b^2*c^6*x^2 + sq
rt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10)) + 3*sqrt(2)*(b^4*c^10)^(1/
4)*x^5*log(b^2*c^6*x^2 - sqrt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10))
 - 8*b*c*x^2 - 12*b*arctan(c*x^2) - 12*a)/x^5
```

giac [A] time = 0.25, size = 150, normalized size = 0.94

$$-\frac{1}{20}bc^3\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\right.}{\sqrt{|c|}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt
(abs(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c
)) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/
abs(c))/sqrt(abs(c))) - 1/15*(2*b*c*x^2 + 3*b*arctan(c*x^2) + 3*a)/x^5
```

maple [A] time = 0.03, size = 138, normalized size = 0.87

$$-\frac{a}{5x^5}-\frac{b\arctan\left(cx^2\right)}{5x^5}-\frac{2bc}{15x^3}-\frac{bc^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)}{10}-\frac{bc^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{20}-\frac{bc^3\left(\frac{1}{c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x^2)-2/15*b*c/x^3-1/10*b*c^3*(1/c^2)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)-1/20*b*c^3*(1/c^2)^(1/4)*2^(1/2)*ln
((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(
1/c^2)^(1/2)))-1/10*b*c^3*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4
)*x+1)
```

maxima [A] time = 0.41, size = 138, normalized size = 0.87

$$-\frac{1}{60}\left(\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)+6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)+3\sqrt{2}c^{\frac{3}{2}}\log\left(cx^2+\sqrt{2}\sqrt{c}x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt
(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(
c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c^(3
/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^5)*b
 - 1/5*a/x^5
```

mupad [B] time = 0.45, size = 63, normalized size = 0.40

$$-\frac{\frac{2\,b\,c\,x^2}{3}+a}{5\,x^5}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{5\,x^5}+\frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{5}+\frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^6,x)
```

```
[Out] ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)*c^(1/2)*x)*1i)/5 - (b*atan(c*x^2))/(5
*x^5) - (a + (2*b*c*x^2)/3)/(5*x^5) + ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)
*c^(1/2)*x*1i))/5
```

sympy [A] time = 63.23, size = 185, normalized size = 1.16

$$\begin{cases} -\frac{a}{5x^5}+\frac{\sqrt[4]{-1}bc^3\sqrt[4]{\frac{1}{c^2}}\log\left(x-\sqrt[4]{-1}\sqrt[4]{\frac{1}{c^2}}\right)}{5}-\frac{\sqrt[4]{-1}bc^3\sqrt[4]{\frac{1}{c^2}}\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{10}+\frac{\sqrt[4]{-1}bc^3\sqrt[4]{\frac{1}{c^2}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}x}{\sqrt[4]{\frac{1}{c^2}}}\right)}{5}-\frac{(-1)^{\frac{3}{4}}bc^2\,\mathrm{atan}\left(cx^2\right)}{5\sqrt[4]{\frac{1}{c^2}}}-\frac{2bc}{15x^3}- \\ -\frac{a}{5x^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + (-1)**(1/4)*b*c**3*(c**(-2))**(1/4)*log(x - (-1)**
(1/4)*(c**(-2))**(1/4))/5 - (-1)**(1/4)*b*c**3*(c**(-2))**(1/4)*log(x**2 +
I*sqrt(c**(-2)))/10 + (-1)**(1/4)*b*c**3*(c**(-2))**(1/4)*atan((-1)**(3/4)*
x/(c**(-2))**(1/4))/5 - (-1)**(3/4)*b*c**2*atan(c*x**2)/(5*(c**(-2))**(1/4)
) - 2*b*c/(15*x**3) - b*atan(c*x**2)/(5*x**5), Ne(c, 0)), (-a/(5*x**5), Tru
e))
```

3.74 $\int x^7 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=124

$$-\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{8c^4}+\frac{abx^2}{4c^3}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{bx^6\left(a+b\tan^{-1}\left(cx^2\right)\right)}{12c}+\frac{b^2x^2\tan^{-1}\left(cx^2\right)}{4c^3}+\frac{b^2x^4}{24c^2}-\frac{b^2 \mathrm{l}}{}$$

```
[Out] 1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctan(c*x^2)/c^3-1/12*b*x^6*(
a+b*arctan(c*x^2))/c-1/8*(a+b*arctan(c*x^2))^2/c^4+1/8*x^8*(a+b*arctan(c*x^
2))^2-1/6*b^2*ln(c^2*x^4+1)/c^4
```

Rubi [C] time = 1.63, antiderivative size = 731, normalized size of antiderivative = 5.90, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{16c^4}-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{16c^4}+\frac{abx^2}{8c^3}-\frac{bx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{32c^2}+\frac{1}{192}ib\left(-\frac{3(1}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(8*c^3) - (((23*I)/192)*b^2*x^2)/c^3 + (b^2*x^4)/(128*c^2) - (((7
*I)/576)*b^2*x^6)/c + (b^2*x^8)/256 - (3*b^2*(1 - I*c*x^2)^2)/(32*c^4) + (b
^2*(1 - I*c*x^2)^3)/(36*c^4) - (b^2*(1 - I*c*x^2)^4)/(256*c^4) - (b^2*Log[I
 - c*x^2])/(24*c^4) - (b^2*(1 - I*c*x^2)*Log[1 - I*c*x^2])/(16*c^4) - (b^2*
Log[1 - I*c*x^2]^2)/(32*c^4) - (b*x^4*((2*I)*a - b*Log[1 - I*c*x^2]))/(32*c
^2) + ((I/48)*b*x^6*((2*I)*a - b*Log[1 - I*c*x^2]))/c + (b*x^8*((2*I)*a - b
*Log[1 - I*c*x^2]))/64 + (x^8*(2*a + I*b*Log[1 - I*c*x^2])^2)/32 + (I/192)*
b*(2*a + I*b*Log[1 - I*c*x^2])*((48*(1 - I*c*x^2))/c^4 - (36*(1 - I*c*x^2)^
2)/c^4 + (16*(1 - I*c*x^2)^3)/c^4 - (3*(1 - I*c*x^2)^4)/c^4 - (12*Log[1 - I
*c*x^2])/c^4) + (b*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/(16
*c^4) + ((I/24)*b^2*x^6*Log[1 + I*c*x^2])/c - (b^2*(1 + I*c*x^2)*Log[1 + I*
c*x^2])/(8*c^4) - (b^2*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/(16*c^4) - (b
*x^8*((2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/16 + (b^2*Log[1 + I*c
*x^2]^2)/(32*c^4) - (b^2*x^8*Log[1 + I*c*x^2]^2)/32 + (5*b^2*Log[I + c*x^2]
)/(192*c^4) - (b^2*PolyLog[2, (1 - I*c*x^2)/2])/(16*c^4) - (b^2*PolyLog[2,
(1 + I*c*x^2)/2])/(16*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^7\left(a+b\tan^{-1}\left(cx^2\right)\right)^2dx=\int\left(\frac{1}{4}x^7\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^7\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\right.$$
$$=\frac{1}{4}\int x^7\left(2a+ib\log\left(1-icx^2\right)\right)^2dx+\frac{1}{2}b\int x^7\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+i\right.$$
$$=\frac{1}{8}\text{Subst}\left(\int x^3(2a+ib\log(1-icx))^2dx,x,x^2\right)+\frac{1}{4}b\,\text{Subst}\left(\int x^3(-2ia+b\log(1\right.$$
$$=\frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{16}bx^8\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{32}$$
$$=\frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{16}bx^8\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{32}$$
$$=\frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{192}ib\left(2a+ib\log\left(1-icx^2\right)\right)\left(\frac{48\left(1-icx^2\right)}{c^4}-\frac{36}{}\right.$$
$$=\frac{abx^2}{8c^3}-\frac{bx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{32c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^2\right)\right)}{48c}+\frac{1}{64}bx^8\left(2ia-\right.$$
$$=\frac{abx^2}{8c^3}-\frac{bx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{32c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^2\right)\right)}{48c}+\frac{1}{64}bx^8\left(2ia-\right.$$
$$=\frac{abx^2}{8c^3}-\frac{55ib^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{ib^2x^6}{576c}+\frac{b^2x^8}{256}-\frac{3b^2\left(1-icx^2\right)^2}{32c^4}+\frac{b^2\left(1-icx^2\right)^3}{36c^4}-\frac{b^2(}{}$$
$$=\frac{abx^2}{8c^3}-\frac{55ib^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{ib^2x^6}{576c}+\frac{b^2x^8}{256}-\frac{3b^2\left(1-icx^2\right)^2}{32c^4}+\frac{b^2\left(1-icx^2\right)^3}{36c^4}-\frac{b^2(}{}$$

Mathematica [A] time = 0.13, size = 121, normalized size = 0.98

$$\frac{cx^2\left(3a^2c^3x^6-2abc^2x^4+6ab+b^2cx^2\right)-2b\tan^{-1}\left(cx^2\right)\left(a\left(3-3c^4x^8\right)+bcx^2\left(c^2x^4-3\right)\right)+3b^2\left(c^4x^8-1\right)\tan^{-1}\left(\right.}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (c*x^2*(6*a*b + b^2*c*x^2 - 2*a*b*c^2*x^4 + 3*a^2*c^3*x^6) - 2*b*(b*c*x^2*(
-3 + c^2*x^4) + a*(3 - 3*c^4*x^8))*ArcTan[c*x^2] + 3*b^2*(-1 + c^4*x^8)*Arc
Tan[c*x^2]^2 - 4*b^2*Log[1 + c^2*x^4])/(24*c^4)
```

fricas [A] time = 0.45, size = 137, normalized size = 1.10

$$\frac{3\,a^2c^4x^8-2\,abc^3x^6+b^2c^2x^4+6\,abcx^2+3\left(b^2c^4x^8-b^2\right)\arctan\left(cx^2\right)^2+6\,ab\arctan\left(\frac{1}{cx^2}\right)-4\,b^2\log\left(c^2x^4+1\right)+}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^2*c^4*x^8 - 2*a*b*c^3*x^6 + b^2*c^2*x^4 + 6*a*b*c*x^2 + 3*(b^2*c^
4*x^8 - b^2)*arctan(c*x^2)^2 + 6*a*b*arctan(1/(c*x^2)) - 4*b^2*log(c^2*x^4
+ 1) + 2*(3*a*b*c^4*x^8 - b^2*c^3*x^6 + 3*b^2*c*x^2)*arctan(c*x^2))/c^4
```

giac [A] time = 0.18, size = 145, normalized size = 1.17

$$\frac{3\,a^2cx^8+2\left(3\,cx^8\arctan\left(cx^2\right)-\frac{3\arctan\left(cx^2\right)}{c^3}-\frac{c^9x^6-3\,c^7x^2}{c^9}\right)ab+\left(3\,cx^8\arctan\left(cx^2\right)^2-\frac{2\,c^3x^6\arctan\left(cx^2\right)-c^2x^4-6\,cx^2\,\mathrm{a}}{}\right.}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*c*x^8 + 2*(3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x
^6 - 3*c^7*x^2)/c^9)*a*b + (3*c*x^8*arctan(c*x^2)^2 - (2*c^3*x^6*arctan(c*x
^2) - c^2*x^4 - 6*c*x^2*arctan(c*x^2) + 3*arctan(c*x^2)^2 + 4*log(c^2*x^4 +
 1))/c^3)*b^2)/c
```

maple [A] time = 0.05, size = 151, normalized size = 1.22

$$\frac{x^8a^2}{8}+\frac{b^2x^8\arctan\left(c\,x^2\right)^2}{8}-\frac{b^2\arctan\left(c\,x^2\right)x^6}{12c}+\frac{b^2x^2\arctan\left(c\,x^2\right)}{4c^3}-\frac{b^2\arctan\left(c\,x^2\right)^2}{8c^4}+\frac{b^2x^4}{24c^2}-\frac{b^2\ln\left(c^2x^4+1\right)}{6c^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] 1/8*x^8*a^2+1/8*b^2*x^8*arctan(c*x^2)^2-1/12*b^2*arctan(c*x^2)/c*x^6+1/4*b^
2*x^2*arctan(c*x^2)/c^3-1/8*b^2/c^4*arctan(c*x^2)^2+1/24*b^2*x^4/c^2-1/6*b^
2*ln(c^2*x^4+1)/c^4+1/4*a*b*x^8*arctan(c*x^2)-1/12*a*b/c*x^6+1/4*a*b*x^2/c^
3-1/4*a*b/c^4*arctan(c*x^2)
```

maxima [A] time = 0.49, size = 169, normalized size = 1.36

$$\frac{1}{8}\,b^2x^8\arctan\left(cx^2\right)^2+\frac{1}{8}\,a^2x^8+\frac{1}{12}\left(3\,x^8\arctan\left(cx^2\right)-c\left(\frac{c^2x^6-3\,x^2}{c^4}+\frac{3\arctan\left(cx^2\right)}{c^5}\right)\right)ab-\frac{1}{24}\left(2\,c\left(\frac{c^2x^6-3\,x}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*arctan(c*x^2)^2 + 1/8*a^2*x^8 + 1/12*(3*x^8*arctan(c*x^2) - c*(
(c^2*x^6 - 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5))*a*b - 1/24*(2*c*((c^2*x^6 - 3
*x^2)/c^4 + 3*arctan(c*x^2)/c^5)*arctan(c*x^2) - (c^2*x^4 + 3*arctan(c*x^2)
^2 - 3*log(12*c^7*x^4 + 12*c^5) - log(c^2*x^4 + 1))/c^4)*b^2
```

mupad [B] time = 1.02, size = 150, normalized size = 1.21

$$\frac{a^2\,x^8}{8}-\frac{b^2\,\mathrm{atan}\left(c\,x^2\right)^2}{8\,c^4}+\frac{b^2\,x^8\,\mathrm{atan}\left(c\,x^2\right)^2}{8}-\frac{b^2\,\ln\left(c^2\,x^4+1\right)}{6\,c^4}+\frac{b^2\,x^4}{24\,c^2}+\frac{b^2\,x^2\,\mathrm{atan}\left(c\,x^2\right)}{4\,c^3}-\frac{b^2\,x^6\,\mathrm{atan}\left(c\,x^2\right)}{12\,c}+\frac{a\,b\,x^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atan(c*x^2))^2,x)
```

```
[Out] (a^2*x^8)/8 - (b^2*atan(c*x^2)^2)/(8*c^4) + (b^2*x^8*atan(c*x^2)^2)/8 - (b^
2*log(c^2*x^4 + 1))/(6*c^4) + (b^2*x^4)/(24*c^2) + (b^2*x^2*atan(c*x^2))/(4
*c^3) - (b^2*x^6*atan(c*x^2))/(12*c) + (a*b*x^2)/(4*c^3) - (a*b*x^6)/(12*c)
 - (a*b*atan(c*x^2))/(4*c^4) + (a*b*x^8*atan(c*x^2))/4
```

sympy [A] time = 99.00, size = 199, normalized size = 1.60

$$\begin{cases}\frac{a^2x^8}{8}+\frac{abx^8\,\mathrm{atan}\left(cx^2\right)}{4}-\frac{abx^6}{12c}+\frac{abx^2}{4c^3}-\frac{ab\,\mathrm{atan}\left(cx^2\right)}{4c^4}+\frac{b^2x^8\,\mathrm{atan}^2\left(cx^2\right)}{8}-\frac{b^2x^6\,\mathrm{atan}\left(cx^2\right)}{12c}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\,\mathrm{atan}\left(cx^2\right)}{4c^3}-\frac{ib^2\sqrt{\frac{1}{c^2}}\,\mathrm{ata}}{3c^3} \\ \frac{a^2x^8}{8}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**8/8 + a*b*x**8*atan(c*x**2)/4 - a*b*x**6/(12*c) + a*b*x*
*2/(4*c**3) - a*b*atan(c*x**2)/(4*c**4) + b**2*x**8*atan(c*x**2)**2/8 - b**
2*x**6*atan(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atan(c*x**2)/(
4*c**3) - I*b**2*sqrt(c**(-2))*atan(c*x**2)/(3*c**3) - b**2*log(x**2 + I*sq
rt(c**(-2)))/(3*c**4) - b**2*atan(c*x**2)**2/(8*c**4), Ne(c, 0)), (a**2*x**
8/8, True))
```

3.75 $\int x^5 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=154

$$-\frac{i\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3c^3}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{bx^4\left(a+b\tan^{-1}\left(cx^2\right)\right)}{6c}-$$

```
[Out] 1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^3-1/6*b*x^4*(a+b*arctan(c*x^2))/c-1
/6*I*(a+b*arctan(c*x^2))^2/c^3+1/6*x^6*(a+b*arctan(c*x^2))^2-1/3*b*(a+b*arc
tan(c*x^2))*ln(2/(1+I*c*x^2))/c^3-1/6*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c^3
```

Rubi [B] time = 1.37, antiderivative size = 647, normalized size of antiderivative = 4.20, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{12c^3}-\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{12c^3}-\frac{iabx^2}{6c^2}+\frac{1}{72}ib\left(\frac{2i\left(1-icx^2\right)^3}{c^3}-\frac{9i\left(1-icx^2\right)^2}{c^3}+\frac{18}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] ((-I/6)*a*b*x^2)/c^2 + (19*b^2*x^2)/(72*c^2) - (((5*I)/144)*b^2*x^4)/c + (b
^2*x^6)/108 - ((I/16)*b^2*(1 - I*c*x^2)^2)/c^3 + ((I/108)*b^2*(1 - I*c*x^2)
^3)/c^3 + ((I/12)*b^2*Log[I - c*x^2])/c^3 + ((I/12)*b^2*(1 - I*c*x^2)*Log[1
 - I*c*x^2])/c^3 - ((I/24)*b^2*Log[1 - I*c*x^2]^2)/c^3 + ((I/24)*b*x^4*((2*
I)*a - b*Log[1 - I*c*x^2]))/c + (b*x^6*((2*I)*a - b*Log[1 - I*c*x^2]))/36 +
 (x^6*(2*a + I*b*Log[1 - I*c*x^2])^2)/24 + (I/72)*b*(2*a + I*b*Log[1 - I*c*
x^2])*(((18*I)*(1 - I*c*x^2))/c^3 - ((9*I)*(1 - I*c*x^2)^2)/c^3 + ((2*I)*(1
 - I*c*x^2)^3)/c^3 - ((6*I)*Log[1 - I*c*x^2])/c^3) - ((I/12)*b*((2*I)*a - b
*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/c^3 + ((I/12)*b^2*x^4*Log[1 + I*c*
x^2])/c - ((I/12)*b^2*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/c^3 - (b*x^6*(
(2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/12 - ((I/24)*b^2*Log[1 + I*
c*x^2]^2)/c^3 - (b^2*x^6*Log[1 + I*c*x^2]^2)/24 - ((I/72)*b^2*Log[I + c*x^2
])/c^3 + ((I/12)*b^2*PolyLog[2, (1 - I*c*x^2)/2])/c^3 - ((I/12)*b^2*PolyLog
[2, (1 + I*c*x^2)/2])/c^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^5\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^5\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)\right.$$

$$=\frac{1}{4}\int x^5\left(2a+ib\log\left(1-icx^2\right)\right)^2dx+\frac{1}{2}b\int x^5\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+\right.$$

$$=\frac{1}{8}\text{Subst}\left(\int x^2(2a+ib\log(1-icx))^2dx,x,x^2\right)+\frac{1}{4}b\,\text{Subst}\left(\int x^2(-2ia+b\log(1\right.$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{24}$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{24}$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{72}ib\left(2a+ib\log\left(1-icx^2\right)\right)\left(\frac{18i\left(1-icx^2\right)}{c^3}-\frac{9i}{}\right.$$

$$=-\frac{iabx^2}{6c^2}+\frac{ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{24c}+\frac{1}{36}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{1}{24}x^6\left(2\right.$$

$$=-\frac{iabx^2}{6c^2}+\frac{ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{24c}+\frac{1}{36}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{1}{24}x^6\left(2\right.$$

$$=-\frac{iabx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{ib^2x^4}{144c}+\frac{b^2x^6}{108}-\frac{ib^2\left(1-icx^2\right)^2}{16c^3}+\frac{ib^2\left(1-icx^2\right)^3}{108c^3}+\frac{ib^2\left(1-ic\right.}{}$$

$$=-\frac{iabx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{ib^2x^4}{144c}+\frac{b^2x^6}{108}-\frac{ib^2\left(1-icx^2\right)^2}{16c^3}+\frac{ib^2\left(1-icx^2\right)^3}{108c^3}+\frac{ib^2\left(1-ic\right.}{}$$

Mathematica [A] time = 0.33, size = 141, normalized size = 0.92

$$\frac{a^2c^3x^6-abc^2x^4+ab\log\left(c^2x^4+1\right)-b\tan^{-1}\left(cx^2\right)\left(-2ac^3x^6+bc^2x^4+2b\log\left(1+e^{2i\tan^{-1}\left(cx^2\right)}\right)+b\right)+b^2\left(c^3x^6+\right.}{6c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*c*x^2 - a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(I + c^3*x^6)*ArcTan[c*x^2]^2
- b*ArcTan[c*x^2]*(b + b*c^2*x^4 - 2*a*c^3*x^6 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^2])]) + a*b*Log[1 + c^2*x^4] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
2])])/(6*c^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^5\arctan\left(cx^2\right)^2+2\,abx^5\arctan\left(cx^2\right)+a^2x^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^5*arctan(c*x^2)^2 + 2*a*b*x^5*arctan(c*x^2) + a^2*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \arctan\left(cx^2\right) + a\right)^2 x^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*x^5, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + b \arctan\left(c\, x^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] int(x^5*(a+b*arctan(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\, a^2 x^6 + \frac{1}{6} \left(2\, x^6 \arctan\left(cx^2\right) - \left(\frac{x^4}{c^2} - \frac{\log\left(c^2 x^4 + 1\right)}{c^4}\right) c\right) ab + \frac{1}{96} \left(4\, x^6 \arctan\left(cx^2\right)^2 - x^6 \log\left(c^2 x^4 + 1\right)^2 + 96 \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + 1/6*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c
)*a*b + 1/96*(4*x^6*arctan(c*x^2)^2 - x^6*log(c^2*x^4 + 1)^2 + 96*integrate
(1/48*(4*c^2*x^9*log(c^2*x^4 + 1) - 8*c*x^7*arctan(c*x^2) + 36*(c^2*x^9 + x
^5)*arctan(c*x^2)^2 + 3*(c^2*x^9 + x^5)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1),
x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a + b\, \mathrm{atan}\left(c\, x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int(x^5*(a + b*atan(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + b\, \mathrm{atan}\left(cx^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral(x**5*(a + b*atan(c*x**2))**2, x)
```

3.76 $\int x^3 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=90

$$\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{4c^2}-\frac{abx^2}{2c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)^2+\frac{b^2\log\left(c^2x^4+1\right)}{4c^2}-\frac{b^2x^2\tan^{-1}\left(cx^2\right)}{2c}$$

```
[Out] -1/2*a*b*x^2/c-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*(a+b*arctan(c*x^2))^2/c^2+1/
4*x^4*(a+b*arctan(c*x^2))^2+1/4*b^2*ln(c^2*x^4+1)/c^2
```

Rubi [C] time = 1.04, antiderivative size = 612, normalized size of antiderivative = 6.80, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{8c^2}-\frac{\left(1-icx^2\right)^2\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c^2}+\frac{\left(1-icx^2\right)\left(2a+\right.}{8}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (-3*a*b*x^2)/(4*c) + (b^2*x^4)/16 + (b^2*(1 - I*c*x^2)^2)/(32*c^2) + (b^2*(
1 + I*c*x^2)^2)/(32*c^2) - (b^2*Log[I - c*x^2])/(16*c^2) + (3*b^2*(1 - I*c*
x^2)*Log[1 - I*c*x^2])/(8*c^2) + (b*x^4*((2*I)*a - b*Log[1 - I*c*x^2]))/16
+ ((I/16)*b*(1 - I*c*x^2)^2*(2*a + I*b*Log[1 - I*c*x^2]))/c^2 + ((1 - I*c*x
^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/(8*c^2) - ((1 - I*c*x^2)^2*(2*a + I*b*L
og[1 - I*c*x^2])^2)/(16*c^2) - (b*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I
*c*x^2)/2])/(8*c^2) - (b^2*x^4*Log[1 + I*c*x^2])/16 + (3*b^2*(1 + I*c*x^2)*
Log[1 + I*c*x^2])/(8*c^2) - (b^2*(1 + I*c*x^2)^2*Log[1 + I*c*x^2])/(16*c^2)
 + (b^2*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/(8*c^2) - (b*x^4*((2*I)*a -
b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/8 - (b^2*(1 + I*c*x^2)*Log[1 + I*c*x^
2]^2)/(8*c^2) + (b^2*(1 + I*c*x^2)^2*Log[1 + I*c*x^2]^2)/(16*c^2) - (b^2*Lo
g[I + c*x^2])/(16*c^2) + (b^2*PolyLog[2, (1 - I*c*x^2)/2])/(8*c^2) + (b^2*P
olyLog[2, (1 + I*c*x^2)/2])/(8*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx=\int\left(\frac{1}{4}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^3\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\right.$$

$$=\frac{1}{4}\int x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2\,dx+\frac{1}{2}b\int x^3\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+i\right.$$

$$=\frac{1}{8}\text{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\,\text{Subst}\left(\int x(-2ia+b\log(1-\right.$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)+\frac{1}{8}\text{Subst}\left(\int\left(-\frac{i(2a+ib\log(1-ic}{c}\right.\right.$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{i\,\text{Subst}\left(\int(2a+ib\log(1-icx))^2\,d\right.}{8c}$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{8}b\,\text{Subst}\left(\int x(-2ia+b\log(1-ic\right.$$

$$=-\frac{abx^2}{4c}+\frac{1}{16}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c^2}-\frac{(1}{}$$

$$=-\frac{3abx^2}{4c}-\frac{3ib^2x^2}{8c}+\frac{b^2\left(1-icx^2\right)^2}{32c^2}+\frac{b^2\left(1+icx^2\right)^2}{32c^2}+\frac{1}{16}bx^4\left(2ia-b\log\left(1-icx^2\right.\right.$$

$$=-\frac{3abx^2}{4c}+\frac{b^2x^4}{16}+\frac{b^2\left(1-icx^2\right)^2}{32c^2}+\frac{b^2\left(1+icx^2\right)^2}{32c^2}-\frac{b^2\log\left(i-cx^2\right)}{16c^2}+\frac{3b^2\left(1-icx\right.}{}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.94

$$\frac{2b\tan^{-1}\left(cx^2\right)\left(ac^2x^4+a-bcx^2\right)+acx^2\left(acx^2-2b\right)+b^2\log\left(c^2x^4+1\right)+b^2\left(c^2x^4+1\right)\tan^{-1}\left(cx^2\right)^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*c*x^2*(-2*b + a*c*x^2) + 2*b*(a - b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b
^2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 + b^2*Log[1 + c^2*x^4])/(4*c^2)
```

fricas [A] time = 0.45, size = 100, normalized size = 1.11

$$\frac{a^2c^2x^4-2\,abcx^2+\left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2-2\,ab\arctan\left(\frac{1}{cx^2}\right)+b^2\log\left(c^2x^4+1\right)+2\left(abc^2x^4-b^2cx^2\right)\mathrm{arc}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*c^2*x^4 - 2*a*b*c*x^2 + (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - 2*a*
b*arctan(1/(c*x^2)) + b^2*log(c^2*x^4 + 1) + 2*(a*b*c^2*x^4 - b^2*c*x^2)*ar
ctan(c*x^2))/c^2
```

giac [A] time = 1.33, size = 100, normalized size = 1.11

$$\frac{a^2cx^4+\frac{2\left(c^2x^4\arctan\left(cx^2\right)-cx^2+\arctan\left(cx^2\right)\right)ab}{c}+\frac{\left(c^2x^4\arctan\left(cx^2\right)^2-2\,cx^2\arctan\left(cx^2\right)+\arctan\left(cx^2\right)^2+\log\left(c^2x^4+1\right)\right)b^2}{c}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*c*x^4 + 2*(c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*a*b/c +
(c^2*x^4*arctan(c*x^2)^2 - 2*c*x^2*arctan(c*x^2) + arctan(c*x^2)^2 + log(c^
2*x^4 + 1))*b^2/c)/c
```

maple [A] time = 0.04, size = 113, normalized size = 1.26

$$\frac{a^2x^4}{4}+\frac{b^2x^4\arctan\left(c\,x^2\right)^2}{4}-\frac{b^2x^2\arctan\left(c\,x^2\right)}{2c}+\frac{b^2\arctan\left(c\,x^2\right)^2}{4c^2}+\frac{b^2\ln\left(c^2x^4+1\right)}{4c^2}+\frac{ab\,x^4\arctan\left(c\,x^2\right)}{2}-\frac{ab\,x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctan(c*x^2)^2-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*b^2
/c^2*arctan(c*x^2)^2+1/4*b^2*ln(c^2*x^4+1)/c^2+1/2*a*b*x^4*arctan(c*x^2)-1/
2*a*b*x^2/c+1/2*a*b/c^2*arctan(c*x^2)
```

maxima [A] time = 0.49, size = 126, normalized size = 1.40

$$\frac{1}{4}\,b^2x^4\arctan\left(cx^2\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{2}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)ab-\frac{1}{4}\left(2\,c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^2*x^4 + 1/2*(x^4*arctan(c*x^2) - c*(x^2
/c^2 - arctan(c*x^2)/c^3))*a*b - 1/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)*arc
tan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*b^2
```

mupad [B] time = 0.68, size = 112, normalized size = 1.24

$$\frac{a^2\,x^4}{4}+\frac{b^2\,\mathrm{atan}\left(c\,x^2\right)^2}{4\,c^2}+\frac{b^2\,x^4\,\mathrm{atan}\left(c\,x^2\right)^2}{4}+\frac{b^2\,\ln\left(c^2\,x^4+1\right)}{4\,c^2}-\frac{b^2\,x^2\,\mathrm{atan}\left(c\,x^2\right)}{2\,c}-\frac{a\,b\,x^2}{2\,c}+\frac{a\,b\,\mathrm{atan}\left(c\,x^2\right)}{2\,c^2}+\frac{a\,b\,x^4\,\mathrm{atan}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*atan(c*x^2)^2)/(4*c^2) + (b^2*x^4*atan(c*x^2)^2)/4 + (b^
2*log(c^2*x^4 + 1))/(4*c^2) - (b^2*x^2*atan(c*x^2))/(2*c) - (a*b*x^2)/(2*c)
 + (a*b*atan(c*x^2))/(2*c^2) + (a*b*x^4*atan(c*x^2))/2
```

sympy [A] time = 34.91, size = 155, normalized size = 1.72

$$\begin{cases} \frac{a^2x^4}{4} + \frac{abx^4\operatorname{atan}\left(cx^2\right)}{2} - \frac{abx^2}{2c} + \frac{ab\operatorname{atan}\left(cx^2\right)}{2c^2} + \frac{b^2x^4\operatorname{atan}^2\left(cx^2\right)}{4} - \frac{b^2x^2\operatorname{atan}\left(cx^2\right)}{2c} + \frac{ib^2\sqrt{\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{2c} + \frac{b^2\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{2c^2} + \frac{b^2\operatorname{atan}}{4} \\ \frac{a^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x**2)/2 - a*b*x**2/(2*c) + a*b*ata
n(c*x**2)/(2*c**2) + b**2*x**4*atan(c*x**2)**2/4 - b**2*x**2*atan(c*x**2)/(
2*c) + I*b**2*sqrt(c**(-2))*atan(c*x**2)/(2*c) + b**2*log(x**2 + I*sqrt(c**
(-2)))/(2*c**2) + b**2*atan(c*x**2)**2/(4*c**2), Ne(c, 0)), (a**2*x**4/4, T
rue))
```

3.77 $\int x \left(a + b \tan^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2+\frac{i\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{2c}+\frac{b\log\left(\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)}{c}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)}{2c}$$

```
[Out] 1/2*I*(a+b*arctan(c*x^2))^2/c+1/2*x^2*(a+b*arctan(c*x^2))^2+b*(a+b*arctan(c
*x^2))*ln(2/(1+I*c*x^2))/c+1/2*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c
```

Rubi [B] time = 0.55, antiderivative size = 255, normalized size of antiderivative = 2.52, number of steps used = 28, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{4c}+\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{4c}+\frac{ib\log\left(\frac{1}{2}\left(1+icx^2\right)\right)\left(2ia-b\log\left(1-icx^2\right)\right)}{4c}-\frac{1}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] ((I/8)*(1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/c + ((I/4)*b*((2*I)*a
- b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/c + ((I/4)*b^2*Log[(1 - I*c*x^2
)/2]*Log[1 + I*c*x^2])/c - (b*x^2*((2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*
c*x^2])/4 + ((I/8)*b^2*(1 + I*c*x^2)*Log[1 + I*c*x^2]^2)/c - ((I/4)*b^2*Pol
yLog[2, (1 - I*c*x^2)/2])/c + ((I/4)*b^2*PolyLog[2, (1 + I*c*x^2)/2])/c
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\right.$$

$$=\frac{1}{4}\int x\left(2a+ib\log\left(1-icx^2\right)\right)^2 dx+\frac{1}{2}b\int x\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+\right.$$

$$=\frac{1}{8}\operatorname{Subst}\left(\int(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\operatorname{Subst}\left(\int(-2ia+b\log(1-ic\right.$$

$$=-\frac{1}{4}bx^2\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)+\frac{i\operatorname{Subst}\left(\int(2a+ib\log(x))^2\,dx,x\right.}{8c}$$

$$=\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\frac{1}{4}bx^2\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx\right.$$

$$=-\frac{1}{2}iabx^2-\frac{b^2x^2}{4}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\frac{ib^2\left(1+icx^2\right)\log\left(1+\right.}{4c}$$

$$=-\frac{1}{2}b^2x^2+\frac{ib^2\left(1-icx^2\right)\log\left(1-icx^2\right)}{4c}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}+$$

$$=-\frac{1}{4}b^2x^2+\frac{ib^2\left(1-icx^2\right)\log\left(1-icx^2\right)}{4c}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}+$$

$$=\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}+\frac{ib\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(\frac{1}{2}\left(1+icx\right.\right.}{4c}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 1.06

$$\frac{a\left(acx^2-b\log\left(c^2x^4+1\right)\right)+2b\tan^{-1}\left(cx^2\right)\left(acx^2+b\log\left(1+e^{2i\tan^{-1}\left(cx^2\right)}\right)\right)-ib^2\operatorname{Li}_2\left(-e^{2i\tan^{-1}\left(cx^2\right)}\right)+b^2\left(cx^2\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*(-I + c*x^2)*ArcTan[c*x^2]^2 + 2*b*ArcTan[c*x^2]*(a*c*x^2 + b*Log[1 +
E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2 - b*Log[1 + c^2*x^4]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^2])])/(2*c)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2x\arctan\left(cx^2\right)^2+2\,abx\arctan\left(cx^2\right)+a^2x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctan(c*x^2)^2 + 2*a*b*x*arctan(c*x^2) + a^2*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^2\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*x, x)
```

maple [A] time = 0.21, size = 146, normalized size = 1.45

$$\frac{\arctan\left(c\,x^{2}\right)^{2} x^{2} b^{2}}{2}+x^{2} a b \arctan\left(c\,x^{2}\right)-\frac{i \arctan\left(c\,x^{2}\right)^{2} b^{2}}{2c}+\frac{a^{2} x^{2}}{2}-\frac{i \operatorname{polylog}\left(2,-\frac{\left(i c\,x^{2}+1\right)^{2}}{c^{2} x^{4}+1}\right) b^{2}}{2c}+\frac{\arctan\left(c\,x^{2}\right) \ln\left(\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] 1/2*arctan(c*x^2)^2*x^2*b^2+x^2*a*b*arctan(c*x^2)-1/2*I/c*arctan(c*x^2)^2*b
^2+1/2*a^2*x^2-1/2*I/c*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1))*b^2+1/c*arctan
(c*x^2)*ln((1+I*c*x^2)^2/(c^2*x^4+1)+1)*b^2-1/2/c*a*b*ln(c^2*x^4+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^{2}x^{2}+\frac{1}{32}\left(4\,x^{2}\arctan\left(cx^{2}\right)^{2}-x^{2}\log\left(c^{2}x^{4}+1\right)^{2}+384\,c^{2}\int\frac{x^{5}\arctan\left(cx^{2}\right)^{2}}{16\left(c^{2}x^{4}+1\right)}\,dx+32\,c^{2}\int\frac{x^{5}\log\left(c^{2}x^{4}+1\right)^{2}}{16\left(c^{2}x^{4}+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/32*(4*x^2*arctan(c*x^2)^2 - x^2*log(c^2*x^4 + 1)^2 + 384*c^
2*integrate(1/16*x^5*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 32*c^2*integrate(1
/16*x^5*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 128*c^2*integrate(1/16*x^5*l
og(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 4*arctan(c*x^2)^3/c - 256*c*integrate(1
/16*x^3*arctan(c*x^2)/(c^2*x^4 + 1), x) + 32*integrate(1/16*x*log(c^2*x^4 +
 1)^2/(c^2*x^4 + 1), x))*b^2 + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1
))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{atan}\left(c\,x^{2}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int(x*(a + b*atan(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atan}\left(cx^{2}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral(x*(a + b*atan(c*x**2))**2, x)
```

3.78 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x}\,dx$

Optimal. Leaf size=151

$$-\frac{1}{2}ib\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{1}{2}ib\text{Li}_2\left(\frac{2}{icx^2+1}-1\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)($$

```
[Out] -(a+b*arctan(c*x^2))^2*arctanh(-1+2/(1+I*c*x^2))-1/2*I*b*(a+b*arctan(c*x^2)
)*polylog(2,1-2/(1+I*c*x^2))+1/2*I*b*(a+b*arctan(c*x^2))*polylog(2,-1+2/(1+
I*c*x^2))-1/4*b^2*polylog(3,1-2/(1+I*c*x^2))+1/4*b^2*polylog(3,-1+2/(1+I*c*
x^2))
```

Rubi [A] time = 0.32, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5031, 4850, 4988, 4884, 4994, 6610}

$$-\frac{1}{2}ib\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{1}{2}ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}b^2\text{P}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 - 2/(1 + I*c*x^2)] - (I/2)*b*(a + b*ArcTa
n[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + (I/2)*b*(a + b*ArcTan[c*x^2])*P
olyLog[2, -1 + 2/(1 + I*c*x^2)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 +
 (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^2)])/4
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx,x,x^2\right)$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh}{1+c^2x^2}\right.$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)+(bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{}{1}\right.}{1+c^2x^2}\right.$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\frac{2}{1+icx}\right.$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\frac{2}{1+icx}\right.$$

Mathematica [A] time = 0.11, size = 165, normalized size = 1.09

$$\frac{1}{4}b\left(2i\text{Li}_2\left(\frac{cx^2+i}{i-cx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-2i\text{Li}_2\left(\frac{cx^2+i}{cx^2-i}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+b\left(\text{Li}_3\left(\frac{cx^2+i}{i-cx^2}\right)-\text{Li}_3\left(\frac{cx^2+i}{cx^2-i}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 + (2*I)/(-I + c*x^2)] + (b*((2*I)*(a + b*
ArcTan[c*x^2])*PolyLog[2, (I + c*x^2)/(I - c*x^2)] - (2*I)*(a + b*ArcTan[c*
x^2])*PolyLog[2, (I + c*x^2)/(-I + c*x^2)] + b*(PolyLog[3, (I + c*x^2)/(I -
 c*x^2)] - PolyLog[3, (I + c*x^2)/(-I + c*x^2)])))/4
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(cx^2\right)^2+2\,ab\arctan\left(cx^2\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\arctan\left(cx^2\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\log(x)+\frac{1}{16}\int \frac{12\,b^{2}\arctan\left(cx^{2}\right)^{2}+b^{2}\log\left(c^{2}x^{4}+1\right)^{2}+32\,ab\arctan\left(cx^{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^2)^2 + b^2*log(c^2*x^4 + 1)^
2 + 32*a*b*arctan(c*x^2))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x,x)
```

```
[Out] int((a + b*atan(c*x^2))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x, x)
```

3.79 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2}ic\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{2x^2}+bc\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{2}ib^2c\mathrm{Li}_2\left(\frac{2}{1-icx^2}-1\right)$$

```
[Out] -1/2*I*c*(a+b*arctan(c*x^2))^2-1/2*(a+b*arctan(c*x^2))^2/x^2+b*c*(a+b*arcta
n(c*x^2))*ln(2-2/(1-I*c*x^2))-1/2*I*b^2*c*polylog(2,-1+2/(1-I*c*x^2))
```

Rubi [B] time = 0.65, antiderivative size = 290, normalized size of antiderivative = 2.99, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5035, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$\frac{1}{2}ib^2c\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{2}ib^2c\text{PolyLog}\left(2,icx^2\right)-\frac{1}{4}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)+\frac{1}{4}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/(8*x^2) + (
I/4)*b*c*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2] + (I/4)*b^2*c*
Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2] + (b*((2*I)*a - b*Log[1 - I*c*x^2])*L
og[1 + I*c*x^2])/(4*x^2) + (b^2*(1 + I*c*x^2)*Log[1 + I*c*x^2]^2)/(8*x^2) +
 (I/2)*b^2*c*PolyLog[2, (-I)*c*x^2] - (I/2)*b^2*c*PolyLog[2, I*c*x^2] - (I/
4)*b^2*c*PolyLog[2, (1 - I*c*x^2)/2] + (I/4)*b^2*c*PolyLog[2, (1 + I*c*x^2)
/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

```
&& IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^3}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^3}-\frac{b^2\log^2(}{4}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^3}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^3}\,dx$$

$$=\frac{1}{8}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^2}\,dx,x,x^2\right)+\frac{1}{4}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx))}{x^2}\right.$$

$$=-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{4x^2}+\frac{b^2}{}$$

$$=abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+\right.}{4x^2}$$

$$=abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+\right.}{4x^2}$$

$$=2abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{1}{4}ibc\left(2ia-b\log\left(1-icx^2\right)\right)\log$$

$$=2abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{1}{4}ibc\left(2ia-b\log\left(1-icx^2\right)\right)\log$$

$$=2abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{1}{4}ibc\left(2ia-b\log\left(1-icx^2\right)\right)\log$$

Mathematica [A] time = 0.16, size = 127, normalized size = 1.31

$$-\frac{a^2}{2x^2}+abc\left(-\frac{1}{2}\log\left(c^2x^4+1\right)+\log\left(cx^2\right)-\frac{\tan^{-1}\left(cx^2\right)}{cx^2}\right)+\frac{1}{2}b^2c\left(-i\left(\tan^{-1}\left(cx^2\right)^2+\text{Li}_2\left(e^{2i\tan^{-1}\left(cx^2\right)}\right)\right)-\frac{\tan^{-1}\left(c\right.}{cx^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + a*b*c*(-(ArcTan[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 + c^2*x
^4]/2) + (b^2*c*(-(ArcTan[c*x^2]^2/(c*x^2)) + 2*ArcTan[c*x^2]*Log[1 - E^((2
*I)*ArcTan[c*x^2])] - I*(ArcTan[c*x^2]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x^2
])])))/2
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(cx^2\right)^2+2\,ab\arctan\left(cx^2\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^3, x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\,x^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^3,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^4+1\right) - \log\left(x^4\right)\right) + \frac{2\arctan\left(cx^2\right)}{x^2}\right)ab + \frac{\frac{1}{4}\left(8\,x^2\int -\frac{12\,c^2x^4\log\left(c^2x^4+1\right) - 56\,cx^2\arctan\left(cx^2\right) - 36\left(c^2x^4+1\right)\text{arc}}{4\left(c^2x^7+x^3\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a*b + 1/32*(32
*x^2*integrate(-1/16*(4*c^2*x^4*log(c^2*x^4 + 1) - 8*c*x^2*arctan(c*x^2) -
12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x
^7 + x^3), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x^2 - 1/2*a^2/x
^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^3,x)
```

```
[Out] int((a + b*atan(c*x^2))^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**3, x)
```

3.80 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{4}c^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{bc\left(a+b\tan^{-1}\left(cx^2\right)\right)}{2x^2}-\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{4x^4}-\frac{1}{4}b^2c^2\log\left(c^2x^4+1\right)+b^2c^2\log(x)$$

```
[Out] -1/2*b*c*(a+b*arctan(c*x^2))/x^2-1/4*c^2*(a+b*arctan(c*x^2))^2-1/4*(a+b*arc
tan(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)
```

Rubi [C] time = 1.14, antiderivative size = 419, normalized size of antiderivative = 4.82, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)+\frac{1}{8}bc^2\log\left(\frac{1}{2}\left(1+icx^2\right)\right)\left(2ia-b\log\left(1-icx^2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[I - c*x^2])/4 + ((I/8)*b*c*((2*I)*a - b*Log[1
 - I*c*x^2]))/x^2 - (b*c*(1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2]))/(8*x^2
) - (c^2*(2*a + I*b*Log[1 - I*c*x^2])^2)/16 - (2*a + I*b*Log[1 - I*c*x^2])^
2/(16*x^4) + (b*c^2*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/8
+ ((I/4)*b^2*c*Log[1 + I*c*x^2])/x^2 - (b^2*c^2*Log[(1 - I*c*x^2)/2]*Log[1
+ I*c*x^2])/8 + (b*((2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/(8*x^4)
 + (b^2*c^2*Log[1 + I*c*x^2]^2)/16 + (b^2*Log[1 + I*c*x^2]^2)/(16*x^4) - (b
^2*c^2*Log[I + c*x^2])/8 - (b^2*c^2*PolyLog[2, (1 - I*c*x^2)/2])/8 - (b^2*c
^2*PolyLog[2, (1 + I*c*x^2)/2])/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^5}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^5}-\frac{b^2\log\ldots}{\ldots}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^5}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^5}\ldots$$

$$=\frac{1}{8}\,\mathrm{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^3}\,dx,x,x^2\right)+\frac{1}{4}b\,\mathrm{Subst}\left(\int \frac{(-2ia+b\log(1-ic\ldots}{x^3}\ldots\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(1\ldots\right.}{16x\ldots}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(1\ldots\right.}{16x\ldots}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(1\ldots\right.}{16x\ldots}$$

$$=-\frac{1}{2}iabc^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\ldots\right.\right.}{8x^2}$$

$$=\frac{1}{4}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)}{8x^2}\ldots$$

$$=\frac{1}{2}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)}{8x^2}\ldots$$

Mathematica [A] time = 0.08, size = 98, normalized size = 1.13

$$-\frac{a^2+2b\tan^{-1}\left(cx^2\right)\left(ac^2x^4+a+bcx^2\right)+2abcx^2-4b^2c^2x^4\log(x)+b^2c^2x^4\log\left(c^2x^4+1\right)+b^2\left(c^2x^4+1\right)\tan\ldots}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] -1/4*(a^2 + 2*a*b*c*x^2 + 2*b*(a + b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b^2
*(1 + c^2*x^4)*ArcTan[c*x^2]^2 - 4*b^2*c^2*x^4*Log[x] + b^2*c^2*x^4*Log[1 +
 c^2*x^4])/x^4
```

fricas [A] time = 0.44, size = 115, normalized size = 1.32

$$\frac{2\,abc^2x^4\arctan\left(\frac{1}{cx^2}\right)-b^2c^2x^4\log\left(c^2x^4+1\right)+4\,b^2c^2x^4\log(x)-2\,abcx^2-\left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2-a^2-2\,(}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c^2*x^4*arctan(1/(c*x^2)) - b^2*c^2*x^4*log(c^2*x^4 + 1) + 4*b^2
*c^2*x^4*log(x) - 2*a*b*c*x^2 - (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - a^2 -
 2*(b^2*c*x^2 + a*b)*arctan(c*x^2))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\arctan\left(cx^2\right)+a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^5, x)
```

maple [A] time = 0.06, size = 118, normalized size = 1.36

$$-\frac{a^2}{4x^4}-\frac{b^2\arctan\left(c\,x^2\right)^2}{4x^4}-\frac{b^2c^2\arctan\left(c\,x^2\right)^2}{4}-\frac{b^2c\arctan\left(c\,x^2\right)}{2x^2}+b^2c^2\ln(x)-\frac{b^2c^2\ln\left(c^2x^4+1\right)}{4}-\frac{ab\arctan\left(c\,x^2\right)}{2x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arctan(c*x^2)^2-1/4*b^2*c^2*arctan(c*x^2)^2-1/2*b^
2*c*arctan(c*x^2)/x^2+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)-1/2*a*b/x^4*a
rctan(c*x^2)-1/2*a*b*c^2*arctan(c*x^2)-1/2*c*a*b/x^2
```

maxima [A] time = 0.52, size = 110, normalized size = 1.26

$$-\frac{1}{2}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)ab+\frac{1}{4}\left(\left(\arctan\left(cx^2\right)^2-\log\left(c^2x^4+1\right)+4\,\log(x)\right)c^2-2\left(c\arctan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a*b + 1/4*((arctan(c
*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*c*
arctan(c*x^2))*b^2 - 1/4*b^2*arctan(c*x^2)^2/x^4 - 1/4*a^2/x^4
```

mupad [B] time = 0.61, size = 152, normalized size = 1.75

$$b^2\,c^2\,\ln(x)-\frac{b^2\,c^2\,\mathrm{atan}\left(c\,x^2\right)^2}{4}-\frac{b^2\,\mathrm{atan}\left(c\,x^2\right)^2}{4\,x^4}-\frac{b^2\,c^2\,\ln\left(c^2\,x^4+1\right)}{4}-\frac{a^2}{4\,x^4}-\frac{b^2\,c\,\mathrm{atan}\left(c\,x^2\right)}{2\,x^2}-\frac{a\,b\,c}{2\,x^2}-\frac{a\,b\,c^2\,\mathrm{atan}\left(\frac{a^2\,c}{a^2+2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^5,x)
```

```
[Out] b^2*c^2*log(x) - (b^2*c^2*atan(c*x^2)^2)/4 - (b^2*atan(c*x^2)^2)/(4*x^4) -
(b^2*c^2*log(c^2*x^4 + 1))/4 - a^2/(4*x^4) - (b^2*c*atan(c*x^2))/(2*x^2) -
```

```
(a*b*c)/(2*x^2) - (a*b*c^2*atan((a^2*c*x^2)/(a^2 + 25*b^2) + (25*b^2*c*x^2)
/(a^2 + 25*b^2)))/2 - (a*b*atan(c*x^2))/(2*x^4)
```

sympy [A] time = 52.41, size = 167, normalized size = 1.92

$$\begin{cases} -\frac{a^2}{4x^4} - \frac{abc^2\operatorname{atan}\left(cx^2\right)}{2} - \frac{abc}{2x^2} - \frac{ab\operatorname{atan}\left(cx^2\right)}{2x^4} + b^2c^2\log\left(x\right) - \frac{b^2c^2\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{2} - \frac{b^2c^2\operatorname{atan}^2\left(cx^2\right)}{4} - \frac{ib^2c\operatorname{atan}\left(cx^2\right)}{2\sqrt{\frac{1}{c^2}}} - \frac{b^2c\operatorname{atan}\ldots}{2x\ldots} \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b*c**2*atan(c*x**2)/2 - a*b*c/(2*x**2) - a*b*
atan(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x**2 + I*sqrt(c**(
-2)))/2 - b**2*c**2*atan(c*x**2)**2/4 - I*b**2*c*atan(c*x**2)/(2*sqrt(c**(-
2))) - b**2*c*atan(c*x**2)/(2*x**2) - b**2*atan(c*x**2)**2/(4*x**4), Ne(c,
0)), (-a**2/(4*x**4), True))
```

3.81 $\int x^2 \left(a + b \tan^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=1393

$$\frac{1}{12}\left(2a + ib\log\left(1 - icx^2\right)\right)^2 x^3 - \frac{1}{12}b^2\log^2\left(icx^2 + 1\right)x^3 + \frac{2}{9}iabx^3 - \frac{1}{9}b^2\log\left(1 - icx^2\right)x^3 - \frac{1}{9}ib\left(2a + ib\log\left(1 - icx^2\right)\right.$$

```
[Out] 2/9*I*a*b*x^3+1/12*x^3*(2*a+I*b*ln(1-I*c*x^2))^2-2/3*(-1)^(3/4)*b^2*arctanh
((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(3/4
)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(
-1)^(3/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/
2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-1/3
*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/
2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-2/3*I*b^2*x*ln(1-I*c*x^2)/c-1/3*I*a*b
*x^3*ln(1+I*c*x^2)-2/3*(-1)^(1/4)*a*b*arctanh((-1)^(3/4)*x*c^(1/2))/c^(3/2)
-1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(3/2)-1/3
*(-1)^(1/4)*b*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))/c^(3/2)+
1/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(3/2)+1/3*(
-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(3/2)-2/3*(-1)^
(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(3/
2)+2/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(
1/2)))/c^(3/2)-1/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*(
(-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+2/3*(-1)^(3/4)*b^2*
arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(3/2)+2/3*I*
b^2*x*ln(1+I*c*x^2)/c+4/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))/c^(3/
2)+1/3*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(3/2)-4/3*(-1)^(3/4)
*b^2*arctanh((-1)^(3/4)*x*c^(1/2))/c^(3/2)-1/3*(-1)^(3/4)*b^2*arctanh((-1)^
(3/4)*x*c^(1/2))^2/c^(3/2)+1/6*b^2*x^3*ln(1-I*c*x^2)*ln(1+I*c*x^2)+1/3*(-1)
^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(1/4)*b
^2*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(1/4)*b^2*polyl
og(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+1/3
*(-1)^(3/4)*b^2*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(3
/4)*b^2*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(3/4)*b^2*
polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2
)-1/6*(-1)^(3/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/
4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(1/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*
x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1/9*I*b*x^3*(2*a+I*b*ln(1-I*c*
x^2))-1/9*b^2*x^3*ln(1-I*c*x^2)-1/12*b^2*x^3*ln(1+I*c*x^2)^2-4/3*a*b*x/c
```

Rubi [A] time = 2.60, antiderivative size = 1393, normalized size of antiderivative = 1.00, number of steps used = 86, number of rules used = 27, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.687$, Rules used = {5035, 2457, 2476, 2448, 321, 203, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315, 6742, 206, 30, 2557, 205, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (-4*a*b*x)/(3*c) + ((2*I)/9)*a*b*x^3 + (4*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*
Sqrt[c]*x])/(3*c^(3/2)) + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/(
3*c^(3/2)) - (2*(-1)^(1/4)*a*b*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) -
 (4*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) - ((-1)^(3/4)
*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTa
n[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2
*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2
```

```
]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2*(
-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x
)])/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
+ (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]
*x))])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
+ I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) -
 ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*
Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) - (((2*I)/3)*b^2*x*Log
[1 - I*c*x^2])/c - (b^2*x^3*Log[1 - I*c*x^2])/9 - ((-1)^(3/4)*b^2*ArcTanh[(
-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/(3*c^(3/2)) - (I/9)*b*x^3*(2*a + I*b
*Log[1 - I*c*x^2]) - ((-1)^(1/4)*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/(3*c^(3/2)) + (x^3*(2*a + I*b*Log[1 - I*c*x^2])^2)/12 +
(((2*I)/3)*b^2*x*Log[1 + I*c*x^2])/c - (I/3)*a*b*x^3*Log[1 + I*c*x^2] + ((-
1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2)) + (
(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2))
+ (b^2*x^3*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/6 - (b^2*x^3*Log[1 + I*c*x^2]
^2)/12 + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c
^(3/2)) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*
c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1 -
 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1
- 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1
 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/
2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - ((
1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \left(a + b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^2\left(2a + ib\log\left(1 - icx^2\right)\right)^2 + \frac{1}{2}bx^2\left(-2ia + b\log\left(1 - icx^2\right)\right)\log\left(1 + icx^2\right.\right.$$

$$= \frac{1}{4}\int x^2\left(2a + ib\log\left(1 - icx^2\right)\right)^2 dx + \frac{1}{2}b\int x^2\left(-2ia + b\log\left(1 - icx^2\right)\right)\log\left(1\right.$$

$$= \frac{1}{12}x^3\left(2a + ib\log\left(1 - icx^2\right)\right)^2 - \frac{1}{12}b^2x^3\log^2\left(1 + icx^2\right) + \frac{1}{2}b\int\left(-2iax^2\log\left(1\right.\right.$$

$$= \frac{1}{12}x^3\left(2a + ib\log\left(1 - icx^2\right)\right)^2 - \frac{1}{12}b^2x^3\log^2\left(1 + icx^2\right) - (iab)\int x^2\log\left(1 + ic\right.$$

$$= \frac{1}{12}x^3\left(2a + ib\log\left(1 - icx^2\right)\right)^2 - \frac{1}{3}iabx^3\log\left(1 + icx^2\right) + \frac{1}{6}b^2x^3\log\left(1 - icx^2\right)\text{lo}$$

$$= -\frac{2abx}{3c} - \frac{1}{9}ibx^3\left(2a + ib\log\left(1 - icx^2\right)\right) - \frac{\sqrt[4]{-1}\,b\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)\left(2a + ib\,\text{lo}\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} - \frac{2ib^2x}{3c} + \frac{2}{9}iabx^3 - \frac{ib^2x\log\left(1 - icx^2\right)}{3c} - \frac{1}{9}ibx^3\left(2a + ib\log\left(1 - icx^2\right)\right)$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 - \frac{4b^2x^3}{27} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{3c^{3/2}} - \frac{2\sqrt[4]{-1}\,ab\tanh^{-1}\left(\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 - \frac{4b^2x^3}{27} + \frac{8(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{9c^{3/2}} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 - \frac{4b^2x^3}{27} + \frac{8(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{9c^{3/2}} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 + \frac{14(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{9c^{3/2}} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 + \frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 + \frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 + \frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 + \frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}} + \frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

Mathematica [F] time = 5.18, size = 0, normalized size = 0.00

$$\int x^2\left(a + b\tan^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[x^2*(a + b*ArcTan[c*x^2])^2, x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\arctan\left(cx^2\right)^2+2\,abx^2\arctan\left(cx^2\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arctan(c*x^2)^2 + 2*a*b*x^2*arctan(c*x^2) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^2\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2*x^2, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\arctan\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^2))^2,x)
[Out] int(x^2*(a+b*arctan(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2x^3+\frac{1}{6}\left(4\,x^3\arctan\left(cx^2\right)-c\left(\frac{8\,x}{c^2}-\frac{\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\,\log\left(cx^2+\sqrt{2}\,\sqrt{c}\,x+1\right)}{\sqrt{c}}-}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2*
sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 + sq
rt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/s
qrt(c))/c^2))*a*b + 1/48*(4*x^3*arctan(c*x^2)^2 - x^3*log(c^2*x^4 + 1)^2 +
48*integrate(1/48*(8*c^2*x^6*log(c^2*x^4 + 1) - 16*c*x^4*arctan(c*x^2) + 36
*(c^2*x^6 + x^2)*arctan(c*x^2)^2 + 3*(c^2*x^6 + x^2)*log(c^2*x^4 + 1)^2)/(c
^2*x^4 + 1), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(cx^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x**2))**2, x)
```

3.82 $\int \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=1191

$$xa^2-\frac{2(-1)^{3/4}b\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)a}{\sqrt{c}}+\frac{2(-1)^{3/4}b\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)a}{\sqrt{c}}+ibx\log\left(1-icx^2\right)a-ibx\log\left(icx^2+1\right)a+\cdots$$

```
[Out] (-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/
2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^
(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+
I*a*b*x*ln(1-I*c*x^2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*
x^2)/c^(1/2)-(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(
1/2)-(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(1/2)+(-1)
^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(1/2)-2*(-1)^(3/4)
*a*b*arctan((-1)^(3/4)*x*c^(1/2))/c^(1/2)+2*(-1)^(3/4)*a*b*arctanh((-1)^(3/
4)*x*c^(1/2))/c^(1/2)+2*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1
-(-1)^(1/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2
))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+2*(-1)^(1/4)*b^2*arctanh((-1)^(3/
4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*arcta
nh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-I*a*b*x*ln(
1+I*c*x^2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/
4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*arctanh((-1)
^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2
)))/c^(1/2)+(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)-(-1)^(1/4
)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)+(-1)^(3/4)*b^2*polylog(2,1-2/
(1-(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(3/4)*b^2*polylog(2,1-2/(1+(-1)^(1/4
)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2))
)/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+1/
2*b^2*x*ln(1-I*c*x^2)*ln(1+I*c*x^2)-1/2*(-1)^(3/4)*b^2*polylog(2,1-2^(1/2)*
((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(1/4)*b^2
*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/
2)-1/2*(-1)^(1/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3
/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)
*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)-1/4*b^2*x*ln(1-I*c*x^2)^2-1/4
*b^2*x*ln(1+I*c*x^2)^2+a^2*x
```

Rubi [A] time = 1.78, antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 69, number of rules used = 23, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.917, Rules used = {5029, 2448, 321, 203, 2450, 2476, 2470, 12, 4920, 4854, 2402, 2315, 206, 2556, 205, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] a^2*x - (2*(-1)^(3/4)*a*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)^(3/
4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(3/4)*a*b*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]
*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 -
(-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[
c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)
*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log
[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
```

```
+ (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 +
 (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + I*a*b*x*Log[
1 - I*c*x^2] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2
])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2]
)/Sqrt[c] - (b^2*x*Log[1 - I*c*x^2]^2)/4 - I*a*b*x*Log[1 + I*c*x^2] - ((-1)
^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + ((-1)^(
1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + (b^2*x*L
og[1 - I*c*x^2]*Log[1 + I*c*x^2])/2 - (b^2*x*Log[1 + I*c*x^2]^2)/4 + ((-1)^
(3/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(3/
4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(3/4)*
b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[
c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt
[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]
*x)])/Sqrt[c] - ((-1)^(1/4)*b^2*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[
c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(1/4)*b^2*PolyLog[2
, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*
Sqrt[c]) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*
x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5029

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x],
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int \left(a^2+iab\log\left(1-icx^2\right)-\frac{1}{4}b^2\log^2\left(1-icx^2\right)-iab\log\left(1+icx^2\right)+\frac{1}{2}b^2\log\left(1-i\right.\right.$$

$$= a^2x+(iab)\int \log\left(1-icx^2\right)dx-(iab)\int \log\left(1+icx^2\right)dx-\frac{1}{4}b^2\int \log^2\left(1-icx^2\right)$$

$$= a^2x+iabx\log\left(1-icx^2\right)-\frac{1}{4}b^2x\log^2\left(1-icx^2\right)-iabx\log\left(1+icx^2\right)+\frac{1}{2}b^2x\log\left(1\right.$$

$$= a^2x+iabx\log\left(1-icx^2\right)-\frac{1}{4}b^2x\log^2\left(1-icx^2\right)-iabx\log\left(1+icx^2\right)+\frac{1}{2}b^2x\log\left(1\right.$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+iabx\log\left(\right.$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+iabx\log\left(\right.$$

$$= a^2x-4b^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+iai$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{2\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan}{}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab}{}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab}{}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab}{}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab}{}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab}{}$$

Mathematica [C] time = 31.54, size = 5504, normalized size = 4.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2,x]
[Out] Result too large to show
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\arctan\left(cx^2\right)^2+2\,ab\arctan\left(cx^2\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \arctan \left(cx^{2}\right) + a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(a + b \arctan \left(c\,x^{2}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(cx^{2}+\sqrt{2}\,\sqrt{c}\,x+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{2}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^
(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(
3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x
^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*a*b + 1/16*(4*x*a
rctan(c*x^2)^2 - x*log(c^2*x^4 + 1)^2 + 16*integrate(1/16*(8*c^2*x^4*log(c^
2*x^4 + 1) - 16*c*x^2*arctan(c*x^2) + 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 + (c
^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2,x)
```

```
[Out] int((a + b*atan(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atan}{\left(cx^{2} \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2, x)
```

3.83 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=1164

$$\sqrt[4]{-1}\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 b^2-(-1)^{3/4}\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 b^2+\frac{\log^2\left(icx^2+1\right)b^2}{4x}-2(-1)^{3/4}\sqrt{c}\tan^{-1}\left((-1)^3\right.$$

```
[Out] -1/4*(2*a+I*b*ln(1-I*c*x^2))^2/x-2*(-1)^(1/4)*a*b*arctanh((-1)^(3/4)*x*c^(1
/2))*c^(1/2)-2*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/
4)*x*c^(1/2)))*c^(1/2)+2*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(
1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+2*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1
/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-2*(-1)^(3/4)*b^2*arctanh((-1)^(
3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+I*a*b*ln(1+I*c*x^2)/
x-(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)*c^(1/2)-(-1)^(
1/4)*b*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))*c^(1/2)+(-1)^(3
/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)*c^(1/2)+(-1)^(3/4)*b^2*a
rctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)*c^(1/2)-(-1)^(3/4)*b^2*arctan((-
1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/
2)))*c^(1/2)+(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)
^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*arctanh(
(-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(
1/2)))*c^(1/2)-(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)
^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*polylog(
2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*polylog(2,1-2/(1+(-1
)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2*c
^(1/2)-(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))^2*c^(1/2)+(-1)^(1/4)*b^
2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+(-1)^(1/4)*b^2*polylog(2,
1-2/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)-1/2*b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x
-1/2*(-1)^(1/4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4
)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*
c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1-(
1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(1
/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)
))*c^(1/2)+1/4*b^2*ln(1+I*c*x^2)^2/x
```

Rubi [A] time = 1.62, antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2457, 203, 2470, 12, 4920, 4854, 2402, 2315, 2455, 6742, 206, 30, 2557, 205, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

$$\sqrt[4]{-1}\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 b^2-(-1)^{3/4}\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 b^2+\frac{\log^2\left(icx^2+1\right)b^2}{4x}-2(-1)^{3/4}\sqrt{c}\tan^{-1}\left((-1)^3\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

```
[Out] (-1)^(1/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(1/4)*a*b*Sq
rt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]^2 - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*L
og[2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/
4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)
^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)] - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^
(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^2*Sqrt[c]
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
```

```
+ (-1)^(3/4)*Sqrt[c]*x))] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[
c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)]
- (-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1
)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2] - (-1)^(1/4)*b*Sqrt[c]*ArcTan[
(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]) - (2*a + I*b*Log[1 - I*c
*x^2])^2/(4*x) + (I*a*b*Log[1 + I*c*x^2])/x + (-1)^(3/4)*b^2*Sqrt[c]*ArcTan
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-
1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] - (b^2*Log[1 - I*c*x^2]*Log[1 + I*c*x^
2])/(2*x) + (b^2*Log[1 + I*c*x^2]^2)/(4*x) + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog
[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1
 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - ((-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1 - (
Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/2 + (-1)^(3/
4)*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^
2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)] - ((-1)^(3/4)*b^2*Sq
rt[c]*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/2 - ((-1)^(3/4)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/
4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/2 - ((-1)^(1/4)*b^2*Sqrt[c]*Pol
yLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
```

```
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx = \int\left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^2}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^2}-\frac{b^2\log^2\left(\right.}{4x}\right.$$

$$=\frac{1}{4}\int\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^2}dx+\frac{1}{2}b\int\frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^2}dx$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x}+\frac{b^2\log^2\left(1+icx^2\right)}{4x}+\frac{1}{2}b\int\left(-\frac{2ia\log\left(1+icx^2\right)}{x^2}+\frac{b\log}{}\right.$$

$$=-\sqrt[4]{-1}\,b\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)\left(2a+ib\log\left(1-icx^2\right)\right)-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x}+$$

$$=-\sqrt[4]{-1}\,b\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)\left(2a+ib\log\left(1-icx^2\right)\right)-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x}+$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

$$=\sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\mathrm{ta}$$

Mathematica [B] time = 30.75, size = 5535, normalized size = 4.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^2,x]
[Out] Result too large to show
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\arctan\left(cx^2\right)^2+2\,ab\arctan\left(cx^2\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^2, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\,x^2\right)\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{\sqrt{c}} - \frac{\sqrt{2}\log\left(\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqr
t(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt
(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^
2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*a*b - 1/16*(4*arct
an(c*x^2)^2 - 16*x*integrate(-1/16*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*a
rctan(c*x^2) - 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4
 + 1)^2)/(c^2*x^6 + x^2), x) - log(c^2*x^4 + 1)^2)*b^2/x - a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x^2\right)\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^2,x)
```

```
[Out] int((a + b*atan(c*x^2))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx^2\right)\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**2, x)
```

3.84 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1360

$$\frac{1}{3}(-1)^{3/4}c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 b^2-\frac{1}{3}\sqrt[4]{-1}\,c^{3/2}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 b^2+\frac{\log^2\left(icx^2+1\right)b^2}{12x^3}-\frac{4}{3}\sqrt[4]{-1}\,c^{3/2}\tan^{-1}\left(\right.$$

```
[Out] -1/12*(2*a+I*b*ln(1-I*c*x^2))^2/x^3-2/3*a*b*c/x+2/3*(-1)^(3/4)*a*b*c^(3/2)*
arctanh((-1)^(3/4)*x*c^(1/2))-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)
*x*c^(1/2))*ln(1-I*c*x^2)-1/3*(-1)^(3/4)*b*c^(3/2)*arctan((-1)^(3/4)*x*c^(1
/2))*(2*a+I*b*ln(1-I*c*x^2))-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x
*c^(1/2))*ln(1+I*c*x^2)+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(
1/2))*ln(1+I*c*x^2)+2/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))
*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))-2/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4
)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arct
an((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*
c^(1/2)))+2/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1-
(-1)^(3/4)*x*c^(1/2)))-2/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1
/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)
^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2
)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-
1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*ar
ctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*
x*c^(1/2)))-1/3*I*b^2*c*ln(1-I*c*x^2)/x+1/3*I*a*b*ln(1+I*c*x^2)/x^3+2/3*I*b
^2*c*ln(1+I*c*x^2)/x-4/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2)
)+1/3*(-1)^(3/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))^2-4/3*(-1)^(1/4)*
b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctan
h((-1)^(3/4)*x*c^(1/2))^2-1/3*b*c*(2*a+I*b*ln(1-I*c*x^2))/x-1/6*b^2*ln(1-I*
c*x^2)*ln(1+I*c*x^2)/x^3+1/3*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1-2/(1-(-1)^(
1/4)*x*c^(1/2)))+1/3*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1-2/(1+(-1)^(1/4)*x*c
^(1/2)))-1/6*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/
2))/(1+(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,1-2/(1-(
-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,1-2/(1+(-1)^(3/4
)*x*c^(1/2)))-1/6*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*
c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,1-(
1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(3/4)*b^2*
c^(3/2)*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)
))+1/12*b^2*ln(1+I*c*x^2)^2/x^3
```

Rubi [A] time = 2.23, antiderivative size = 1360, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 25, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.562, Rules used = {5035, 2457, 2476, 2455, 203, 205, 2470, 12, 4920, 4854, 2402, 2315, 325, 6742, 206, 30, 2557, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

```
[Out] (-2*a*b*c)/(3*x) - (4*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/
3 + ((-1)^(3/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(3/
4)*a*b*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - (4*(-1)^(1/4)*b^2*c^(3/2)
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2
*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
```

```
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^
(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/3 +
((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^
(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/3 + ((-1)^(1/4)*b^2*c^(3/
2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*S
qrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/3 - ((I/3)*b^2*c*Log[1 - I*c*x^2])/x - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/3 - (b*c*(2*a + I*b*Log[1 - I*c*x^
2]))/(3*x) - ((-1)^(3/4)*b*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/3 - (2*a + I*b*Log[1 - I*c*x^2])^2/(12*x^3) + ((I/3)*a*b
*Log[1 + I*c*x^2])/x^3 + (((2*I)/3)*b^2*c*Log[1 + I*c*x^2])/x - ((-1)^(1/4)
*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 + ((-1)^(1/4)
*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 - (b^2*Log[1
 - I*c*x^2]*Log[1 + I*c*x^2])/(6*x^3) + (b^2*Log[1 + I*c*x^2]^2)/(12*x^3) +
 ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 +
((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 - (
(-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1
 + (-1)^(1/4)*Sqrt[c]*x)])/6 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 +
 (-1)^(3/4)*Sqrt[c]*x)])/3 - ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 + (Sqrt[2
]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/6 - ((-1)^(1/4)*b^
2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/
4)*Sqrt[c]*x)])/6 - ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-
1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
```

```
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^4}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^4}-\frac{b^2\log^2\left(\right.}{4x}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^4}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^4}\,dx$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3}+\frac{b^2\log^2\left(1+icx^2\right)}{12x^3}+\frac{1}{2}b\int \left(-\frac{2ia\log\left(1+icx^2\right)}{x^4}+\frac{b\log}{}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3}+\frac{b^2\log^2\left(1+icx^2\right)}{12x^3}-(iab)\int \frac{\log\left(1+icx^2\right)}{x^4}\,dx+\frac{1}{2}b^2\int$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3}+\frac{iab\log\left(1+icx^2\right)}{3x^3}-\frac{b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)}{6x^3}+$$

$$=-\frac{2abc}{3x}-\frac{bc\left(2a+ib\log\left(1-icx^2\right)\right)}{3x}-\frac{1}{3}(-1)^{3/4}bc^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)\left(2a+ib\log\right.$$

$$=-\frac{2abc}{3x}-\frac{2}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{2}{3}(-1)^{3/4}abc^{3/2}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{2}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{2}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

$$=-\frac{2abc}{3x}-\frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}$$

Mathematica [F] time = 2.94, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^4,x]
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^4, x]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2 \arctan\left(cx^2\right)^2 + 2\,ab \arctan\left(cx^2\right) + a^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x^4, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\,x^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^4,x)
[Out] int((a+b*arctan(c*x^2))^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(\left(c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{2}\,\mathrm{l}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="maxima")
[Out] -1/6*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))
/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/
c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(
c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)*a*b
 + 1/48*(48*x^3*integrate(-1/48*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*arct
an(c*x^2) - 36*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 3*(c^2*x^4 + 1)*log(c^2*x^4
+ 1)^2)/(c^2*x^8 + x^4), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x
^3 - 1/3*a^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^4,x)
[Out] int((a + b*atan(c*x^2))^2/x^4, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^2\right)\right)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**4,x)
[Out] Integral((a + b*atan(c*x**2))**2/x**4, x)
```

3.85 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1444

$$-\frac{1}{5}\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2c^{5/2}+\frac{1}{5}(-1)^{3/4}b^2\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2c^{5/2}-\frac{4}{15}(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)c^{5/2}$$

```
[Out] -1/5*I*b*c^2*(2*a+I*b*ln(1-I*c*x^2))/x+1/5*I*a*b*ln(1+I*c*x^2)/x^5+2/15*I*b
^2*c*ln(1+I*c*x^2)/x^3-1/20*(2*a+I*b*ln(1-I*c*x^2))^2/x^5-8/15*b^2*c^2/x+2/
5*(-1)^(1/4)*a*b*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))+1/5*(-1)^(3/4)*b^2*c
^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)+1/5*(-1)^(1/4)*b*c^(5/2)
*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))-1/5*(-1)^(3/4)*b^2*c^
(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)-1/5*(-1)^(3/4)*b^2*c^(5/2)
*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)+2/5*(-1)^(3/4)*b^2*c^(5/2)*arc
tan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))-2/5*(-1)^(3/4)*b^2
*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))+1/5*(-
1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*
c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-2/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^
(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))+2/5*(-1)^(3/4)*b^2*c^(5/2)*
arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)
*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2
))/(1+(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*
x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/5*
(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*
x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-1/15*I*b^2*c*ln(1-I*c*x^2)/x^3-2/15*a*
b*c/x^3-4/15*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))-1/5*(-1)^(
1/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))^2+4/15*(-1)^(3/4)*b^2*c^(5/2)
*arctanh((-1)^(3/4)*x*c^(1/2))+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4
)*x*c^(1/2))^2-1/5*b^2*c^2*ln(1-I*c*x^2)/x-1/15*b*c*(2*a+I*b*ln(1-I*c*x^2))
/x^3-1/10*b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x^5-1/5*(-1)^(1/4)*b^2*c^(5/2)*po
lylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))-1/5*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,
1-2/(1+(-1)^(1/4)*x*c^(1/2)))+1/10*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1-2^(1/
2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5
/2)*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5/2)*poly
log(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))+1/10*(-1)^(3/4)*b^2*c^(5/2)*polylog(2,1
+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/10*(-1)^(3/4)*b
^2*c^(5/2)*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/
2)))+1/10*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2)
)/(1+(-1)^(1/4)*x*c^(1/2)))+1/20*b^2*ln(1+I*c*x^2)^2/x^5+2/5*I*a*b*c^2/x
```

Rubi [A] time = 2.43, antiderivative size = 1444, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 25, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.562, Rules used = {5035, 2457, 2476, 2455, 325, 203, 205, 2470, 12, 4920, 4854, 2402, 2315, 6742, 206, 30, 2557, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

```
[Out] (-2*a*b*c)/(15*x^3) + (((2*I)/5)*a*b*c^2)/x - (8*b^2*c^2)/(15*x) - (4*(-1)^
(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/15 - ((-1)^(1/4)*b^2*c^(5/2
)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*(-1)^(1/4)*a*b*c^(5/2)*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x])/5 + (4*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c
]*x])/15 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*
(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*S
qrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[
```

```
2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4
)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]
*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/5 + (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*A
rcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 +
(-1)^(3/4)*Sqrt[c]*x))])/5 - ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1
 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 - ((I/15)*b^2*c*Lo
g[1 - I*c*x^2])/x^3 - (b^2*c^2*Log[1 - I*c*x^2])/(5*x) + ((-1)^(3/4)*b^2*c^
(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/5 - (b*c*(2*a + I*b*L
og[1 - I*c*x^2]))/(15*x^3) - ((I/5)*b*c^2*(2*a + I*b*Log[1 - I*c*x^2]))/x +
 ((-1)^(1/4)*b*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*
x^2]))/5 - (2*a + I*b*Log[1 - I*c*x^2])^2/(20*x^5) + ((I/5)*a*b*Log[1 + I*c
*x^2])/x^5 + (((2*I)/15)*b^2*c*Log[1 + I*c*x^2])/x^3 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - (b^2*Log[1 - I*c*x^
2]*Log[1 + I*c*x^2])/(10*x^5) + (b^2*Log[1 + I*c*x^2]^2)/(20*x^5) - ((-1)^(
1/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/5 - ((-1)^(1
/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(1/
4)*b^2*c^(5/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^
(1/4)*Sqrt[c]*x)])/10 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^
(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(
3/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 + (Sqrt[2]*((-1)
^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/10 + ((-1)^(3/4)*b^2*c^(5
/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/10 + ((-1)^(1/4)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3
/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^6}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^6}-\frac{b^2\log^2\left(\right.}{4\!:}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^6}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^6}\,dx$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5}+\frac{b^2\log^2\left(1+icx^2\right)}{20x^5}+\frac{1}{2}b\int \left(-\frac{2ia\log\left(1+icx^2\right)}{x^6}+\frac{b\log}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5}+\frac{b^2\log^2\left(1+icx^2\right)}{20x^5}-(iab)\int \frac{\log\left(1+icx^2\right)}{x^6}\,dx+\frac{1}{2}b^2\int$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5}+\frac{iab\log\left(1+icx^2\right)}{5x^5}-\frac{b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)}{10x^5}+\frac{b}{}$$

$$= -\frac{2abc}{15x^3}-\frac{bc\left(2a+ib\log\left(1-icx^2\right)\right)}{15x^3}-\frac{ibc^2\left(2a+ib\log\left(1-icx^2\right)\right)}{5x}+\frac{1}{5}\sqrt[4]{-1}\,bc^{5/2}\tan$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{4b^2c^2}{15x}-\frac{2}{5}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{2}{5}(-1)^{3/4}b^2c^{5/2}\tanh$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{4b^2c^2}{15x}-\frac{8}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{4b^2c^2}{15x}-\frac{8}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}-\frac{2}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}-\frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}-\frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}-\frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}-\frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

$$= -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}-\frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\tan^{-1}$$

Mathematica [F] time = 2.89, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^6, x]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \arctan\left(cx^2\right)^2 + 2\,ab\arctan\left(cx^2\right) + a^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\arctan\left(cx^2\right) + a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^6, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\arctan\left(c\,x^2\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^6,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{30}\left(\left(6\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\,cx + \sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right) + 6\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\,cx - \sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right) + 3\sqrt{2}\,c^{\frac{3}{2}}\log\left(cx^2 + \sqrt{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/30*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt
(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(
c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c^(3
/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^5)*a
*b + 1/80*(80*x^5*integrate(-1/80*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*ar
ctan(c*x^2) - 60*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 5*(c^2*x^4 + 1)*log(c^2*x^
4 + 1)^2)/(c^2*x^10 + x^6), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^
2/x^5 - 1/5*a^2/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x^2\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^6,x)
```

```
[Out] int((a + b*atan(c*x^2))^2/x^6, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^2\right)\right)^2}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**6,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**6, x)
```

3.86 $\int x^3 \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 dx$

Optimal. Leaf size=149

$$-\frac{3b^2\log\left(\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)}{2c^2}+\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{4c^2}-\frac{3ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{4c^2}-\frac{3bx^2\left(a+b\tan^{-1}\left(cx^2\right)\right)}{4c}$$

```
[Out] -3/4*I*b*(a+b*arctan(c*x^2))^2/c^2-3/4*b*x^2*(a+b*arctan(c*x^2))^2/c+1/4*(a
+b*arctan(c*x^2))^3/c^2+1/4*x^4*(a+b*arctan(c*x^2))^3-3/2*b^2*(a+b*arctan(c
*x^2))*ln(2/(1+I*c*x^2))/c^2-3/4*I*b^3*polylog(2,1-2/(1+I*c*x^2))/c^2
```

Rubi [B] time = 4.74, antiderivative size = 951, normalized size of antiderivative = 6.38, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2395, 2394, 2393, 2391, 2375, 2317, 2430, 2425}

$$\frac{3}{32}ib^2\left(2ia-b\log\left(1-icx^2\right)\right)\log^2\left(icx^2+1\right)x^4+\frac{3}{32}ib\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(icx^2+1\right)x^4+\frac{3b^2\left(2ia-b\log\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (((3*I)/64)*b^2*(1 - I*c*x^2)^2*((2*I)*a - b*Log[1 - I*c*x^2]))/c^2 + (((3*
I)/64)*b*(1 - I*c*x^2)^2*((2*I)*a - b*Log[1 - I*c*x^2])^2)/c^2 + (3*b^2*(1
- I*c*x^2)^2*(2*a + I*b*Log[1 - I*c*x^2]))/(64*c^2) - (((3*I)/16)*b*(1 - I*
c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/c^2 + (((3*I)/64)*b*(1 - I*c*x^2)^2*
(2*a + I*b*Log[1 - I*c*x^2])^2)/c^2 + ((1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c
*x^2])^3)/(16*c^2) - ((1 - I*c*x^2)^2*(2*a + I*b*Log[1 - I*c*x^2])^3)/(32*c
^2) - (((3*I)/8)*b^2*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/c
^2 + (((3*I)/32)*b*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[(1 + I*c*x^2)/2])/c
^2 + (((3*I)/32)*b*(2*a + I*b*Log[1 - I*c*x^2])^2*Log[(1 + I*c*x^2)/2])/c^2
 - (((3*I)/8)*b^3*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/c^2 + (3*b^2*x^2*(
(2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/(8*c) + ((3*I)/32)*b*x^4*((
2*I)*a - b*Log[1 - I*c*x^2])^2*Log[1 + I*c*x^2] - (((3*I)/32)*b*(2*a + I*b*
Log[1 - I*c*x^2])^2*Log[1 + I*c*x^2])/c^2 - (((3*I)/16)*b^3*(1 + I*c*x^2)*L
og[1 + I*c*x^2]^2)/c^2 + ((3*I)/32)*b^2*x^4*((2*I)*a - b*Log[1 - I*c*x^2])*
Log[1 + I*c*x^2]^2 - (3*b^2*(2*a + I*b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2]^2
)/(32*c^2) + ((I/16)*b^3*(1 + I*c*x^2)*Log[1 + I*c*x^2]^3)/c^2 - ((I/32)*b^
3*(1 + I*c*x^2)^2*Log[1 + I*c*x^2]^3)/c^2 + (((3*I)/8)*b^3*PolyLog[2, (1 -
I*c*x^2)/2])/c^2 - (((3*I)/16)*b^2*((2*I)*a - b*Log[1 - I*c*x^2])*PolyLog[2
, (1 - I*c*x^2)/2])/c^2 - (3*b^2*(2*a + I*b*Log[1 - I*c*x^2])*PolyLog[2, (1
 - I*c*x^2)/2])/(16*c^2) - (((3*I)/8)*b^3*PolyLog[2, (1 + I*c*x^2)/2])/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
```

```
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)^3 dx = \int\left(\frac{1}{8}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^3+\frac{3}{8}ibx^3\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)\right.$$
$$=\frac{1}{8}\int x^3\left(2a+ib\log\left(1-icx^2\right)\right)^3 dx+\frac{1}{8}(3ib)\int x^3\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(\right.$$
$$=\frac{1}{16}\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx,x,x^2\right)+\frac{1}{16}(3ib)\,\text{Subst}\left(\int x(-2ia+b\log\right.$$
$$=\frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx^2\right)\right)1$$
$$=\frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx^2\right)\right)1$$
$$=\frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx^2\right)\right)1$$
$$=\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c^2}-\frac{\left(1-icx^2\right)^2\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32c^2}+\frac{3ib\left(2\right.}{}$$
$$=\frac{3ib\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{32c^2}-\frac{3ib\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c^2}+$$
$$=\frac{9iab^2x^2}{8c}+\frac{9b^3x^2}{16c}-\frac{3ib^3\left(1-icx^2\right)^2}{128c^2}+\frac{3ib^3\left(1+icx^2\right)^2}{128c^2}+\frac{3ib\left(1-icx^2\right)\left(2ia-b\log\right.}{32c^2}$$
$$=\frac{9iab^2x^2}{8c}+\frac{9b^3x^2}{8c}-\frac{3ib^3\left(1-icx^2\right)^2}{128c^2}+\frac{3ib^3\left(1+icx^2\right)^2}{128c^2}-\frac{9ib^3\left(1-icx^2\right)\log\left(1-i\right.}{16c^2}$$
$$=\frac{3iab^2x^2}{4c}+\frac{15b^3x^2}{16c}-\frac{9ib^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{16c^2}+\frac{3ib^2\left(1-icx^2\right)^2\left(2ia-b\log\left(\right.\right.}{64c^2}$$
$$=\frac{3iab^2x^2}{4c}+\frac{3b^3x^2}{4c}-\frac{3ib^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{8c^2}+\frac{3ib^2\left(1-icx^2\right)^2\left(2ia-b\log\left(1\right.\right.}{64c^2}$$

Mathematica [A] time = 0.17, size = 170, normalized size = 1.14

$$\frac{a\left(acx^2\left(acx^2-3b\right)+3b^2\log\left(c^2x^4+1\right)\right)+3b^2\tan^{-1}\left(cx^2\right)^2\left(ac^2x^4+a+b\left(-cx^2+i\right)\right)+3b\tan^{-1}\left(cx^2\right)\left(a\left(ac^2x^4\right.\right.}{4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^4 + b*(I - c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 + c^2*x^4)*
ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a - 2*b*c*x^2 + a*c^2*x^4) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2*(-3*b + a*c*x^2) + 3*b^2*Log
[1 + c^2*x^4]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])])/(4*c^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\arctan\left(cx^2\right)^3+3\,ab^2x^3\arctan\left(cx^2\right)^2+3\,a^2bx^3\arctan\left(cx^2\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c*x^2)^3 + 3*a*b^2*x^3*arctan(c*x^2)^2 + 3*a^2*b*x^
3*arctan(c*x^2) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^2\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*x^3, x)
```

maple [C] time = 1.56, size = 690, normalized size = 4.63

$$-\frac{3a\,b^2x^4\ln\left(-ic\,x^2+1\right)^2}{16}+\frac{3a\,b^2\ln\left(c^2x^4+1\right)}{4c^2}-\frac{3a\,b^2\ln\left(-ic\,x^2+1\right)^2}{16c^2}-\frac{3b^2\left(ix^4b\ln\left(-ic\,x^2+1\right)c^2+2a\,c^2x^4-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2))^3,x)
```

```
[Out] -3/16*a*b^2*x^4*ln(1-I*c*x^2)^2+3/4/c^2*a*b^2*ln(c^2*x^4+1)-3/16/c^2*a*b^2*
ln(1-I*c*x^2)^2-3/32*b^2*(I*x^4*b*ln(1-I*c*x^2)*c^2+2*a*c^2*x^4-2*b*c*x^2+I
*b*ln(1-I*c*x^2)+2*I*b+2*a)/c^2*ln(1+I*c*x^2)^2-1/32*I*b^3*x^4*ln(1-I*c*x^2
)^3-3/4*I/c*a*b^2*x^2*ln(1-I*c*x^2)+3/8*I*a^2*b*x^4*ln(1-I*c*x^2)+1/32*I*b^
3*(c^2*x^4+1)/c^2*ln(1+I*c*x^2)^3+3/4*a^2*b/c^2*arctan(c*x^2)+3/16*b^3/c*x^
2*ln(1-I*c*x^2)^2+3/4*I/c*b^2*Sum((ln(x-_alpha)*ln(1-I*c*x^2)+2*c*(-1/2*ln(
x-_alpha)*(ln((1/2-1/2*I)*(I*(I/c)^(1/2)+(I/c)^(1/2)+x-_alpha)/(I/c)^(1/2))
+ln((-1/2-1/2*I)*(I*(I/c)^(1/2)-(I/c)^(1/2)-x+_alpha)/(I/c)^(1/2)))/c-1/2*(
dilog((1/2-1/2*I)*(I*(I/c)^(1/2)+(I/c)^(1/2)+x-_alpha)/(I/c)^(1/2))+dilog((
-1/2-1/2*I)*(I*(I/c)^(1/2)-(I/c)^(1/2)-x+_alpha)/(I/c)^(1/2)))/c))*b/c,_alp
ha=RootOf(c*_Z^2-RootOf(_Z^2+1,index=1)))-1/32*I*b^3/c^2*ln(1-I*c*x^2)^3+(3
/32*I*b^3*(c^2*x^4+1)/c^2*ln(1-I*c*x^2)^2+3/8*b^2*x^2*(a*c*x^2-b)/c*ln(1-I*
c*x^2)-3/8*I*b*(a^2*c^2*x^4-2*a*b*c*x^2+b^2*ln(1-I*c*x^2)+I*ln(1-I*c*x^2)*a
*b)/c^2)*ln(1+I*c*x^2)-3/4/c*b*x^2*a^2+1/4*x^4*a^3+3/16*I/c^2*b^3*ln(1-I*c*
x^2)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}ab^2x^4\arctan\left(cx^2\right)^2+\frac{1}{4}a^3x^4+\frac{3}{4}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)a^2b-\frac{3}{4}\left(2c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^3*x^4 + 3/4*(x^4*arctan(c*x^2) - c*(x
^2/c^2 - arctan(c*x^2)/c^3))*a^2*b - 3/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)
*arctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*a*b^2 + 1/
128*(4*x^4*arctan(c*x^2)^3 - 3*x^4*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 128*i
ntegrate(1/64*(12*c^2*x^7*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^5*arctan(
c*x^2)^2 + 56*(c^2*x^7 + x^3)*arctan(c*x^2)^3 + 3*(c*x^5 + 2*(c^2*x^7 + x^3
)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + b\,\mathrm{atan}\left(c\,x^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^2))^3,x)
```

```
[Out] int(x^3*(a + b*atan(c*x^2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b\,\mathrm{atan}\left(cx^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x**2))**3, x)
```

3.87 $\int x \left(a+b \tan^{-1}\left(c x^2\right)\right)^3 \, dx$

Optimal. Leaf size=144

$$\frac{3 i b^2 \text{Li}_2\left(1-\frac{2}{i c x^2+1}\right) \left(a+b \tan^{-1}\left(c x^2\right)\right)}{2 c}+\frac{1}{2} x^2 \left(a+b \tan^{-1}\left(c x^2\right)\right)^3+\frac{i \left(a+b \tan^{-1}\left(c x^2\right)\right)^3}{2 c}+\frac{3 b \log \left(\frac{2}{1+i c x^2}\right) \left(a+\right.}{2 c}$$

```
[Out] 1/2*I*(a+b*arctan(c*x^2))^3/c+1/2*x^2*(a+b*arctan(c*x^2))^3+3/2*b*(a+b*arct
an(c*x^2))^2*ln(2/(1+I*c*x^2))/c+3/2*I*b^2*(a+b*arctan(c*x^2))*polylog(2,1-
2/(1+I*c*x^2))/c+3/4*b^3*polylog(3,1-2/(1+I*c*x^2))/c
```

Rubi [B] time = 2.54, antiderivative size = 545, normalized size of antiderivative = 3.78, number of steps used = 82, number of rules used = 23, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.643, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{3 b^2 \text{PolyLog}\left(2,\frac{1}{2} \left(1-i c x^2\right)\right) \left(2 i a-b \log \left(1-i c x^2\right)\right)}{4 c}-\frac{3 b^3 \text{PolyLog}\left(3,\frac{1}{2} \left(1-i c x^2\right)\right)}{4 c}-\frac{3 b^3 \text{PolyLog}\left(3,\frac{1}{2} \left(1+\right.\right.}{4 c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (3*b*(1 - I*c*x^2)*((2*I)*a - b*Log[1 - I*c*x^2])^2)/(16*c) + (3*b*(1 - I*c
*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/(16*c) + ((I/16)*(1 - I*c*x^2)*(2*a +
 I*b*Log[1 - I*c*x^2])^3)/c + (3*b*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[(1
+ I*c*x^2)/2])/(8*c) - (3*b*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[1 + I*c*x^
2])/(16*c) + ((3*I)/16)*b*x^2*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[1 + I*c*
x^2] + (3*b^3*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2]^2)/(8*c) + (3*b^2*((2*I
)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2]^2)/(16*c) + ((3*I)/16)*b^2*x^2*(
(2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2]^2 + (b^3*(1 + I*c*x^2)*Log[1
 + I*c*x^2]^3)/(16*c) - (3*b^2*((2*I)*a - b*Log[1 - I*c*x^2])*PolyLog[2, (1
 - I*c*x^2)/2])/(4*c) + (3*b^3*Log[1 + I*c*x^2]*PolyLog[2, (1 + I*c*x^2)/2]
)/(4*c) - (3*b^3*PolyLog[3, (1 - I*c*x^2)/2])/(4*c) - (3*b^3*PolyLog[3, (1
+ I*c*x^2)/2])/(4*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\,dx=\int\left(\frac{1}{8}x\left(2a+ib\log\left(1-icx^2\right)\right)^3+\frac{3}{8}ibx\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)-\right.$$

$$=\frac{1}{8}\int x\left(2a+ib\log\left(1-icx^2\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+\right.$$

$$=\frac{1}{16}\operatorname{Subst}\left(\int(2a+ib\log(1-icx))^3\,dx,x,x^2\right)+\frac{1}{16}(3ib)\operatorname{Subst}\left(\int(-2ia+b\log(1-\right.$$

$$=\frac{3}{16}ibx^2\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{16}ib^2x^2\left(2ia-b\log\left(1-icx^2\right)\right)\mathrm{lo}$$

$$=\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c}+\frac{3}{16}ibx^2\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx\right.$$

$$=\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c}+\frac{3}{16}$$

$$=-\frac{3}{4}ab^2x^2-\frac{3}{8}ib^3x^2+\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i\left(1-icx^2\right)\left(2a+ib\,\mathrm{l}\right.}{16c}$$

$$=-\frac{3}{4}ab^2x^2+\frac{3b^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{8c}+\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+$$

$$=\frac{3}{8}ib^3x^2+\frac{3b^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{8c}+\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3}{}$$

$$=\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i}{}$$

$$=\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i}{}$$

Mathematica [A] time = 0.13, size = 224, normalized size = 1.56

$$\frac{2a^3cx^2-3a^2b\log\left(c^2x^4+1\right)+6a^2bcx^2\tan^{-1}\left(cx^2\right)-6ib^2\mathrm{Li}_2\left(-e^{2i\tan^{-1}\left(cx^2\right)}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-6iab^2\tan^{-1}\left(cx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (2*a^3*c*x^2 + 6*a^2*b*c*x^2*ArcTan[c*x^2] - (6*I)*a*b^2*ArcTan[c*x^2]^2 +
6*a*b^2*c*x^2*ArcTan[c*x^2]^2 - (2*I)*b^3*ArcTan[c*x^2]^3 + 2*b^3*c*x^2*Arc
Tan[c*x^2]^3 + 12*a*b^2*ArcTan[c*x^2]*Log[1 + E^((2*I)*ArcTan[c*x^2])] + 6*
b^3*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])] - 3*a^2*b*Log[1 + c^2*
x^4] - (6*I)*b^2*(a + b*ArcTan[c*x^2])*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])])/(4*c)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^3 x \arctan\left(cx^2\right)^3 + 3\,ab^2 x \arctan\left(cx^2\right)^2 + 3\,a^2 bx \arctan\left(cx^2\right) + a^3 x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c*x^2)^3 + 3*a*b^2*x*arctan(c*x^2)^2 + 3*a^2*b*x*arct
an(c*x^2) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \arctan\left(cx^2\right) + a\right)^3 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*x, x)
```

maple [B] time = 0.24, size = 306, normalized size = 2.12

$$\frac{x^2 a^3}{2} - \frac{i b^3 \arctan\left(c\,x^2\right)^3}{2c} + \frac{b^3 x^2 \arctan\left(c\,x^2\right)^3}{2} + \frac{3 b^3 \arctan\left(c\,x^2\right)^2 \ln\left(\frac{\left(i c\,x^2+1\right)^2}{c^2 x^4+1}+1\right)}{2c} - \frac{3 i b^3 \arctan\left(c\,x^2\right) \operatorname{polylog}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2))^3,x)
```

```
[Out] 1/2*x^2*a^3-1/2*I/c*b^3*arctan(c*x^2)^3+1/2*b^3*x^2*arctan(c*x^2)^3+3/2/c*b
^3*arctan(c*x^2)^2*ln((1+I*c*x^2)^2/(c^2*x^4+1)+1)-3/2*I/c*b^3*arctan(c*x^2
)*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1))+3/4/c*b^3*polylog(3,-(1+I*c*x^2)^2/
(c^2*x^4+1))-3/2*I/c*arctan(c*x^2)^2*a*b^2+3/2*x^2*a*b^2*arctan(c*x^2)^2+3/
c*ln((1+I*c*x^2)^2/(c^2*x^4+1)+1)*arctan(c*x^2)*a*b^2-3/2*I/c*polylog(2,-(1
+I*c*x^2)^2/(c^2*x^4+1))*a*b^2+3/2*x^2*a^2*b*arctan(c*x^2)-3/4/c*a^2*b*ln(c
^2*x^4+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\,b^3 x^2 \arctan\left(cx^2\right)^3 - \frac{3}{64}\,b^3 x^2 \arctan\left(cx^2\right) \log\left(c^2 x^4 + 1\right)^2 + \frac{7\,b^3 \arctan\left(cx^2\right)^4}{64\,c} + 28\,b^3 c^2 \int \frac{x^5 \arctan\left(cx^2\right)^3}{32\left(c^2 x^4 + 1\right)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^2*arctan(c*x^2)^3 - 3/64*b^3*x^2*arctan(c*x^2)*log(c^2*x^4 + 1)^
2 + 7/64*b^3*arctan(c*x^2)^4/c + 28*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2
)^3/(c^2*x^4 + 1), x) + 3*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2*
x^4 + 1)^2/(c^2*x^4 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^5*arctan(c*x^2
```

```
)^2/(c^2*x^4 + 1), x) + 12*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2
*x^4 + 1)/(c^2*x^4 + 1), x) + 1/2*a^3*x^2 + 1/2*a*b^2*arctan(c*x^2)^3/c - 1
2*b^3*c*integrate(1/32*x^3*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 3*b^3*c*inte
grate(1/32*x^3*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3*b^3*integrate(1/32*
x*arctan(c*x^2)*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3/4*(2*c*x^2*arctan(
c*x^2) - log(c^2*x^4 + 1))*a^2*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\operatorname{atan}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^2))^3,x)
```

```
[Out] int(x*(a + b*atan(c*x^2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{atan}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c*x**2))**3, x)
```

3.88 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x}\,dx$

Optimal. Leaf size=229

$$-\frac{3}{4}b^2\text{Li}_3\left(1-\frac{2}{icx^2+1}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{3}{4}b^2\text{Li}_3\left(\frac{2}{icx^2+1}-1\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{3}{4}ib\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)($$

```
[Out] -(a+b*arctan(c*x^2))^3*arctanh(-1+2/(1+I*c*x^2))-3/4*I*b*(a+b*arctan(c*x^2)
)^2*polylog(2,1-2/(1+I*c*x^2))+3/4*I*b*(a+b*arctan(c*x^2))^2*polylog(2,-1+2
/(1+I*c*x^2))-3/4*b^2*(a+b*arctan(c*x^2))*polylog(3,1-2/(1+I*c*x^2))+3/4*b^
2*(a+b*arctan(c*x^2))*polylog(3,-1+2/(1+I*c*x^2))+3/8*I*b^3*polylog(4,1-2/(
1+I*c*x^2))-3/8*I*b^3*polylog(4,-1+2/(1+I*c*x^2))
```

Rubi [A] time = 0.53, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5031, 4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{3}{4}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{3}{4}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{3}{4}ib\text{P}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 - 2/(1 + I*c*x^2)] - ((3*I)/4)*b*(a + b*A
rcTan[c*x^2])^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + ((3*I)/4)*b*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, -1 + 2/(1 + I*c*x^2)] - (3*b^2*(a + b*ArcTan[c*x^2])*
PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 + (3*b^2*(a + b*ArcTan[c*x^2])*PolyLog[3
, -1 + 2/(1 + I*c*x^2)])/4 + ((3*I)/8)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^2)]
- ((3*I)/8)*b^3*PolyLog[4, -1 + 2/(1 + I*c*x^2)]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx,x,x^2\right)$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tanh}{1+c^2x^2}\right.$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)+\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log}{1+c^2x^2}\right.$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+ic}\right.$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+ic}\right.$$

$$=\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+ic}\right.$$

Mathematica [A] time = 0.21, size = 245, normalized size = 1.07

$$\frac{3}{8}ib\left(2\text{Li}_2\left(\frac{cx^2+i}{i-cx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-2\text{Li}_2\left(\frac{cx^2+i}{cx^2-i}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)^2+b\left(-2i\text{Li}_3\left(\frac{cx^2+i}{i-cx^2}\right)\left(a+b\tan^{-1}\left(\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 + (2*I)/(-I + c*x^2)] + ((3*I)/8)*b*(2*(a
 + b*ArcTan[c*x^2])^2*PolyLog[2, (I + c*x^2)/(I - c*x^2)] - 2*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, (I + c*x^2)/(-I + c*x^2)] + b*((-2*I)*(a + b*ArcTan[c
*x^2])*PolyLog[3, (I + c*x^2)/(I - c*x^2)] + (2*I)*(a + b*ArcTan[c*x^2])*Po
lyLog[3, (I + c*x^2)/(-I + c*x^2)] + b*(-PolyLog[4, (I + c*x^2)/(I - c*x^2)
] + PolyLog[4, (I + c*x^2)/(-I + c*x^2)])))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3 \arctan\left(cx^2\right)^3 + 3\,ab^2 \arctan\left(cx^2\right)^2 + 3\,a^2 b \arctan\left(cx^2\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\,x^2\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \log(x) + \frac{1}{32} \int \frac{28\,b^3 \arctan\left(cx^2\right)^3 + 3\,b^3 \arctan\left(cx^2\right) \log\left(c^2x^4 + 1\right)^2 + 96\,ab^2 \arctan\left(cx^2\right)^2 + 96\,a^2 b \arcta}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^2)^3 + 3*b^3*arctan(c*x^2)*l
og(c^2*x^4 + 1)^2 + 96*a*b^2*arctan(c*x^2)^2 + 96*a^2*b*arctan(c*x^2))/x, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \,\mathrm{atan}\left(c\,x^2\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^3/x,x)
```

```
[Out] int((a + b*atan(c*x^2))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \,\mathrm{atan}\left(cx^2\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x, x)
```

3.89 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{3}{2}ib^2c\text{Li}_2\left(\frac{2}{1-icx^2}-1\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{2}ic\left(a+b\tan^{-1}\left(cx^2\right)\right)^3-\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{2x^2}+\frac{3}{2}bc\log\left(2-\frac{}{1-}\right.$$

```
[Out] -1/2*I*c*(a+b*arctan(c*x^2))^3-1/2*(a+b*arctan(c*x^2))^3/x^2+3/2*b*c*(a+b*a
rctan(c*x^2))^2*ln(2-2/(1-I*c*x^2))-3/2*I*b^2*c*(a+b*arctan(c*x^2))*polylog
(2,-1+2/(1-I*c*x^2))+3/4*b^3*c*polylog(3,-1+2/(1-I*c*x^2))
```

Rubi [F] time = 0.83, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] (3*b*c*Log[I*c*x^2]*(2*a + I*b*Log[1 - I*c*x^2])^2)/16 - ((1 - I*c*x^2)*(2*
a + I*b*Log[1 - I*c*x^2])^3)/(16*x^2) - (3*b^3*c*Log[(-I)*c*x^2]*Log[1 + I*
c*x^2]^2)/16 - ((I/16)*b^3*(1 + I*c*x^2)*Log[1 + I*c*x^2]^3)/x^2 + ((3*I)/8
)*b^2*c*(2*a + I*b*Log[1 - I*c*x^2])*PolyLog[2, 1 - I*c*x^2] - (3*b^3*c*Log
[1 + I*c*x^2]*PolyLog[2, 1 + I*c*x^2])/8 + (3*b^3*c*PolyLog[3, 1 - I*c*x^2]
)/8 + (3*b^3*c*PolyLog[3, 1 + I*c*x^2])/8 + ((3*I)/16)*b*Defer[Subst][Defer
[Int][(((-2*I)*a + b*Log[1 - I*c*x])^2*Log[1 + I*c*x])/x^2, x], x, x^2] - (
(3*I)/16)*b^2*Defer[Subst][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])*Log[1
+ I*c*x]^2)/x^2, x], x, x^2]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx=\int\left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{8x^3}+\frac{3ib\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)}{8x^3}-\frac{3ib}{}\right.$$

$$=\frac{1}{8}\int\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{x^3}dx+\frac{1}{8}(3ib)\int\frac{\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+\right.}{x^3}$$

$$=\frac{1}{16}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^2}dx,x,x^2\right)+\frac{1}{16}(3ib)\,\text{Subst}\left(\int\frac{(-2ia+b\log}{}\right.$$

$$=-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2}-\frac{ib^3\left(1+icx^2\right)\log^3\left(1+icx^2\right)}{16x^2}+\frac{1}{16}(3ib)\,\text{Su}$$

$$=\frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2}-$$

$$=\frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2}-$$

$$=\frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2}-$$

$$=\frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2}-$$

Mathematica [A] time = 0.46, size = 239, normalized size = 1.73

$$\frac{1}{4}\left(-\frac{2a^3}{x^2}-3a^2bc\log\left(c^2x^4+1\right)-\frac{6a^2b\tan^{-1}\left(cx^2\right)}{x^2}+12a^2bc\log(x)+6ab^2c\left(\tan^{-1}\left(cx^2\right)\left(\left(-\frac{1}{cx^2}-i\right)\tan^{-1}\left(cx^2\right)\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] ((-2*a^3)/x^2 - (6*a^2*b*ArcTan[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*c
*Log[1 + c^2*x^4] + 6*a*b^2*c*(ArcTan[c*x^2]*((-I - 1/(c*x^2))*ArcTan[c*x^2
] + 2*Log[1 - E^((2*I)*ArcTan[c*x^2])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^
2])]) + 2*b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^2]^3 - ArcTan[c*x^2]^3/(c*x^2
) + 3*ArcTan[c*x^2]^2*Log[1 - E^((-2*I)*ArcTan[c*x^2])] + (3*I)*ArcTan[c*x^
2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*
x^2])])/2))/4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(cx^2\right)^3+3\,ab^2\arctan\left(cx^2\right)^2+3\,a^2b\arctan\left(cx^2\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\arctan\left(cx^2\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x^3, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\arctan\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x^3,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{4}\left(c\left(\log\left(c^2x^4+1\right)-\log\left(x^4\right)\right)+\frac{2\arctan\left(cx^2\right)}{x^2}\right)a^2b-\frac{a^3}{2\,x^2}-\frac{\frac{15}{2}\,b^3\arctan\left(cx^2\right)^3-\frac{21}{8}\,b^3\arctan\left(cx^2\right)\log\left(c^2x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a^2*b - 1/2*a^
3/x^2 - 1/64*(4*b^3*arctan(c*x^2)^3 - 3*b^3*arctan(c*x^2)*log(c^2*x^4 + 1)^
2 - 64*x^2*integrate(-1/32*(12*b^3*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) -
 28*(b^3*c^2*x^4 + b^3)*arctan(c*x^2)^3 - 12*(8*a*b^2*c^2*x^4 + b^3*c*x^2 +
 8*a*b^2)*arctan(c*x^2)^2 + 3*(b^3*c*x^2 - (b^3*c^2*x^4 + b^3)*arctan(c*x^2
))*log(c^2*x^4 + 1)^2)/(c^2*x^7 + x^3), x))/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^3/x^3,x)
```

```
[Out] int((a + b*atan(c*x^2))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x**3, x)
```

3.90 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$

Optimal. Leaf size=149

$$\frac{3}{2}b^2c^2\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{3}{4}ibc^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{1}{4}c^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^3-\frac{3bc\left(a+b\tan^{-1}\right.}{4x^2}$$

```
[Out] -3/4*I*b*c^2*(a+b*arctan(c*x^2))^2-3/4*b*c*(a+b*arctan(c*x^2))^2/x^2-1/4*c^
2*(a+b*arctan(c*x^2))^3-1/4*(a+b*arctan(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*arct
an(c*x^2))*ln(2-2/(1-I*c*x^2))-3/4*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^2))
```

Rubi [F] time = 1.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (3*b*c*(1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^
2)/(32*x^2) + ((3*I)/32)*b*c^2*Log[I*c*x^2]*(2*a + I*b*Log[1 - I*c*x^2])^2
- (c^2*(2*a + I*b*Log[1 - I*c*x^2])^3)/32 - (2*a + I*b*Log[1 - I*c*x^2])^3/
(32*x^4) + (3*b^3*c*(1 + I*c*x^2)*Log[1 + I*c*x^2]^2)/(32*x^2) + ((3*I)/32)
*b^3*c^2*Log[(-I)*c*x^2]*Log[1 + I*c*x^2]^2 - (I/32)*b^3*c^2*Log[1 + I*c*x^
2]^3 - ((I/32)*b^3*Log[1 + I*c*x^2]^3)/x^4 + ((3*I)/16)*b^3*c^2*PolyLog[2,
(-I)*c*x^2] - ((3*I)/16)*b^3*c^2*PolyLog[2, I*c*x^2] - (3*b^2*c^2*(2*a + I*
b*Log[1 - I*c*x^2])*PolyLog[2, 1 - I*c*x^2])/16 + ((3*I)/16)*b^3*c^2*Log[1
+ I*c*x^2]*PolyLog[2, 1 + I*c*x^2] + ((3*I)/16)*b^3*c^2*PolyLog[3, 1 - I*c*
x^2] - ((3*I)/16)*b^3*c^2*PolyLog[3, 1 + I*c*x^2] + ((3*I)/16)*b*Defer[Subs
t][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])^2*Log[1 + I*c*x])/x^3, x], x,
x^2] - ((3*I)/16)*b^2*Defer[Subst][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x]
)*Log[1 + I*c*x]^2)/x^3, x], x, x^2]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{8x^5} + \frac{3ib\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)}{8x^5} - \frac{3ib}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{x^5}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+i\right.}{x^5}$$

$$= \frac{1}{16}\operatorname{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^3}\,dx, x, x^2\right) + \frac{1}{16}(3ib)\operatorname{Subst}\left(\int \frac{(-2ia+b\log}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4} - \frac{ib^3\log^3\left(1+icx^2\right)}{32x^4} + \frac{1}{16}(3ib)\operatorname{Subst}\left(\int \frac{(-2ia+b\log}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4} - \frac{ib^3\log^3\left(1+icx^2\right)}{32x^4} + \frac{1}{32}(3ib)\operatorname{Subst}\left(\int \frac{(2a+ib\log}{x\left(-\frac{i}{c}+\frac{ix}{c}\right.}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4} - \frac{ib^3\log^3\left(1+icx^2\right)}{32x^4} + \frac{1}{32}(3ib)\operatorname{Subst}\left(\int \frac{(2a+ib\log}{\left(-\frac{i}{c}+\frac{ix}{c}\right.}\right.$$

$$= -\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2} - \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4} + \frac{3b^3c\left(1+i\right.}{}$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2} + \frac{3}{32}ibc^2\log\left(icx^2\right)\left(2a+\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2} + \frac{3}{32}ibc^2\log\left(icx^2\right)\left(2a+\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2} + \frac{3}{32}ibc^2\log\left(icx^2\right)\left(2a+\right.$$

Mathematica [A] time = 0.35, size = 196, normalized size = 1.32

$$-\frac{a\left(a\left(a+3bcx^2\right)-6b^2c^2x^4\log\left(\frac{cx^2}{\sqrt{c^2x^4+1}}\right)\right)+3b^2\tan^{-1}\left(cx^2\right)^2\left(ac^2x^4+a+bcx^2\left(1+icx^2\right)\right)+3b\tan^{-1}\left(cx^2\right)\left(\right.}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

```
[Out] -1/4*(3*b^2*(a + a*c^2*x^4 + b*c*x^2*(1 + I*c*x^2))*ArcTan[c*x^2]^2 + b^3*(
1 + c^2*x^4)*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a + 2*b*c*x^2 + a*c^2*
x^4) - 2*b^2*c^2*x^4*Log[1 - E^((2*I)*ArcTan[c*x^2])]) + a*(a*(a + 3*b*c*x^
2) - 6*b^2*c^2*x^4*Log[(c*x^2)/Sqrt[1 + c^2*x^4]]) + (3*I)*b^3*c^2*x^4*Poly
Log[2, E^((2*I)*ArcTan[c*x^2])])/x^4
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^3\arctan\left(cx^2\right)^3+3\,ab^2\arctan\left(cx^2\right)^2+3\,a^2b\arctan\left(cx^2\right)+a^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="fricas")
```

[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*x^2) + a^3)/x^5, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="giac")

[Out] integrate((b*arctan(c*x^2) + a)^3/x^5, x)

maple [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\,x^2\right)\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctan(c*x^2))^3/x^5,x)

[Out] int((a+b*arctan(c*x^2))^3/x^5,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{4}\left(\left(c \arctan\left(cx^2\right) + \frac{1}{x^2}\right)c + \frac{\arctan\left(cx^2\right)}{x^4}\right)a^2b + \frac{3}{4}\left(\left(\arctan\left(cx^2\right)^2 - \log\left(c^2x^4 + 1\right) + 4\log(x)\right)c^2 - 2\left(c \arctan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="maxima")

[Out] -3/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a^2*b + 3/4*((arctan(c*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*c*arctan(c*x^2))*a*b^2 - 3/4*a*b^2*arctan(c*x^2)^2/x^4 + 1/128*(128*x^4*integrate(-1/64*(12*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^2*arctan(c*x^2)^2 - 56*(c^2*x^4 + 1)*arctan(c*x^2)^3 + 3*(c*x^2 - 2*(c^2*x^4 + 1)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^9 + x^5), x) - 4*arctan(c*x^2)^3 + 3*arctan(c*x^2)*log(c^2*x^4 + 1)^2)*b^3/x^4 - 1/4*a^3/x^4

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x^2\right)\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atan(c*x^2))^3/x^5,x)

[Out] int((a + b*atan(c*x^2))^3/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx^2\right)\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atan(c*x**2))**3/x**5,x)

[Out] Integral((a + b*atan(c*x**2))**3/x**5, x)

3.91 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$$

Mathematica [A] time = 1.96, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\arctan\left(cx^2\right)^3 + 3\,ab^2\arctan\left(cx^2\right)^2 + 3\,a^2b\arctan\left(cx^2\right) + a^3\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*(d*x)^m, x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx)^{m+1}a^3}{d(m+1)}+\frac{\frac{15}{2}b^3d^mxx^m\arctan\left(cx^2\right)^3-\frac{21}{8}b^3d^mxx^m\arctan\left(cx^2\right)\log\left(c^2x^4+1\right)^2+(m+1)\int\frac{168\,b^3c^2d^mx^4x^m\arctan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^2)^3 - 3*b
^3*d^m*x*x^m*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 32*(m + 1)*integrate(1/32*(
24*b^3*c^2*d^m*x^4*x^m*arctan(c*x^2)*log(c^2*x^4 + 1) + 28*(b^3*d^m*m + (b^
3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2)^3 - 24*(b^3*c*d
^m*x^2 - 4*a*b^2*d^m*m - 4*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^4 - 4*a*b^2*
d^m)*x^m*arctan(c*x^2)^2 + 96*(a^2*b*d^m*m + (a^2*b*c^2*d^m*m + a^2*b*c^2*d
^m)*x^4 + a^2*b*d^m)*x^m*arctan(c*x^2) + 3*(2*b^3*c*d^m*x^2*x^m + (b^3*d^m*
m + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2))*log(c^2
*x^4 + 1)^2)/((c^2*m + c^2)*x^4 + m + 1), x))/(m + 1)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Timed out
```

3.92 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$$

Mathematica [A] time = 1.28, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \arctan\left(cx^2\right)^2 + 2\,ab \arctan\left(cx^2\right) + a^2\right)(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \arctan\left(cx^2\right) + a\right)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2*(d*x)^m, x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b \arctan\left(c\,x^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx)^{m+1}a^2}{d(m+1)} + \frac{7\,b^2d^m x x^m \arctan\left(cx^2\right)^2 - \frac{3}{4}\,b^2d^m x x^m \log\left(c^2x^4+1\right)^2 + (m+1)\int \frac{24\,b^2c^2d^m x^4x^m \log\left(c^2x^4+1\right)+36\left(\left(b^2c^2d^m m+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^2)^2 - b^2
*d^m*x*x^m*log(c^2*x^4 + 1)^2 + 16*(m + 1)*integrate(1/16*(8*b^2*c^2*d^m*x^
4*x^m*log(c^2*x^4 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d^m*m
+ b^2*d^m)*x^m*arctan(c*x^2)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d
^m*m + b^2*d^m)*x^m*log(c^2*x^4 + 1)^2 - 16*(b^2*c*d^m*x^2 - 2*(a*b*c^2*d^m
*m + a*b*c^2*d^m)*x^4 - 2*a*b*d^m*m - 2*a*b*d^m)*x^m*arctan(c*x^2))/((c^2*m
 + c^2)*x^4 + m + 1), x))/(m + 1)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Timed out
```

3.93 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=75

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};-c^2x^4\right)}{d^3(m+1)(m+3)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3/4
+1/4*m],[7/4+1/4*m],-c^2*x^4)/d^3/(1+m)/(3+m)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5033, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};-c^2x^4\right)}{d^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*Hy
pergeometric2F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)])/(d^3*(1 + m)*(3 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tan^{-1}\left(cx^2\right)\right)dx&=\frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int\frac{x(dx)^{1+m}}{1+c^2x^4}\,dx}{d(1+m)}\\&=\frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int\frac{(dx)^{2+m}}{1+c^2x^4}\,dx}{d^2(1+m)}\\&=\frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};-c^2x^4\right)}{d^3(1+m)(3+m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.87

$$-\frac{x(dx)^m\left(2bcx^2\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};-c^2x^4\right)-(m+3)\left(a+b\tan^{-1}\left(cx^2\right)\right)\right)}{(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -((x*(d*x)^m*(-((3 + m)*(a + b*ArcTan[c*x^2])) + 2*b*c*x^2*Hypergeometric2F
1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)]))/((1 + m)*(3 + m)))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\arctan\left(cx^2\right)+a\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^2\right)+a\right)(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int(dx)^m\left(a+b\arctan\left(c\,x^2\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(d^mxx^m\arctan\left(cx^2\right)-2\left(cd^mm+cd^m\right)\int\frac{x^2x^m}{\left(c^2m+c^2\right)x^4+m+1}\,dx\right)b}{m+1}+\frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] (d^m*x*x^m*arctan(c*x^2) - 2*(c*d^m*m + c*d^m)*integrate(x^2*x^m/((c^2*m +
c^2)*x^4 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(d\,x)^m\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2)),x)
```

```
[Out] Timed out
```

3.94 $\int \frac{(dx)^m}{a+b\tan^{-1}(cx^2)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx = \int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx$$

Mathematica [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\arctan\left(cx^2\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x^2) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\arctan\left(cx^2\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

maple [**A**] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\arctan\left(c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\arctan\left(cx^2\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{atan}\left(c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^2)),x)
```

```
[Out] int((d*x)^m/(a + b*atan(c*x^2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2)),x)
```

```
[Out] Timed out
```

3.95 $\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\arctan\left(cx^2\right)^2+2\,ab\arctan\left(cx^2\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b\arctan\left(cx^2\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a)^2, x)
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\arctan\left(c\,x^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
[Out] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2d^mx^4+d^m\right)x^m-\left(b^2cx\arctan\left(cx^2\right)+abcx\right)\int \frac{\left(\left(c^2d^mm+3\,c^2d^m\right)x^4+d^mm-d^m\right)x^m}{b^2cx^2\arctan\left(cx^2\right)+abcx^2}\,dx}{2\left(b^2cx\arctan\left(cx^2\right)+abcx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
[Out] -1/2*((c^2*d^m*x^4 + d^m)*x^m - 2*(b^2*c*x*arctan(c*x^2) + a*b*c*x)*integra
te(1/2*((c^2*d^m*m + 3*c^2*d^m)*x^4 + d^m*m - d^m)*x^m/(b^2*c*x^2*arctan(c*
x^2) + a*b*c*x^2), x))/(b^2*c*x*arctan(c*x^2) + a*b*c*x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^2))^2,x)
[Out] int((d*x)^m/(a + b*atan(c*x^2))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2))**2,x)
[Out] Timed out
```

3.96 $\int x^{11} \left(a + b \tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=54

$$\frac{1}{12}x^{12}\left(a + b\tan^{-1}\left(cx^3\right)\right) - \frac{b\tan^{-1}\left(cx^3\right)}{12c^4} + \frac{bx^3}{12c^3} - \frac{bx^9}{36c}$$

```
[Out] 1/12*b*x^3/c^3-1/36*b*x^9/c-1/12*b*arctan(c*x^3)/c^4+1/12*x^12*(a+b*arctan(
c*x^3))
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5033, 275, 302, 203}

$$\frac{1}{12}x^{12}\left(a + b\tan^{-1}\left(cx^3\right)\right) + \frac{bx^3}{12c^3} - \frac{b\tan^{-1}\left(cx^3\right)}{12c^4} - \frac{bx^9}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*ArcTan[c*x^3])/(12*c^4) + (x^12*(a +
 b*ArcTan[c*x^3]))/12
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{4}(bc)\int\frac{x^{14}}{1+c^2x^6}\,dx\\ &= \frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\frac{x^4}{1+c^2x^2}\,dx,x,x^3\right)\\ &= \frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)dx,x\right.\\ &= \frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{12c^3}\\ &= \frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\tan^{-1}\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.09

$$\frac{ax^{12}}{12}-\frac{b\tan^{-1}\left(cx^3\right)}{12c^4}+\frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{1}{12}bx^{12}\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) + (a*x^12)/12 - (b*ArcTan[c*x^3])/(12*c^4
) + (b*x^12*ArcTan[c*x^3])/12
```

fricas [A] time = 0.41, size = 51, normalized size = 0.94

$$\frac{3\,ac^4x^{12}-bc^3x^9+3\,bcx^3+3\left(bc^4x^{12}-b\right)\arctan\left(cx^3\right)}{36\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/36*(3*a*c^4*x^12 - b*c^3*x^9 + 3*b*c*x^3 + 3*(b*c^4*x^12 - b)*arctan(c*x^
3))/c^4
```

giac [A] time = 0.16, size = 60, normalized size = 1.11

$$\frac{3\,acx^{12}+\left(3\,cx^{12}\arctan\left(cx^3\right)-\frac{3\,\arctan\left(cx^3\right)}{c^3}-\frac{c^9x^9-3\,c^7x^3}{c^9}\right)b}{36\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/36*(3*a*c*x^12 + (3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9*x^9
 - 3*c^7*x^3)/c^9)*b)/c
```

maple [A] time = 0.03, size = 50, normalized size = 0.93

$$\frac{x^{12}a}{12}+\frac{b\,x^{12}\arctan\left(c\,x^3\right)}{12}-\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}-\frac{b\arctan\left(c\,x^3\right)}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/12*x^12*a+1/12*b*x^12*arctan(c*x^3)-1/36*b*x^9/c+1/12*b*x^3/c^3-1/12*b*ar
ctan(c*x^3)/c^4
```

maxima [A] time = 0.42, size = 54, normalized size = 1.00

$$\frac{1}{12}\,ax^{12}+\frac{1}{36}\left(3\,x^{12}\arctan\left(cx^{3}\right)-c\left(\frac{c^{2}x^{9}-3\,x^{3}}{c^{4}}+\frac{3\,\arctan\left(cx^{3}\right)}{c^{5}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/12*a*x^12 + 1/36*(3*x^12*arctan(c*x^3) - c*((c^2*x^9 - 3*x^3)/c^4 + 3*arc
tan(c*x^3)/c^5))*b
```

mupad [B] time = 0.45, size = 49, normalized size = 0.91

$$\frac{a\,x^{12}}{12}+\frac{b\,x^3}{12\,c^3}-\frac{b\,x^9}{36\,c}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{12\,c^4}+\frac{b\,x^{12}\,\mathrm{atan}\left(c\,x^3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^12)/12 + (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*atan(c*x^3))/(12*c^4)
+ (b*x^12*atan(c*x^3))/12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atan(c*x**3)),x)
```

```
[Out] Timed out
```

3.97 $\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=47

$$\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^2x^6+1\right)}{18c^3}-\frac{bx^6}{18c}$$

```
[Out] -1/18*b*x^6/c+1/9*x^9*(a+b*arctan(c*x^3))+1/18*b*ln(c^2*x^6+1)/c^3
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 43}

$$\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^2x^6+1\right)}{18c^3}-\frac{bx^6}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*x^6)/(18*c) + (x^9*(a + b*ArcTan[c*x^3]))/9 + (b*Log[1 + c^2*x^6])/(18*
c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^8\left(a+b\tan^{-1}\left(cx^3\right)\right) dx &= \frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{3}(bc)\int \frac{x^{11}}{1+c^2x^6}\,dx\\
&= \frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int \frac{x}{1+c^2x}\,dx,x,x^6\right)\\
&= \frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int \left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right) dx,x,x^6\right)\\
&= -\frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.11

$$\frac{ax^9}{9}+\frac{b\log\left(c^2x^6+1\right)}{18c^3}-\frac{bx^6}{18c}+\frac{1}{9}bx^9\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/18*(b*x^6)/c + (a*x^9)/9 + (b*x^9*ArcTan[c*x^3])/9 + (b*Log[1 + c^2*x^6]
)/(18*c^3)
```

fricas [A] time = 0.42, size = 51, normalized size = 1.09

$$\frac{2\,bc^3x^9\arctan\left(cx^3\right)+2\,ac^3x^9-bc^2x^6+b\log\left(c^2x^6+1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/18*(2*b*c^3*x^9*arctan(c*x^3) + 2*a*c^3*x^9 - b*c^2*x^6 + b*log(c^2*x^6 +
 1))/c^3
```

giac [A] time = 0.81, size = 47, normalized size = 1.00

$$\frac{2\,acx^9+\left(2\,cx^9\arctan\left(cx^3\right)-x^6+\frac{\log\left(c^2x^6+1\right)}{c^2}\right)b}{18\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/18*(2*a*c*x^9 + (2*c*x^9*arctan(c*x^3) - x^6 + log(c^2*x^6 + 1)/c^2)*b)/c
```

maple [A] time = 0.03, size = 45, normalized size = 0.96

$$\frac{x^9a}{9}+\frac{b\,x^9\arctan\left(c\,x^3\right)}{9}-\frac{b\,x^6}{18c}+\frac{b\ln\left(c^2x^6+1\right)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/9*x^9*a+1/9*b*x^9*arctan(c*x^3)-1/18*b*x^6/c+1/18*b*ln(c^2*x^6+1)/c^3
```

maxima [A] time = 0.31, size = 48, normalized size = 1.02

$$\frac{1}{9}\,ax^9+\frac{1}{18}\left(2\,x^9\arctan\left(cx^3\right)-\left(\frac{x^6}{c^2}-\frac{\log\left(c^2x^6+1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9 + 1/18*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c)
*b
```

mupad [B] time = 0.38, size = 44, normalized size = 0.94

$$\frac{a\,x^9}{9}+\frac{b\,\ln\left(c^2\,x^6+1\right)}{18\,c^3}-\frac{b\,x^6}{18\,c}+\frac{b\,x^9\,\mathrm{atan}\left(c\,x^3\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^9)/9 + (b*log(c^2*x^6 + 1))/(18*c^3) - (b*x^6)/(18*c) + (b*x^9*atan(c*
x^3))/9
```

sympy [A] time = 177.26, size = 133, normalized size = 2.83

$$\begin{cases} \frac{ax^9}{9} + \frac{bx^9 \operatorname{atan}\left(cx^3\right)}{9} - \frac{bx^6}{18c} - \frac{ib\sqrt{\frac{1}{c^2}} \operatorname{atan}\left(cx^3\right)}{9c^2} + \frac{b\log\left(x - \sqrt[6]{-1}\sqrt[6]{\frac{1}{c^2}}\right)}{9c^3} + \frac{b\log\left(4x^2 + 4\sqrt[6]{-1}x\sqrt[6]{\frac{1}{c^2}} + 4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{9c^3} & \text{for } c \neq 0 \\ \frac{ax^9}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**9/9 + b*x**9*atan(c*x**3)/9 - b*x**6/(18*c) - I*b*sqrt(c**(
-2))*atan(c*x**3)/(9*c**2) + b*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(9*c**
3) + b*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-
2))**(1/3))/(9*c**3), Ne(c, 0)), (a*x**9/9, True))
```

3.98 $\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right) \, dx$

Optimal. Leaf size=43

$$\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}-\frac{bx^3}{6c}$$

```
[Out] -1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2+1/6*x^6*(a+b*arctan(c*x^3))
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 321, 203}

$$\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}-\frac{bx^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3]),x]
[Out] -(b*x^3)/(6*c) + (b*ArcTan[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3]))/6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right) dx &= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}(bc)\int \frac{x^8}{1+c^2x^6}\,dx \\
&= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{x^2}{1+c^2x^2}\,dx,x,x^3\right) \\
&= -\frac{bx^3}{6c}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\,\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{6c} \\
&= -\frac{bx^3}{6c}+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.12

$$\frac{ax^6}{6}+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}-\frac{bx^3}{6c}+\frac{1}{6}bx^6\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/6*(b*x^3)/c + (a*x^6)/6 + (b*ArcTan[c*x^3])/(6*c^2) + (b*x^6*ArcTan[c*x^
3])/6
```

fricas [A] time = 0.42, size = 38, normalized size = 0.88

$$\frac{ac^2x^6-bcx^3+\left(bc^2x^6+b\right)\arctan\left(cx^3\right)}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*c^2*x^6 - b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3))/c^2
```

giac [A] time = 0.17, size = 43, normalized size = 1.00

$$\frac{acx^6+\frac{\left(c^2x^6\arctan\left(cx^3\right)-cx^3+\arctan\left(cx^3\right)\right)b}{c}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/6*(a*c*x^6 + (c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*b/c)/c
```

maple [A] time = 0.03, size = 41, normalized size = 0.95

$$\frac{x^6a}{6}+\frac{b\,x^6\arctan\left(c\,x^3\right)}{6}-\frac{b\,x^3}{6c}+\frac{b\arctan\left(c\,x^3\right)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctan(c*x^3)-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2
```

maxima [A] time = 0.41, size = 43, normalized size = 1.00

$$\frac{1}{6}ax^6+\frac{1}{6}\left(x^6\arctan\left(cx^3\right)-c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/6*(x^6*arctan(c*x^3) - c*(x^3/c^2 - arctan(c*x^3)/c^3))*b
```

mupad [B] time = 0.40, size = 40, normalized size = 0.93

$$\frac{a\,x^6}{6}-\frac{b\,x^3}{6\,c}+\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{6\,c^2}+\frac{b\,x^6\,\mathrm{atan}\left(c\,x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^6)/6 - (b*x^3)/(6*c) + (b*atan(c*x^3))/(6*c^2) + (b*x^6*atan(c*x^3))/6
```

sympy [A] time = 89.85, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^6}{6}+\frac{bx^6\,\mathrm{atan}\left(cx^3\right)}{6}-\frac{bx^3}{6c}+\frac{b\,\mathrm{atan}\left(cx^3\right)}{6c^2} & \text{for}\ c\neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**3)/6 - b*x**3/(6*c) + b*atan(c*x**3)
/(6*c**2), Ne(c, 0)), (a*x**6/6, True))
```

3.99 $\int x^2 \left(a + b \tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=36

$$\frac{1}{3}x^3\left(a + b\tan^{-1}\left(cx^3\right)\right) - \frac{b\log\left(c^2x^6 + 1\right)}{6c}$$

```
[Out] 1/3*x^3*(a+b*arctan(c*x^3))-1/6*b*ln(c^2*x^6+1)/c
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5033, 260}

$$\frac{1}{3}x^3\left(a + b\tan^{-1}\left(cx^3\right)\right) - \frac{b\log\left(c^2x^6 + 1\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (x^3*(a + b*ArcTan[c*x^3]))/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a + b\tan^{-1}\left(cx^3\right)\right) dx &= \frac{1}{3}x^3\left(a + b\tan^{-1}\left(cx^3\right)\right) - (bc)\int \frac{x^5}{1 + c^2x^6}\, dx\\ &= \frac{1}{3}x^3\left(a + b\tan^{-1}\left(cx^3\right)\right) - \frac{b\log\left(1 + c^2x^6\right)}{6c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.14

$$\frac{ax^3}{3} - \frac{b\log\left(c^2x^6 + 1\right)}{6c} + \frac{1}{3}bx^3\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcTan[c*x^3])/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

fricas [A] time = 0.42, size = 39, normalized size = 1.08

$$\frac{2\,bcx^3\arctan\left(cx^3\right) + 2\,acx^3 - b\log\left(c^2x^6 + 1\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c*x^3*arctan(c*x^3) + 2*a*c*x^3 - b*log(c^2*x^6 + 1))/c
```

giac [A] time = 0.71, size = 40, normalized size = 1.11

$$\frac{2\,acx^3 + \left(2\,cx^3 \arctan\left(cx^3\right) - \log\left(c^2x^6 + 1\right)\right)b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/6*(2*a*c*x^3 + (2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b)/c
```

maple [A] time = 0.03, size = 36, normalized size = 1.00

$$\frac{x^3a}{3} + \frac{b\,x^3 \arctan\left(c\,x^3\right)}{3} - \frac{b\ln\left(c^2x^6 + 1\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c*x^3)-1/6*b*ln(c^2*x^6+1)/c
```

maxima [A] time = 0.31, size = 38, normalized size = 1.06

$$\frac{1}{3}\,ax^3 + \frac{\left(2\,cx^3 \arctan\left(cx^3\right) - \log\left(c^2x^6 + 1\right)\right)b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b/c
```

mupad [B] time = 0.10, size = 35, normalized size = 0.97

$$\frac{a\,x^3}{3} - \frac{b\,\ln\left(c^2\,x^6 + 1\right)}{6\,c} + \frac{b\,x^3\,\mathrm{atan}\left(c\,x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^3)/3 - (b*log(c^2*x^6 + 1))/(6*c) + (b*x^3*atan(c*x^3))/3
```

sympy [A] time = 47.59, size = 117, normalized size = 3.25

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{atan}\left(cx^3\right)}{3} + \frac{ib\sqrt{\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{3} - \frac{b\log\left(x - \sqrt[6]{-1}\,\sqrt[6]{\frac{1}{c^2}}\right)}{3c} - \frac{b\log\left(4x^2 + 4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}} + 4\sqrt[3]{-1}\,\sqrt[3]{\frac{1}{c^2}}\right)}{3c} & \text{for}\ c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x**3)/3 + I*b*sqrt(c**(-2))*atan(c*x**3
)/3 - b*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(3*c) - b*log(4*x**2 + 4*(-1)
**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(3*c), Ne(c, 0
)), (a*x**3/3, True))
```

3.100 $\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x}\,dx$

Optimal. Leaf size=39

$$a\log(x)+\frac{1}{6}ib\text{Li}_2\left(-icx^3\right)-\frac{1}{6}ib\text{Li}_2\left(icx^3\right)$$

```
[Out] a*ln(x)+1/6*I*b*polylog(2,-I*c*x^3)-1/6*I*b*polylog(2,I*c*x^3)
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5031, 4848, 2391}

$$\frac{1}{6}ib\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\text{PolyLog}\left(2,icx^3\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,x^3\right)\\ &= a\log(x)+\frac{1}{6}(ib)\,\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,x^3\right)-\frac{1}{6}(ib)\,\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,\right.\\ &= a\log(x)+\frac{1}{6}ib\text{Li}_2\left(-icx^3\right)-\frac{1}{6}ib\text{Li}_2\left(icx^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$a\log(x)+\frac{1}{6}ib\text{Li}_2\left(-icx^3\right)-\frac{1}{6}ib\text{Li}_2\left(icx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\arctan\left(cx^3\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="fricas")
[Out] integral((b*arctan(c*x^3) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\arctan\left(cx^3\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)/x, x)
```

maple [C] time = 0.10, size = 63, normalized size = 1.62

$$a\ln(x)+b\ln(x)\arctan\left(c\,x^3\right)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2\_Z^6+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^3}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x,x)
[Out] a*ln(x)+b*ln(x)*arctan(c*x^3)-1/2*b/c*sum(1/_R1^3*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^6*c^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\arctan\left(cx^3\right)}{x}\,dx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="maxima")
[Out] b*integrate(arctan(c*x^3)/x, x) + a*log(x)
```

mupad [B] time = 0.35, size = 32, normalized size = 0.82

$$a\,\ln(x)-\frac{b\,\left(\mathrm{Li}_2\left(1-c\,x^3\,1\mathrm{i}\right)-\mathrm{Li}_2\left(1\mathrm{i}\,c\,x^3+1\right)\right)\,1\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x,x)
[Out] a*log(x) - (b*(dilog(1 - c*x^3*1i) - dilog(c*x^3*1i + 1))*1i)/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{atan}\left(cx^3\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x,x)
```

```
[Out] Integral((a + b*atan(c*x**3))/x, x)
```

3.101 $\int \frac{a+b\tan^{-1}(cx^3)}{x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{a+b\tan^{-1}(cx^3)}{3x^3}-\frac{1}{6}bc\log(c^2x^6+1)+bc\log(x)$$

```
[Out] 1/3*(-a-b*arctan(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(c^2*x^6+1)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5033, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}(cx^3)}{3x^3}-\frac{1}{6}bc\log(c^2x^6+1)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^4,x]
```

```
[Out] -(a + b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^4}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+(bc)\int \frac{1}{x\left(1+c^2x^6\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx,x,x^6\right)$$
$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^6\right)-\frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx,x,\right.$$
$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.13

$$-\frac{a}{3x^3}-\frac{1}{6}bc\log\left(c^2x^6+1\right)-\frac{b\tan^{-1}\left(cx^3\right)}{3x^3}+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6]
)/6
```

fricas [A] time = 0.47, size = 43, normalized size = 1.10

$$-\frac{bcx^3\log\left(c^2x^6+1\right)-6\,bcx^3\log(x)+2\,b\arctan\left(cx^3\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3*log(c^2*x^6 + 1) - 6*b*c*x^3*log(x) + 2*b*arctan(c*x^3) + 2*a
)/x^3
```

giac [A] time = 2.01, size = 60, normalized size = 1.54

$$-\frac{bc^3x^3\log\left(c^2x^6+1\right)-2\,bc^3x^3\log\left(cx^3\right)+2\,bc^2\arctan\left(cx^3\right)+2\,ac^2}{6\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(b*c^3*x^3*log(c^2*x^6 + 1) - 2*b*c^3*x^3*log(c*x^3) + 2*b*c^2*arctan(
c*x^3) + 2*a*c^2)/(c^2*x^3)
```

maple [A] time = 0.03, size = 39, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b\arctan\left(c\,x^3\right)}{3x^3}-\frac{bc\ln\left(c^2x^6+1\right)}{6}+bc\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x^3)-1/6*b*c*ln(c^2*x^6+1)+b*c*ln(x)
```

maxima [A] time = 0.32, size = 41, normalized size = 1.05

$$-\frac{1}{6}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\arctan\left(cx^3\right)}{x^3}\right)b-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="maxima")
[Out] -1/6*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*b - 1/3*a/x^3
```

mupad [B] time = 0.38, size = 38, normalized size = 0.97

$$b\,c\,\ln(x)-\frac{a}{3\,x^3}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{3\,x^3}-\frac{b\,c\,\ln\left(c^2\,x^6+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^4,x)
[Out] b*c*log(x) - a/(3*x^3) - (b*atan(c*x^3))/(3*x^3) - (b*c*log(c^2*x^6 + 1))/6
```

sympy [A] time = 96.09, size = 126, normalized size = 3.23

$$\begin{cases}-\frac{a}{3x^3}+bc\log\left(x\right)-\frac{bc\log\left(x-\sqrt[6]{-1}\sqrt[6]{\frac{1}{c^2}}\right)}{3}-\frac{bc\log\left(4x^2+4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{3}+\frac{ib\,\mathrm{atan}\left(cx^3\right)}{3\sqrt{\frac{1}{c^2}}}-\frac{b\,\mathrm{atan}\left(cx^3\right)}{3x^3} & \text{for}\ c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**4,x)
[Out] Piecewise((-a/(3*x**3) + b*c*log(x) - b*c*log(x - (-1)**(1/6)*(c**(-2))**(1
/6))/3 - b*c*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*
(c**(-2))**(1/3))/3 + I*b*atan(c*x**3)/(3*sqrt(c**(-2))) - b*atan(c*x**3)/(
3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

3.102 $\int \frac{a+b\tan^{-1}(cx^3)}{x^7}\,dx$

Optimal. Leaf size=41

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}-\frac{1}{6}bc^2\tan^{-1}\left(cx^3\right)-\frac{bc}{6x^3}$$

```
[Out] -1/6*b*c/x^3-1/6*b*c^2*arctan(c*x^3)+1/6*(-a-b*arctan(c*x^3))/x^6
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 325, 203}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}-\frac{1}{6}bc^2\tan^{-1}\left(cx^3\right)-\frac{bc}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^7,x]
```

```
[Out] -(b*c)/(6*x^3) - (b*c^2*ArcTan[c*x^3])/6 - (a + b*ArcTan[c*x^3])/(6*x^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^7}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{2}(bc)\int \frac{1}{x^4\left(1+c^2x^6\right)}\,dx\\ &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx,x,x^3\right)\\ &= -\frac{bc}{6x^3}-\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}-\frac{1}{6}\left(bc^3\right)\mathrm{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^3\right)\\ &= -\frac{bc}{6x^3}-\frac{1}{6}bc^2\tan^{-1}\left(cx^3\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 1.17

$$-\frac{a}{6x^6}-\frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^6\right)}{6x^3}-\frac{b\tan^{-1}\left(cx^3\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^7,x]
```

```
[Out] -1/6*a/x^6 - (b*ArcTan[c*x^3])/(6*x^6) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^6)])/(6*x^3)
```

fricas [A] time = 0.41, size = 30, normalized size = 0.73

$$-\frac{bcx^3+\left(bc^2x^6+b\right)\arctan\left(cx^3\right)+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3) + a)/x^6
```

giac [B] time = 0.17, size = 74, normalized size = 1.80

$$\frac{bc^5ix^6\log\left(cix^3+1\right)-bc^5ix^6\log\left(-cix^3+1\right)-2\,bc^4x^3-2\,bc^3\arctan\left(cx^3\right)-2\,ac^3}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(b*c^5*i*x^6*log(c*i*x^3 + 1) - b*c^5*i*x^6*log(-c*i*x^3 + 1) - 2*b*c^
4*x^3 - 2*b*c^3*arctan(c*x^3) - 2*a*c^3)/(c^3*x^6)
```

maple [A] time = 0.04, size = 39, normalized size = 0.95

$$-\frac{a}{6x^6}-\frac{b\arctan\left(c\,x^3\right)}{6x^6}-\frac{b\,c^2\arctan\left(c\,x^3\right)}{6}-\frac{bc}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctan(c*x^3)-1/6*b*c^2*arctan(c*x^3)-1/6*b*c/x^3
```

maxima [A] time = 0.42, size = 35, normalized size = 0.85

$$-\frac{1}{6}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*b - 1/6*a/x^6
```

mupad [B] time = 0.39, size = 41, normalized size = 1.00

$$-\frac{\frac{b\,c\,x^3}{3}+\frac{a}{3}}{2\,x^6}-\frac{b\,c^2\,\mathrm{atan}\left(c\,x^3\right)}{6}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^7,x)
```

```
[Out] - (a/3 + (b*c*x^3)/3)/(2*x^6) - (b*c^2*atan(c*x^3))/6 - (b*atan(c*x^3))/(6*
x^6)
```

sympy [A] time = 88.00, size = 42, normalized size = 1.02

$$-\frac{a}{6x^6}-\frac{bc^2\,\mathrm{atan}\left(cx^3\right)}{6}-\frac{bc}{6x^3}-\frac{b\,\mathrm{atan}\left(cx^3\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**7,x)
```

```
[Out] -a/(6*x**6) - b*c**2*atan(c*x**3)/6 - b*c/(6*x**3) - b*atan(c*x**3)/(6*x**6
)
```

3.103 $\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^{10}}\,dx$

Optimal. Leaf size=55

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{bc}{18x^6}$$

```
[Out] -1/18*b*c/x^6+1/9*(-a-b*arctan(c*x^3))/x^9-1/3*b*c^3*ln(x)+1/18*b*c^3*ln(c^
2*x^6+1)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 44}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^10,x]
```

```
[Out] -(b*c)/(18*x^6) - (a + b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^6])/18
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^{10}}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{3}(bc)\int \frac{1}{x^7\left(1+c^2x^6\right)}\,dx\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^6\right)\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^6\right)\\
&= -\frac{bc}{18x^6}-\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.09

$$-\frac{a}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{bc}{18x^6}-\frac{b\tan^{-1}\left(cx^3\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^10,x]
```

```
[Out] -1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3
 + (b*c^3*Log[1 + c^2*x^6])/18
```

fricas [A] time = 0.44, size = 54, normalized size = 0.98

$$\frac{bc^3x^9\log\left(c^2x^6+1\right)-6\,bc^3x^9\log(x)-bcx^3-2\,b\arctan\left(cx^3\right)-2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(b*c^3*x^9*log(c^2*x^6 + 1) - 6*b*c^3*x^9*log(x) - b*c*x^3 - 2*b*arcta
n(c*x^3) - 2*a)/x^9
```

giac [A] time = 0.16, size = 69, normalized size = 1.25

$$\frac{bc^7x^9\log\left(c^2x^6+1\right)-2\,bc^7x^9\log\left(cx^3\right)-bc^5x^3-2\,bc^4\arctan\left(cx^3\right)-2\,ac^4}{18\,c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="giac")
```

```
[Out] 1/18*(b*c^7*x^9*log(c^2*x^6 + 1) - 2*b*c^7*x^9*log(c*x^3) - b*c^5*x^3 - 2*b
*c^4*arctan(c*x^3) - 2*a*c^4)/(c^4*x^9)
```

maple [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{a}{9x^9}-\frac{b\arctan\left(c\,x^3\right)}{9x^9}+\frac{b\,c^3\ln\left(c^2x^6+1\right)}{18}-\frac{bc}{18x^6}-\frac{b\,c^3\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^10,x)
```

```
[Out] -1/9*a/x^9-1/9*b/x^9*arctan(c*x^3)+1/18*b*c^3*ln(c^2*x^6+1)-1/18*b*c/x^6-1/
3*b*c^3*ln(x)
```

maxima [A] time = 0.33, size = 53, normalized size = 0.96

$$\frac{1}{18}\left(\left(c^2\log\left(c^2x^6+1\right)-c^2\log\left(x^6\right)-\frac{1}{x^6}\right)c-\frac{2\,\arctan\left(cx^3\right)}{x^9}\right)b-\frac{a}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="maxima")
```

```
[Out] 1/18*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9
)*b - 1/9*a/x^9
```

mupad [B] time = 0.40, size = 50, normalized size = 0.91

$$\frac{b\,c^3\,\ln\left(c^2\,x^6+1\right)}{18}-\frac{a}{9\,x^9}-\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{9\,x^9}-\frac{b\,c}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^10,x)
```

```
[Out] (b*c^3*log(c^2*x^6 + 1))/18 - a/(9*x^9) - (b*c^3*log(x))/3 - (b*atan(c*x^3)
)/(9*x^9) - (b*c)/(18*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**10,x)
```

```
[Out] Timed out
```

3.104 $\int x^3 \left(a + b \tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=174

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}-\frac{b}{}$$

```
[Out] -3/4*b*x/c+1/4*b*arctan(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctan(c*x^3))+1/8*
b*arctan(2*c^(1/3)*x-3^(1/2))/c^(4/3)+1/8*b*arctan(2*c^(1/3)*x+3^(1/2))/c^(
4/3)-1/16*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)+1/16*b*ln(1
+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
```

Rubi [A] time = 0.32, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 209, 634, 618, 204, 628, 203}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTan[c*x
^3]))/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[Sqrt[3]
 + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/
3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/(16*c^(4/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{4}(3bc)\int\frac{x^6}{1+c^2x^6}\,dx\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\int\frac{1}{1+c^2x^6}\,dx}{4c}\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{4c}+\frac{b\int\frac{1-\frac{1}{2}\sqrt{3}\sqrt[3]{c}\,x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{4c}+\frac{b\int -}{1+}\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\left(\sqrt{3}\,b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{16c^{4/3}}+\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right.}{16c^{4/3}}\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{8c^{4/3}}+\frac{b\tan}{}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 179, normalized size = 1.03

$$\frac{ax^4}{4}-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{8c^{4/3}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (a*x^4)/4 + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (b*x^4*Arc
Tan[c*x^3])/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[S
qrt[3] + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
```

```
c^(2/3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3
)*x^2])/(16*c^(4/3))
```

fricas [B] time = 0.46, size = 399, normalized size = 2.29

$$4\,bcx^4\arctan\left(cx^3\right)+4\,acx^4+\sqrt{3}\,c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(b^2x^2+\sqrt{3}\,bcx\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}+c^2\left(\frac{b^6}{c^8}\right)^{\frac{1}{3}}\right)-\sqrt{3}\,c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(b^2x^2-\sqrt{3}\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*c*x^4*arctan(c*x^3) + 4*a*c*x^4 + sqrt(3)*c*(b^6/c^8)^(1/6)*log(b
^2*x^2 + sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1/3)) - sqrt(3)*c*(
b^6/c^8)^(1/6)*log(b^2*x^2 - sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^
(1/3)) - 4*c*(b^6/c^8)^(1/6)*arctan(-(2*b*c^7*x*(b^6/c^8)^(5/6) - 2*sqrt(b^
2*x^2 + sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1/3))*c^7*(b^6/c^8)^
(5/6) + sqrt(3)*b^6)/b^6) - 4*c*(b^6/c^8)^(1/6)*arctan(-(2*b*c^7*x*(b^6/c^8
)^(5/6) - 2*sqrt(b^2*x^2 - sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1
/3))*c^7*(b^6/c^8)^(5/6) - sqrt(3)*b^6)/b^6) - 8*c*(b^6/c^8)^(1/6)*arctan(-
(b*c^7*x*(b^6/c^8)^(5/6) - sqrt(b^2*x^2 + c^2*(b^6/c^8)^(1/3))*c^7*(b^6/c^8
)^(5/6))/b^6) - 12*b*x)/c
```

giac [A] time = 3.78, size = 167, normalized size = 0.96

$$\frac{1}{16}\,bc^7\left(\frac{\sqrt{3}\,\log\left(x^2+\frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\,\log\left(x^2-\frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\,\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\,\arctan\left(\left(2\,x-\right.\right.}{c^8|c|^{\frac{1}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*c^7*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^8
*abs(c)^(1/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))
/(c^8*abs(c)^(1/3)) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(
c^8*abs(c)^(1/3)) + 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^
8*abs(c)^(1/3)) + 4*arctan(x*abs(c)^(1/3))/(c^8*abs(c)^(1/3))) + 1/4*(b*c*x
^4*arctan(c*x^3) + a*c*x^4 - 3*b*x)/c
```

maple [A] time = 0.16, size = 165, normalized size = 0.95

$$\frac{x^4a}{4}+\frac{b\,x^4\arctan\left(c\,x^3\right)}{4}-\frac{3bx}{4c}+\frac{b\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}\right)}{4c}-\frac{b\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{16c}+\frac{b\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctan(c*x^3)-3/4*b*x/c+1/4*b/c*(1/c^2)^(1/6)*arctan(x/
(1/c^2)^(1/6))-1/16*b/c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*
x+(1/c^2)^(1/3))+1/8*b/c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))+1/
16*b/c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+
1/8*b/c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))
```

maxima [A] time = 0.43, size = 148, normalized size = 0.85

$$\frac{1}{4}\,ax^4+\frac{1}{16}\left(4\,x^4\arctan\left(cx^3\right)+c\left(\frac{\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\,\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\,\arctan\left(\frac{2c^{\frac{2}{3}}x+\sqrt{3}c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/16*(4*x^4*arctan(c*x^3) + c*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(
3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1
)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c
^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3)
)/c^(1/3))/c^2 - 12*x/c^2))*b
```

mupad [B] time = 1.16, size = 114, normalized size = 0.66

$$\frac{a\,x^4}{4}-\frac{b\left(\mathrm{atan}\left({(-1)}^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{(-1)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{8\,c^{4/3}}+\frac{b\,x^4\,\mathrm{atan}\left(c\,x^3\right)}{4}-\frac{3\,b\,x}{4\,c}-\frac{\sqrt{3}\,\dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^4)/4 - (b*(atan((-1)^(2/3)*c^(1/3)*x) - atan((c^(1/3)*x*(3^(1/2)*1i +
1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(8*c^(4/3)) +
(b*x^4*atan(c*x^3))/4 - (3*b*x)/(4*c) - (3^(1/2)*b*(atan((c^(1/3)*x*(3^(1/2
)*1i + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/(8*c^(4/3))
```

sympy [A] time = 58.10, size = 311, normalized size = 1.79

$$\begin{cases}\frac{ax^4}{4}+\frac{bx^4\,\mathrm{atan}\left(cx^3\right)}{4}-\frac{3bx}{4c}-\frac{3\sqrt[6]{-1}\,b\sqrt[6]{\frac{1}{c^2}}\,\log\left(4x^2-4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{16c}+\frac{3\sqrt[6]{-1}\,b\sqrt[6]{\frac{1}{c^2}}\,\log\left(4x^2+4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{16c}-\dots \\ \frac{ax^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x**3)/4 - 3*b*x/(4*c) - 3*(-1)**(1/6)*b
*(c**(-2))**(1/6)*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(
1/3)*(c**(-2))**(1/3))/(16*c) + 3*(-1)**(1/6)*b*(c**(-2))**(1/6)*log(4*x**2
 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(16*c
) - (-1)**(1/6)*sqrt(3)*b*(c**(-2))**(1/6)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*
(c**(-2))**(1/6)) - sqrt(3)/3)/(8*c) - (-1)**(1/6)*sqrt(3)*b*(c**(-2))**(1/
6)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(8*c) - (
-1)**(2/3)*b*atan(c*x**3)/(4*c**2*(c**(-2))**(1/3)), Ne(c, 0)), (a*x**4/4,
True))
```

3.105 $\int \left(a + b\tan^{-1}\left(cx^3\right)\right)dx$

Optimal. Leaf size=101

$$ax + \frac{b\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}} + bx\tan^{-1}\left(cx^3\right)$$

```
[Out] a*x+b*x*arctan(c*x^3)+1/2*b*ln(1+c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1-c^(2/3)*x^
2+c^(4/3)*x^4)/c^(1/3)+1/2*b*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)/
c^(1/3)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5027, 275, 292, 31, 634, 617, 204, 628}

$$ax + \frac{b\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}} - \frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + bx\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] + (Sqrt[3]*b*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(
2*c^(1/3)) + (b*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 - c^(2/3)*x^2
+ c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= ax+b\int \tan^{-1}\left(cx^3\right)dx\\
&= ax+bx\tan^{-1}\left(cx^3\right)-(3bc)\int \frac{x^3}{1+c^2x^6}\,dx\\
&= ax+bx\tan^{-1}\left(cx^3\right)-\frac{1}{2}(3bc)\,\mathrm{Subst}\left(\int \frac{x}{1+c^2x^3}\,dx,x,x^2\right)\\
&= ax+bx\tan^{-1}\left(cx^3\right)+\frac{1}{2}\left(b\sqrt[3]{c}\right)\mathrm{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{2}\left(b\sqrt[3]{c}\right)\mathrm{Subst}\left(\int \frac{}{1}\right.\\
&= ax+bx\tan^{-1}\left(cx^3\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\,\mathrm{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{4\sqrt[3]{c}}-\frac{1}{4}\left(3b\right.\\
&= ax+bx\tan^{-1}\left(cx^3\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}-\frac{(3b)\,\mathrm{Subst}\left(\int\right.}{}\\
&= ax+bx\tan^{-1}\left(cx^3\right)+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1-c^{2/3}x^2+\right.}{4\sqrt[3]{c}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 131, normalized size = 1.30

$$
ax-\frac{b\left(-2\log\left(c^{2/3}x^2+1\right)+\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)-2\sqrt{3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)-\right.}{4\sqrt[3]{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*c^(1/3)*x] - 2*
Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c^(2/3)*x^2] + Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]))
/(4*c^(1/3))
```

fricas [A] time = 0.46, size = 234, normalized size = 2.32

$$\left[\frac{4\,bcx\arctan\left(cx^{3}\right)+\sqrt{3}\,bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2\,c^{2}x^{6}-3\,c^{\frac{2}{3}}x^{2}-\sqrt{3}\left(2\,c^{\frac{5}{3}}x^{4}+cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}-1}{c^{2}x^{6}+1}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}-c^{\frac{4}{3}}x^{2}+\right.}{4\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x*arctan(c*x^3) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 -
 3*c^(2/3)*x^2 - sqrt(3)*(2*c^(5/3)*x^4 + c*x^2 - c^(1/3))*sqrt(-1/c^(2/3))
 - 1)/(c^2*x^6 + 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2
/3)) + 2*b*c^(2/3)*log(c*x^2 + c^(1/3)))/c, 1/4*(4*b*c*x*arctan(c*x^3) + 2*
sqrt(3)*b*c^(2/3)*arctan(-1/3*sqrt(3)*(2*c*x^2 - c^(1/3))/c^(1/3)) + 4*a*c*
x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2
+ c^(1/3)))/c]
```

giac [A] time = 0.16, size = 95, normalized size = 0.94

$$-\frac{1}{4}\left(\left(c\left(\frac{2\sqrt{3}\,|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{2}-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{2}}+\frac{|c|^{\frac{2}{3}}\log\left(x^{4}-\frac{x^{2}}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{2}}-\frac{2\log\left(x^{2}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)-4\,x\arctan\left(c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))
*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/
3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 4*x*arctan(c*x^3))*b
+ a*x
```

maple [A] time = 0.03, size = 98, normalized size = 0.97

$$ax+bx\arctan\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x^3),x)
```

```
[Out] a*x+b*x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/(
1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/2*b*3^(1/2)/c/(1/c^2
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))
```

maxima [A] time = 0.42, size = 92, normalized size = 0.91

$$-\frac{1}{4}\left(\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}-\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)-4x\arctan\left(cx^3\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^
(4/3) + log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(4/3) - 2*log((c^(2/3)*x^2 + 1
)/c^(2/3))/c^(4/3)) - 4*x*arctan(c*x^3))*b + a*x
```

mupad [B] time = 2.30, size = 91, normalized size = 0.90

$$a\,x+b\,x\,\mathrm{atan}\left(c\,x^3\right)+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{2\,c^{1/3}}-\frac{\ln\left(2-4\,c^{2/3}\,x^2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}-\frac{\ln\left(4\,c^{2/3}\,x^2-2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b+\sqrt{\right.}}{4\,c^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c*x^3),x)
```

```
[Out] a*x + b*x*atan(c*x^3) + (b*log(c^(2/3)*x^2 + 1))/(2*c^(1/3)) - (log(3^(1/2)
*2i - 4*c^(2/3)*x^2 + 2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2i
+ 4*c^(2/3)*x^2 - 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3))
```

sympy [A] time = 31.24, size = 898, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x**3),x)
```

```
[Out] a*x + b*Piecewise((0, Eq(c, 0)), (-oo*I*x, Eq(c, -I/x**3)), (oo*I*x, Eq(c,
I/x**3)), (4*(-1)**(2/3)*c**4*x**6*(c**(-2))**(5/3)*log(x - (-1)**(1/6)*(c*
*(-2))**(1/6))/(4*c*x**6 + 4/c) - 3*(-1)**(2/3)*c**4*x**6*(c**(-2))**(5/3)*
log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1
/3))/(4*c*x**6 + 4/c) + (-1)**(2/3)*c**4*x**6*(c**(-2))**(5/3)*log(4*x**2 +
 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(4*c*x*
*6 + 4/c) + 2*(-1)**(2/3)*sqrt(3)*c**4*x**6*(c**(-2))**(5/3)*atan(2*(-1)**(
5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(4*c*x**6 + 4/c) - 2*(-1)*
*(2/3)*sqrt(3)*c**4*x**6*(c**(-2))**(5/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(
c**(-2))**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) + 4*(-1)**(2/3)*c**4*x**6*(c
**(-2))**(5/3)*log(2)/(4*c*x**6 + 4/c) + 4*(-1)**(1/6)*c**3*x**6*(c**(-2))*
*(7/6)*atan(c*x**3)/(4*c*x**6 + 4/c) + 4*(-1)**(2/3)*c**2*(c**(-2))**(5/3)*
log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(4*c*x**6 + 4/c) - 3*(-1)**(2/3)*c**2
*(c**(-2))**(5/3)*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(
1/3)*(c**(-2))**(1/3))/(4*c*x**6 + 4/c) + (-1)**(2/3)*c**2*(c**(-2))**(5/3)
*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(
1/3))/(4*c*x**6 + 4/c) + 2*(-1)**(2/3)*sqrt(3)*c**2*(c**(-2))**(5/3)*atan(2
*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(4*c*x**6 + 4/c) -
 2*(-1)**(2/3)*sqrt(3)*c**2*(c**(-2))**(5/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(
3*(c**(-2))**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) + 4*(-1)**(2/3)*c**2*(c**
(-2))**(5/3)*log(2)/(4*c*x**6 + 4/c) + 4*c*x**7*atan(c*x**3)/(4*c*x**6 + 4/
c) + 4*(-1)**(1/6)*c*(c**(-2))**(7/6)*atan(c*x**3)/(4*c*x**6 + 4/c) + 4*x*a
tan(c*x**3)/(4*c**2*x**6 + 4), True))
```

3.106 $\int \frac{a+b\tan^{-1}(cx^3)}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{2}bc^{2/3}\tan^{-1}$$

```
[Out] 1/2*b*c^(2/3)*arctan(c^(1/3)*x)+1/2*(-a-b*arctan(c*x^3))/x^2+1/4*b*c^(2/3)*
arctan(2*c^(1/3)*x-3^(1/2))+1/4*b*c^(2/3)*arctan(2*c^(1/3)*x+3^(1/2))-1/8*b
*c^(2/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/8*b*c^(2/3)*ln(1+c^(
2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5033, 209, 634, 618, 204, 628, 203}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{2}bc^{2/3}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^3,x]
```

```
[Out] (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (a + b*ArcTan[c*x^3])/(2*x^2) - (b*c^(2/3
)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*
x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8 + (S
qrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^3}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(3bc)\int \frac{1}{1+c^2x^6}\,dx\\ &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{1+c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{1-\frac{1}{2}\sqrt{3}\sqrt[3]{c}\,x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx+\frac{1}{2}(bc\\ &= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\left(\sqrt{3}\,bc^{2/3}\right)\int \frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx+\frac{1}{8}\\ &= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}\\ &= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{4}bc^{2/3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)+\frac{1}{4}bc^{2/3}\tan^{-1}\left(\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 170, normalized size = 1.03

$$-\frac{a}{2x^2}-\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{1}{4}bc^{2/3}\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (b*ArcTan[c*x^3])/(2*x^2) -
(b*c^(2/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2
*c^(1/3)*x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2
])/8 + (Sqrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

fricas [B] time = 0.48, size = 505, normalized size = 3.06

$$\frac{\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}x^2\log\left(4\,b^2c^2x^2+4\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}bcx+4\left(b^6c^4\right)^{\frac{1}{3}}\right)-\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}x^2\log\left(4\,b^2c^2x^2-4\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}bcx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(4*b^2*c^2*x^2 + 4*sqrt(3)*(b^6*c^4)^(
1/6)*b*c*x + 4*(b^6*c^4)^(1/3)) - sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(4*b^2*c^2
*x^2 - 4*sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + 4*(b^6*c^4)^(1/3)) + sqrt(3)*(b^6*
c^4)^(1/6)*x^2*log(b^2*c^2*x^2 + sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^
(1/3)) - sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(b^2*c^2*x^2 - sqrt(3)*(b^6*c^4)^(1
/6)*b*c*x + (b^6*c^4)^(1/3)) - 8*(b^6*c^4)^(1/6)*x^2*arctan(-(sqrt(3)*b^6*c
^4 + 2*(b^6*c^4)^(5/6)*b*c*x - 2*(b^6*c^4)^(5/6)*sqrt(b^2*c^2*x^2 + sqrt(3)
*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^(1/3)))/(b^6*c^4)) - 8*(b^6*c^4)^(1/6)*x
^2*arctan((sqrt(3)*b^6*c^4 - 2*(b^6*c^4)^(5/6)*b*c*x + 2*(b^6*c^4)^(5/6)*sq
rt(b^2*c^2*x^2 - sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^(1/3)))/(b^6*c^4
)) - 16*(b^6*c^4)^(1/6)*x^2*arctan(-((b^6*c^4)^(5/6)*b*c*x - (b^6*c^4)^(5/6
)*sqrt(b^2*c^2*x^2 + (b^6*c^4)^(1/3)))/(b^6*c^4)) - 8*b*arctan(c*x^3) - 8*a
)/x^2
```

giac [A] time = 0.18, size = 137, normalized size = 0.83

$$\frac{1}{8}\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2x-\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right.\right.}{|c|^{\frac{1}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3
) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3)
 + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 2*arc
tan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 4*arctan(x*ab
s(c)^(1/3))/abs(c)^(1/3))*b*c - 1/2*(b*arctan(c*x^3) + a)/x^2
```

maple [A] time = 0.11, size = 148, normalized size = 0.90

$$-\frac{a}{2x^2}-\frac{b\arctan\left(c\,x^3\right)}{2x^2}+\frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}\right)}{2}-\frac{bc\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{8}+\frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x^3)+1/2*b*c*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(
1/6))-1/8*b*c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^
(1/3))+1/4*b*c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))+1/8*b*c*3^(1
/2)*(1/c^2)^(1/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/4*b*c*(1/
c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))
```

maxima [A] time = 0.42, size = 137, normalized size = 0.83

$$\frac{1}{8}\left(\left(\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*lo
g(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3
) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2
*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3))*c - 4*arctan(c*x^3)/x^2)*b
- 1/2*a/x^2
```

mupad [B] time = 0.91, size = 107, normalized size = 0.65

$$-\frac{a}{2\,x^2}-\frac{b\,c^{2/3}\left(\frac{\operatorname{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)}{2}-\frac{\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)}{2}+\operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{2}-\frac{b\operatorname{atan}\left(c\,x^3\right)}{2\,x^2}-\frac{\sqrt{3}\,b\,c^{2/3}\left(\operatorname{atan}\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^3,x)
```

```
[Out] - a/(2*x^2) - (b*c^(2/3)*(atan((-1)^(2/3)*c^(1/3)*x)/2 - atan((c^(1/3)*x*(3
^(1/2)*1i + 1))/2)/2 + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/2
- (b*atan(c*x^3))/(2*x^2) - (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2)*1i
 + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/4
```

sympy [A] time = 73.75, size = 301, normalized size = 1.82

$$\begin{cases} -\frac{a}{2x^2}-\frac{(-1)^{\frac{2}{3}}b\operatorname{atan}\left(cx^3\right)}{2\sqrt[3]{\frac{1}{c^2}}}-\frac{b\operatorname{atan}\left(cx^3\right)}{2x^2}-\frac{3\sqrt[6]{-1}\,b\log\left(4x^2-4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{8c\left(\frac{1}{c^2}\right)^{\frac{5}{6}}}+\frac{3\sqrt[6]{-1}\,b\log\left(4x^2+4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{8c\left(\frac{1}{c^2}\right)^{\frac{5}{6}}}-\cdots \\ -\frac{a}{2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - (-1)**(2/3)*b*atan(c*x**3)/(2*(c**(-2))**(1/3)) -
b*atan(c*x**3)/(2*x**2) - 3*(-1)**(1/6)*b*log(4*x**2 - 4*(-1)**(1/6)*x*(c**
(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(8*c*(c**(-2))**(5/6)) + 3*(
-1)**(1/6)*b*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*
(c**(-2))**(1/3))/(8*c*(c**(-2))**(5/6)) - (-1)**(1/6)*sqrt(3)*b*atan(2*(-1
)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(4*c*(c**(-2))**(5/6))
 - (-1)**(1/6)*sqrt(3)*b*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6))
+ sqrt(3)/3)/(4*c*(c**(-2))**(5/6)), Ne(c, 0)), (-a/(2*x**2), True))
```

3.107 $\int \frac{a+b\tan^{-1}(cx^3)}{x^6}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right)+\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)$$

```
[Out] -3/10*b*c/x^2+1/5*(-a-b*arctan(c*x^3))/x^5+1/10*b*c^(5/3)*ln(1+c^(2/3)*x^2)
-1/20*b*c^(5/3)*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)+1/10*b*c^(5/3)*arctan(1/3*(1-
2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5033, 275, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)+\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] (-3*b*c)/(10*x^2) - (a + b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5/3)*ArcT
an[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 -
 (b*c^(5/3)*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^6}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{5}(3bc)\int \frac{1}{x^3\left(1+c^2x^6\right)}\,dx \\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}(3bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^3\right)}\,dx,x,x^2\right) \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}\left(3bc^3\right)\text{Subst}\left(\int \frac{x}{1+c^2x^3}\,dx,x,x^2\right) \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{10}\left(bc^{7/3}\right)\text{Subs} \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}\left(bc^{5/3}\right)\text{Subst}\left(\int \frac{-c^{2/3}+2}{1-c^{2/3}x+}\right. \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right) \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 183, normalized size = 1.59

$$-\frac{a}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{10}\sqrt{3}\,bc$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (3*b*c)/(10*x^2) - (b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5
/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[Sqrt[3] +
```

```
2*c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 - (b*c^(5/3)*Log[1
- Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/20 - (b*c^(5/3)*Log[1 + Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/20
```

fricas [A] time = 0.43, size = 121, normalized size = 1.05

$$-\frac{2\sqrt{3}\,b\left(c^2\right)^{\frac{1}{3}}cx^5\arctan\left(\frac{2}{3}\sqrt{3}\left(c^2\right)^{\frac{1}{3}}x^2-\frac{1}{3}\sqrt{3}\right)+b\left(c^2\right)^{\frac{1}{3}}cx^5\log\left(c^2x^4-\left(c^2\right)^{\frac{2}{3}}x^2+\left(c^2\right)^{\frac{1}{3}}\right)-2\,b\left(c^2\right)^{\frac{1}{3}}cx^5\log\left(c\right.}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(3)*b*(c^2)^(1/3)*c*x^5*arctan(2/3*sqrt(3)*(c^2)^(1/3)*x^2 - 1
/3*sqrt(3)) + b*(c^2)^(1/3)*c*x^5*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/
3)) - 2*b*(c^2)^(1/3)*c*x^5*log(c^2*x^2 + (c^2)^(2/3)) + 6*b*c*x^3 + 4*b*ar
ctan(c*x^3) + 4*a)/x^5
```

giac [A] time = 4.33, size = 108, normalized size = 0.94

$$-\frac{1}{20}\,bc^3\left(\frac{2\sqrt{3}\,|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)-\frac{3\,bcx^3+2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2
/3))*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)
^(4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 1/10*(3*b*c*x^3 +
 2*b*arctan(c*x^3) + 2*a)/x^5
```

maple [A] time = 0.04, size = 105, normalized size = 0.91

$$-\frac{a}{5x^5}-\frac{b\arctan\left(c\,x^3\right)}{5x^5}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{3bc}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x^3)+1/10*b*c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1
/3))-1/20*b*c/(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/10*b*
c*3^(1/2)/(1/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))-3/10*b*
c/x^2
```

maxima [A] time = 0.41, size = 102, normalized size = 0.89

$$-\frac{1}{20}\left(\left(\left(2\sqrt{3}\,c^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)+c^{\frac{2}{3}}\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)-2\,c^{\frac{2}{3}}\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)+\frac{6}{x^2}\right)c+\frac{4\arctan}{x^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2
/3)) + c^(2/3)*log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1) - 2*c^(2/3)*log((c^(2/3)*
x^2 + 1)/c^(2/3)) + 6/x^2)*c + 4*arctan(c*x^3)/x^5)*b - 1/5*a/x^5
```

mupad [B] time = 2.57, size = 118, normalized size = 1.03

$$\frac{b\,c^{5/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{10}-\frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{5\,x^5}-\frac{b\,c^{5/3}\,\ln\left(\sqrt{3}\,c^{2/3}\,x^2-c^{2/3}\,x^2\,1\mathrm{i}+2\mathrm{i}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{20}+\frac{b\,c^{5/3}\,\ln\left(-c^{2/}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^6,x)
```

```
[Out] (b*c^(5/3)*log(c^(2/3)*x^2 + 1))/10 - (a + (3*b*c*x^3)/2)/(5*x^5) - (b*atan
(c*x^3))/(5*x^5) - (b*c^(5/3)*log(3^(1/2)*c^(2/3)*x^2 - c^(2/3)*x^2*1i + 2i
)*(3^(1/2)*1i + 1))/20 + (b*c^(5/3)*log(2i - 3^(1/2)*c^(2/3)*x^2 - c^(2/3)*
x^2*1i)*(3^(1/2)*1i - 1))/20
```

sympy [A] time = 144.51, size = 352, normalized size = 3.06

$$\begin{cases} -\frac{a}{5x^5}+\frac{\sqrt[6]{-1}\,bc^2\sqrt[6]{\frac{1}{c^2}}\,\mathrm{atan}\left(cx^3\right)}{5}+\frac{(-1)^{\frac{2}{3}}bc\log\left(x-\sqrt[6]{-1}\sqrt[6]{\frac{1}{c^2}}\right)}{5\sqrt[3]{\frac{1}{c^2}}}-\frac{3(-1)^{\frac{2}{3}}bc\log\left(4x^2-4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{20\sqrt[3]{\frac{1}{c^2}}}+\frac{(-1)^{\frac{2}{3}}bc\log\left(4x^2+4\sqrt[6]{-1}\right.}{20\sqrt[3]{}} \\ -\frac{a}{5x^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + (-1)**(1/6)*b*c**2*(c**(-2))**(1/6)*atan(c*x**3)/5
 + (-1)**(2/3)*b*c*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(5*(c**(-2))**(1/3
)) - 3*(-1)**(2/3)*b*c*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-
1)**(1/3)*(c**(-2))**(1/3))/(20*(c**(-2))**(1/3)) + (-1)**(2/3)*b*c*log(4*x
**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(2
0*(c**(-2))**(1/3)) + (-1)**(2/3)*sqrt(3)*b*c*atan(2*(-1)**(5/6)*sqrt(3)*x/
(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(10*(c**(-2))**(1/3)) - (-1)**(2/3)*sqrt(
3)*b*c*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(10*(
c**(-2))**(1/3)) - 3*b*c/(10*x**2) - b*atan(c*x**3)/(5*x**5), Ne(c, 0)), (-
a/(5*x**5), True))
```

3.108 $\int x^7 \left(a + b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=176

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}-\frac{b}{}$$

```
[Out] -3/40*b*x^5/c+1/8*b*arctan(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctan(c*x^3))+1
/16*b*arctan(2*c^(1/3)*x-3^(1/2))/c^(8/3)+1/16*b*arctan(2*c^(1/3)*x+3^(1/2)
)/c^(8/3)+1/32*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)-1/32*b
*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)
```

Rubi [A] time = 0.43, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 295, 634, 618, 204, 628, 203}

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTan[
c*x^3]))/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)
*x^2])/(32*c^(8/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^7\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{10}}{1+c^2x^6}\,dx\\
&= -\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\int\frac{x^4}{1+c^2x^6}\,dx}{8c}\\
&= -\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{8c^{7/3}}+\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{7/3}}+\frac{b\int\frac{-}{1}
\\
&= -\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{32c^{8/3}}-\\
&= -\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x\right.}{32c^{8/3}}\\
&= -\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{b\tan}{}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 181, normalized size = 1.03

$$
\frac{ax^8}{8}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (a*x^8)/8 + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (b*x^8*
ArcTan[c*x^3])/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcT
```

```
an[Sqrt[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3
)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c
^(2/3)*x^2])/(32*c^(8/3))
```

fricas [B] time = 0.45, size = 439, normalized size = 2.49

$$20\,bcx^8\arctan\left(cx^3\right)+20\,acx^8-12\,bx^5-5\,\sqrt{3}\,c\left(\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(\sqrt{3}\,b^5c^{13}x\left(\frac{b^6}{c^{16}}\right)^{\frac{5}{6}}+b^6c^{10}\left(\frac{b^6}{c^{16}}\right)^{\frac{2}{3}}+b^{10}x^2\right)+5\,\sqrt{3}\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/160*(20*b*c*x^8*arctan(c*x^3) + 20*a*c*x^8 - 12*b*x^5 - 5*sqrt(3)*c*(b^6/
c^16)^(1/6)*log(sqrt(3)*b^5*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(
2/3) + b^10*x^2) + 5*sqrt(3)*c*(b^6/c^16)^(1/6)*log(-sqrt(3)*b^5*c^13*x*(b^
6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(2/3) + b^10*x^2) - 20*c*(b^6/c^16)^(1/
6)*arctan(-(2*b^5*c^3*x*(b^6/c^16)^(1/6) + sqrt(3)*b^6 - 2*sqrt(sqrt(3)*b^5
*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(2/3) + b^10*x^2)*c^3*(b^6/c
^16)^(1/6))/b^6) - 20*c*(b^6/c^16)^(1/6)*arctan(-(2*b^5*c^3*x*(b^6/c^16)^(1
/6) - sqrt(3)*b^6 - 2*sqrt(-sqrt(3)*b^5*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*
(b^6/c^16)^(2/3) + b^10*x^2)*c^3*(b^6/c^16)^(1/6))/b^6) - 40*c*(b^6/c^16)^(
1/6)*arctan(-(b^5*c^3*x*(b^6/c^16)^(1/6) - sqrt(b^6*c^10*(b^6/c^16)^(2/3) +
 b^10*x^2)*c^3*(b^6/c^16)^(1/6))/b^6))/c
```

giac [A] time = 2.99, size = 171, normalized size = 0.97

$$-\frac{1}{32}\,bc^{15}\left(\frac{\sqrt{3}\,\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{16}|c|^{\frac{5}{3}}}-\frac{\sqrt{3}\,\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{16}|c|^{\frac{5}{3}}}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{18}}-\frac{2\,|c|^{\frac{1}{3}}\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/32*b*c^15*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c
^16*abs(c)^(5/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/
3))/(c^16*abs(c)^(5/3)) - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3)
)*abs(c)^(1/3))/c^18 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*a
bs(c)^(1/3))/c^18 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^18) + 1/40*(5*b
*c*x^8*arctan(c*x^3) + 5*a*c*x^8 - 3*b*x^5)/c
```

maple [A] time = 0.11, size = 167, normalized size = 0.95

$$\frac{x^8a}{8}+\frac{x^8b\arctan\left(c\,x^3\right)}{8}-\frac{3b\,x^5}{40c}+\frac{b\arctan\left(\frac{x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}\right)}{8c^3\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{b\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{32c}+\frac{b\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}\right.}{16c^3\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/8*x^8*a+1/8*x^8*b*arctan(c*x^3)-3/40*b*x^5/c+1/8*b/c^3/(1/c^2)^(1/6)*arct
an(x/(1/c^2)^(1/6))+1/32*b/c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)^(
```

```
1/6)*x+(1/c^2)^(1/3))+1/16*b/c^3/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(
1/2))-1/32*b/c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)
^(1/3))+1/16*b/c^3/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))
```

maxima [A] time = 0.42, size = 152, normalized size = 0.86

$$\frac{1}{8}ax^8+\frac{1}{160}\left(20\,x^8\arctan\left(cx^3\right)-\left(\frac{12\,x^5}{c^2}+\frac{5\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\ldots}{}\right.}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/160*(20*x^8*arctan(c*x^3) - (12*x^5/c^2 + 5*(sqrt(3)*log(c^(2
/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3
)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3
)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c
^(1/3))/c^(1/3))/c^(5/3))/c^2)*c)*b
```

mupad [B] time = 1.00, size = 122, normalized size = 0.69

$$\frac{a\,x^8}{8}-\frac{3\,b\,x^5}{40\,c}-\frac{b\,\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{16\,c^{8/3}}+\frac{b\,x^8\,\mathrm{atan}\left(c\,x^3\ldots\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^8)/8 - (3*b*x^5)/(40*c) - (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^
(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2
)*1i + 1))/2)))/(16*c^(8/3)) + (b*x^8*atan(c*x^3))/8 + (3^(1/2)*b*(atan((-1
)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(
16*c^(8/3))
```

sympy [A] time = 149.06, size = 320, normalized size = 1.82

$$\begin{cases}\frac{ax^8}{8}+\frac{bx^8\,\mathrm{atan}\left(cx^3\right)}{8}-\frac{3bx^5}{40c}-\frac{3(-1)^{\frac{5}{6}}b\log\left(4x^2-4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{\frac{1}{c^2}}}+\frac{3(-1)^{\frac{5}{6}}b\log\left(4x^2+4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{\frac{1}{c^2}}}+\frac{(-1)^{\frac{5}{6}}\sqrt{3}b\ldots}{} \\ \frac{ax^8}{8}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atan(c*x**3)/8 - 3*b*x**5/(40*c) - 3*(-1)**(5/
6)*b*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2)
)**(1/3))/(32*c**3*(c**(-2))**(1/6)) + 3*(-1)**(5/6)*b*log(4*x**2 + 4*(-1)*
*(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(32*c**3*(c**(-
2))**(1/6)) + (-1)**(5/6)*sqrt(3)*b*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2
```

```
))**(1/6)) - sqrt(3)/3)/(16*c**3*(c**(-2))**(1/6)) + (-1)**(5/6)*sqrt(3)*b*
atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(16*c**3*(c*
*(-2))**(1/6)) + (-1)**(1/3)*b*atan(c*x**3)/(8*c**4*(c**(-2))**(2/3)), Ne(c
, 0)), (a*x**8/8, True))
```

3.109 $\int x^4 \left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{3bx^2}{10c}$$

```
[Out] -3/10*b*x^2/c+1/5*x^5*(a+b*arctan(c*x^3))+1/10*b*ln(1+c^(2/3)*x^2)/c^(5/3)-
1/20*b*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)-1/10*b*arctan(1/3*(1-2*c^(2/3)
*x^2)*3^(1/2))*3^(1/2)/c^(5/3)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5033, 275, 321, 200, 31, 634, 617, 204, 628}

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}-\frac{3bx^2}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (x^5*(a + b*ArcTan[c*x^3]))/5 - (Sqrt[3]*b*ArcTan[(1 -
2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{5}(3bc)\int\frac{x^7}{1+c^2x^6}\,dx\\
&= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{10}(3bc)\,\text{Subst}\left(\int\frac{x^3}{1+c^2x^3}\,dx,x,x^2\right)\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\,\text{Subst}\left(\int\frac{1}{1+c^2x^3}\,dx,x,x^2\right)}{10c}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\,\text{Subst}\left(\int\frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)}{10c}+\frac{b\,\text{Subst}\left(\int\frac{}{1-\cdots}\right.}{}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\,\text{Subst}\left(\int\frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\right.}{20c^{5/3}}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x\right.}{20c^{5/3}}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-
\end{aligned}$$

Mathematica [A] time = 0.03, size = 185, normalized size = 1.58

$$\frac{ax^5}{5}+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)}{20c^{5/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)}{20c^{5/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{10c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^3])/5 - (Sqrt[3]*b*ArcTan
[Sqrt[3] - 2*c^(1/3)*x])/(10*c^(5/3)) - (Sqrt[3]*b*ArcTan[Sqrt[3] + 2*c^(1/
```

```
3)*x])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3)) - (b*Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3)) - (b*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(20*c^(5/3))
```

fricas [A] time = 0.45, size = 137, normalized size = 1.17

$$\frac{4\,bc^3x^5\arctan\left(cx^3\right)+4\,ac^3x^5-6\,bc^2x^2+2\,\sqrt{3}\,b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(2\left(c^2\right)^{\frac{2}{3}}x^2-\left(c^2\right)^{\frac{1}{3}}\right)\left(c^2\right)^{\frac{1}{6}}}{3\,c}\right)-b\left(c^2\right)^{\frac{2}{3}}\log\left(c^2x^4-\left(c^2\right)\right.}{20\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^3*x^5*arctan(c*x^3) + 4*a*c^3*x^5 - 6*b*c^2*x^2 + 2*sqrt(3)*b*(
c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(2*(c^2)^(2/3)*x^2 - (c^2)^(1/3))*(c^2)^(1/
6)/c) - b*(c^2)^(2/3)*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c
^2)^(2/3)*log(c^2*x^2 + (c^2)^(2/3)))/c^3
```

giac [A] time = 3.95, size = 119, normalized size = 1.02

$$\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{2\,bcx^5\arctan\left(cx^3\right)+2}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2
/3))/(c^10*abs(c)^(2/3)) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c^
10*abs(c)^(2/3)) + 2*log(x^2 + 1/abs(c)^(2/3))/(c^10*abs(c)^(2/3))) + 1/10*
(2*b*c*x^5*arctan(c*x^3) + 2*a*c*x^5 - 3*b*x^2)/c
```

maple [A] time = 0.03, size = 113, normalized size = 0.97

$$\frac{a\,x^5}{5}+\frac{b\,x^5\arctan\left(c\,x^3\right)}{5}-\frac{3b\,x^2}{10c}+\frac{b\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctan(c*x^3)-3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln(
x^2+(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2
)^(2/3))+1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/
3)*x^2-1))
```

maxima [A] time = 0.42, size = 106, normalized size = 0.91

$$\frac{1}{5}ax^5+\frac{1}{20}\left(4x^5\arctan\left(cx^3\right)-c\left(\frac{6x^2}{c^2}-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{8}{3}}}-\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x^3) - c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1/3
*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(8/3) + log(c^(4/3)*x^4 - c^(
2/3)*x^2 + 1)/c^(8/3) - 2*log((c^(2/3)*x^2 + 1)/c^(2/3))/c^(8/3)))*b
```

mupad [B] time = 1.94, size = 106, normalized size = 0.91

$$\frac{a\,x^5}{5}+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{10\,c^{5/3}}-\frac{3\,b\,x^2}{10\,c}-\frac{\ln\left(1-2\,c^{2/3}\,x^2+\sqrt{3}\,1\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}-\frac{\ln\left(2\,c^{2/3}\,x^2-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^5)/5 + (b*log(c^(2/3)*x^2 + 1))/(10*c^(5/3)) - (3*b*x^2)/(10*c) - (log
(3^(1/2)*1i - 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(3^
(1/2)*1i + 2*c^(2/3)*x^2 - 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*ata
n(c*x^3))/5
```

sympy [A] time = 71.60, size = 359, normalized size = 3.07

$$\begin{cases}\frac{ax^5}{5}-\frac{\sqrt[3]{-1}\,bc^3\left(\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(x-\sqrt[6]{-1}\sqrt[6]{\frac{1}{c^2}}\right)}{5}+\frac{3\sqrt[3]{-1}\,bc^3\left(\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2-4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{20}-\frac{\sqrt[3]{-1}\,bc^3\left(\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2+4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}\right.}{20} \\ \frac{ax^5}{5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**5/5 - (-1)**(1/3)*b*c**3*(c**(-2))**(7/3)*log(x - (-1)**(1/
6)*(c**(-2))**(1/6))/5 + 3*(-1)**(1/3)*b*c**3*(c**(-2))**(7/3)*log(4*x**2 -
 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/20 - (-
1)**(1/3)*b*c**3*(c**(-2))**(7/3)*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(
1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/20 + (-1)**(1/3)*sqrt(3)*b*c**3*(c**
(-2))**(7/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)
/10 - (-1)**(1/3)*sqrt(3)*b*c**3*(c**(-2))**(7/3)*atan(2*(-1)**(5/6)*sqrt(3
)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/10 + (-1)**(5/6)*b*c**2*(c**(-2))**(1
1/6)*atan(c*x**3)/5 + b*x**5*atan(c*x**3)/5 - 3*b*x**2/(10*c), Ne(c, 0)), (
a*x**5/5, True))
```

3.110 $\int x \left(a + b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=165

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{b\tan}{}$$

```
[Out] -1/2*b*arctan(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctan(c*x^3))-1/4*b*arctan(2
*c^(1/3)*x-3^(1/2))/c^(2/3)-1/4*b*arctan(2*c^(1/3)*x+3^(1/2))/c^(2/3)-1/8*b
*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/8*b*ln(1+c^(2/3)*x^2
+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A] time = 0.39, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5033, 295, 634, 618, 204, 628, 203}

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (x^2*(a + b*ArcTan[c*x^3]))/2 + (b*Arc
Tan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3)*x])
/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2
/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(cx^3\right)\right)dx=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}(3bc)\int\frac{x^4}{1+c^2x^6}dx$$

$$=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1+c^{2/3}x^2}dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}}{1+\sqrt{3}\sqrt[3]{c}x}}{2\sqrt[3]{c}}$$

$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}dx}{8c^{2/3}}+\frac{\left(\sqrt{3}\right.}{}$$

$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+$$

$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\tan^{-1}\left(\right.}{4}$$

Mathematica [A] time = 0.03, size = 170, normalized size = 1.03

$$\frac{ax^2}{2}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right.}{4c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (a*x^2)/2 - (b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (b*x^2*ArcTan[c*x^3])/2 + (
b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3
)*x])/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8
*c^(2/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)
)
```

fricas [B] time = 0.47, size = 408, normalized size = 2.47

$$\frac{1}{2}bx^2\arctan\left(cx^3\right)+\frac{1}{2}ax^2+\frac{1}{8}\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^{10}x^2+\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{5}{6}}b^5c^3x+\left(\frac{b^6}{c^4}\right)^{\frac{2}{3}}b^6c^2\right)-\frac{1}{8}\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^{10}x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2*arctan(c*x^3) + 1/2*a*x^2 + 1/8*sqrt(3)*(b^6/c^4)^(1/6)*log(b^10*
x^2 + sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x + (b^6/c^4)^(2/3)*b^6*c^2) - 1/8*sq
rt(3)*(b^6/c^4)^(1/6)*log(b^10*x^2 - sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x + (b
^6/c^4)^(2/3)*b^6*c^2) + 1/2*(b^6/c^4)^(1/6)*arctan(-(2*(b^6/c^4)^(1/6)*b^5
*c*x + sqrt(3)*b^6 - 2*sqrt(b^10*x^2 + sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x +
(b^6/c^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6) + 1/2*(b^6/c^4)^(1/6)*arct
an(-(2*(b^6/c^4)^(1/6)*b^5*c*x - sqrt(3)*b^6 - 2*sqrt(b^10*x^2 - sqrt(3)*(b
^6/c^4)^(5/6)*b^5*c^3*x + (b^6/c^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6)
+ (b^6/c^4)^(1/6)*arctan(-((b^6/c^4)^(1/6)*b^5*c*x - sqrt(b^10*x^2 + (b^6/c
^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6)
```

giac [A] time = 2.11, size = 157, normalized size = 0.95

$$\frac{1}{8}bc^5\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^4|c|^{\frac{5}{3}}}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^6}-\frac{2|c|^{\frac{1}{3}}\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/8*b*c^5*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^4*
abs(c)^(5/3)) - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/a
bs(c)^(2/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c
)^(1/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1
/3))/c^6 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^6) + 1/2*b*x^2*arctan(c*
x^3) + 1/2*a*x^2
```

maple [A] time = 0.11, size = 154, normalized size = 0.93

$$\frac{ax^2}{2}+\frac{bx^2\arctan\left(cx^3\right)}{2}-\frac{b\arctan\left(\frac{x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\frac{bc\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{8}-\frac{b\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctan(c*x^3)-1/2*b/c/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1
/6))-1/8*b*c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(
1/3))-1/4*b/c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))+1/8*b*c*3^(1/
2)*(1/c^2)^(5/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/4*b/c/(1/c
^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))
```

maxima [A] time = 0.43, size = 137, normalized size = 0.83

$$\frac{1}{2}ax^2+\frac{1}{8}\left(4x^2\arctan\left(cx^3\right)+c\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)
*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/
c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(
1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/
c^(5/3)))*b
```

mupad [B] time = 0.69, size = 113, normalized size = 0.68

$$\frac{a\,x^2}{2}+\frac{b\,\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{4\,c^{2/3}}+\frac{b\,x^2\,\mathrm{atan}\left(c\,x^3\right)}{2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^2)/2 + (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^
(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(4*
c^(2/3)) + (b*x^2*atan(c*x^3))/2 - (3^(1/2)*b*(atan((-1)^(2/3)*c^(1/3)*x) -
 atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(4*c^(2/3))
```

sympy [A] time = 39.66, size = 303, normalized size = 1.84

$$\begin{cases} \frac{ax^2}{2}+\frac{bx^2\,\mathrm{atan}\left(cx^3\right)}{2}+\frac{3(-1)^{\frac{5}{6}}b\log\left(4x^2-4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{8c\sqrt[6]{\frac{1}{c^2}}}-\frac{3(-1)^{\frac{5}{6}}b\log\left(4x^2+4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{8c\sqrt[6]{\frac{1}{c^2}}}-\frac{(-1)^{\frac{5}{6}}\sqrt{3}\,b\,\mathrm{atan}\cdots}{4c\cdots} & \\ \frac{ax^2}{2} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x**3)/2 + 3*(-1)**(5/6)*b*log(4*x**2 -
4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(8*c*(c*
*(-2))**(1/6)) - 3*(-1)**(5/6)*b*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1
/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(8*c*(c**(-2))**(1/6)) - (-1)**(5/6)*
sqrt(3)*b*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(4
*c*(c**(-2))**(1/6)) - (-1)**(5/6)*sqrt(3)*b*atan(2*(-1)**(5/6)*sqrt(3)*x/(
3*(c**(-2))**(1/6)) + sqrt(3)/3)/(4*c*(c**(-2))**(1/6)) - (-1)**(1/3)*b*ata
n(c*x**3)/(2*c**2*(c**(-2))**(2/3)), Ne(c, 0)), (a*x**2/2, True))
```

3.111 $\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\,\log\left(c^{2/3}x^2+1\right)-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)$$

```
[Out] (-a-b*arctan(c*x^3))/x+1/2*b*c^(1/3)*ln(1+c^(2/3)*x^2)-1/4*b*c^(1/3)*ln(1-c
^(2/3)*x^2+c^(4/3)*x^4)-1/2*b*c^(1/3)*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))
*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\,\log\left(c^{2/3}x^2+1\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x^3])/x) - (Sqrt[3]*b*c^(1/3)*ArcTan[(1 - 2*c^(2/3)*x^2)/
Sqrt[3]])/2 + (b*c^(1/3)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - c^(2/
3)*x^2 + c^(4/3)*x^4])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^2}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+(3bc)\int \frac{x}{1+c^2x^6}\,dx\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{1}{1+c^2x^3}\,dx,x,x^2\right)\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)+\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{2-}{1-c^{2/3}}\right.\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\right.\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)+\frac{1}{2}\left(\right.\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 170, normalized size = 1.63

$$
-\frac{a}{x}+\frac{1}{2}b\sqrt[3]{c}\,\log\left(c^{2/3}x^2+1\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(c^{2/3}x^2-\sqrt{3}\,\sqrt[3]{c}\,x+1\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(c^{2/3}x^2+\sqrt{3}\,\sqrt[3]{c}\,x+1\right)-\frac{b\tan^{-1}\left(cx^3\right.}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x^3])/x - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] - 2*c^(1/3
)*x])/2 - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/2 + (b*c^(1/3)*
Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^
2])/4 - (b*c^(1/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/4
```

fricas [A] time = 0.44, size = 90, normalized size = 0.87

$$
\frac{2\sqrt{3}\,bc^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}\,c^{\frac{2}{3}}x^2-\frac{1}{3}\sqrt{3}\right)-bc^{\frac{1}{3}}x\log\left(c^2x^4-c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)+2\,bc^{\frac{1}{3}}x\log\left(cx^2+c^{\frac{1}{3}}\right)-4\,b\arctan\left(cx^3\right.}{4\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*c^(1/3)*x*arctan(2/3*sqrt(3)*c^(2/3)*x^2 - 1/3*sqrt(3)) -
b*c^(1/3)*x*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(1/3)*x*log(c*x^2
+ c^(1/3)) - 4*b*arctan(c*x^3) - 4*a)/x
```

giac [A] time = 3.82, size = 91, normalized size = 0.88

$$\frac{1}{4}\,bc\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}+\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{2}{3}}}\right)-\frac{b\arctan\left(cx^3\right)+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/3)
)/abs(c)^(2/3) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3)
+ 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(2/3)) - (b*arctan(c*x^3) + a)/x
```

maple [A] time = 0.03, size = 104, normalized size = 1.00

$$-\frac{a}{x}-\frac{b\arctan\left(c\,x^3\right)}{x}+\frac{b\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^2,x)
```

```
[Out] -a/x-b/x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(2/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))
```

maxima [A] time = 0.42, size = 98, normalized size = 0.94

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right)-\frac{4\arctan\left(cx^3\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(
2/3) - log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(2/3) + 2*log((c^(2/3)*x^2 + 1)
/c^(2/3))/c^(2/3)) - 4*arctan(c*x^3)/x)*b - a/x
```

mupad [B] time = 1.83, size = 99, normalized size = 0.95

$$\frac{b\,c^{1/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{2}-\frac{a}{x}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{x}-\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}-c^{2/3}\,x^2\,2\mathrm{i}+1\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}+c^{2/3}\,x^2\,2\mathrm{i}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^2,x)
```

```
[Out] (b*c^(1/3)*log(c^(2/3)*x^2 + 1))/2 - a/x - (b*atan(c*x^3))/x - (b*c^(1/3)*l
og(1i - c^(2/3)*x^2*2i - 3^(1/2))*(3^(1/2)*1i + 1))/4 + (b*c^(1/3)*log(c^(2
/3)*x^2*2i - 3^(1/2) - 1i)*(3^(1/2)*1i - 1))/4
```

sympy [A] time = 58.27, size = 328, normalized size = 3.15

$$\begin{cases} -\frac{a}{x} + (-1)^{\frac{5}{6}} bc^2 \left(\frac{1}{c^2}\right)^{\frac{5}{6}} \operatorname{atan}\left(cx^3\right) - \sqrt[3]{-1}\, bc \sqrt[3]{\frac{1}{c^2}} \log\left(x - \sqrt[6]{-1} \sqrt[6]{\frac{1}{c^2}}\right) + \frac{3 \sqrt[3]{-1}\, bc \sqrt[3]{\frac{1}{c^2}} \log\left(4x^2 - 4\sqrt[6]{-1}\, x \sqrt[6]{\frac{1}{c^2}} + 4\sqrt[3]{-1} \sqrt[3]{\frac{1}{c^2}}\right.}{4} \\ -\frac{a}{x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**2,x)
```

```
[Out] Piecewise((-a/x + (-1)**(5/6)*b*c**2*(c**(-2))**(5/6)*atan(c*x**3) - (-1)**
(1/3)*b*c*(c**(-2))**(1/3)*log(x - (-1)**(1/6)*(c**(-2))**(1/6)) + 3*(-1)**
(1/3)*b*c*(c**(-2))**(1/3)*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) +
4*(-1)**(1/3)*(c**(-2))**(1/3))/4 - (-1)**(1/3)*b*c*(c**(-2))**(1/3)*log(4*
x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/4
 + (-1)**(1/3)*sqrt(3)*b*c*(c**(-2))**(1/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3
*(c**(-2))**(1/6)) - sqrt(3)/3)/2 - (-1)**(1/3)*sqrt(3)*b*c*(c**(-2))**(1/3
)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/2 - b*atan
(c*x**3)/x, Ne(c, 0)), (-a/x, True))
```

3.112 $\int \frac{a+b\tan^{-1}(cx^3)}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)-\frac{1}{4}bc^{4/3}\tan^{-1}($$

```
[Out] -3/4*b*c/x-1/4*b*c^(4/3)*arctan(c^(1/3)*x)+1/4*(-a-b*arctan(c*x^3))/x^4-1/8
*b*c^(4/3)*arctan(2*c^(1/3)*x-3^(1/2))-1/8*b*c^(4/3)*arctan(2*c^(1/3)*x+3^(
1/2))-1/16*b*c^(4/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/16*b*c^(
4/3)*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5033, 325, 295, 634, 618, 204, 628, 203}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)-\frac{1}{4}bc^{4/3}\tan^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^5,x]
```

```
[Out] (-3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (a + b*ArcTan[c*x^3])/(4
*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(
2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/16
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^5}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}(3bc)\int \frac{1}{x^2\left(1+c^2x^6\right)}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{4}\left(3bc^3\right)\int \frac{x^4}{1+c^2x^6}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{4}\left(bc^{5/3}\right)\int \frac{1}{1+c^{2/3}x^2}\,dx-\frac{1}{4}\left(bc^{5/3}\right)\int \frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{\ldots}}{1-\sqrt{3}\sqrt[3]{c}\,x+c\ldots}$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\left(\sqrt{3}\,bc^{4/3}\right)\int \frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/\ldots}}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2\ldots}}$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/\ldots}\right.$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{8}bc^{4/3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)-\frac{1}{8}bc\ldots$$

Mathematica [A] time = 0.05, size = 179, normalized size = 1.03

$$-\frac{a}{4x^4}-\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)+\frac{1}{8}b\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (b*ArcTan[c*
x^3])/(4*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*Ar
```

```
cTan[Sqrt[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2
/3)*x^2])/16
```

fricas [B] time = 0.52, size = 595, normalized size = 3.42

$$\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(4\,b^{10}c^{14}x^2+4\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8+4\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x\right)-\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(4\,b^{10}c^{14}x^2+4\left(b^6c^8\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(3)*(b^6*c^8)^(1/6)*x^4*log(4*b^10*c^14*x^2 + 4*(b^6*c^8)^(2/3)*b
^6*c^8 + 4*sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x) - sqrt(3)*(b^6*c^8)^(1/6)*x^4
*log(4*b^10*c^14*x^2 + 4*(b^6*c^8)^(2/3)*b^6*c^8 - 4*sqrt(3)*(b^6*c^8)^(5/6
)*b^5*c^7*x) + sqrt(3)*(b^6*c^8)^(1/6)*x^4*log(b^10*c^14*x^2 + (b^6*c^8)^(2
/3)*b^6*c^8 + sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x) - sqrt(3)*(b^6*c^8)^(1/6)*
x^4*log(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8 - sqrt(3)*(b^6*c^8)^(5/6)*b
^5*c^7*x) + 8*(b^6*c^8)^(1/6)*x^4*arctan(-(sqrt(3)*b^6*c^8 + 2*(b^6*c^8)^(1
/6)*b^5*c^7*x - 2*sqrt(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8 + sqrt(3)*(b
^6*c^8)^(5/6)*b^5*c^7*x)*(b^6*c^8)^(1/6))/(b^6*c^8)) + 8*(b^6*c^8)^(1/6)*x^
4*arctan((sqrt(3)*b^6*c^8 - 2*(b^6*c^8)^(1/6)*b^5*c^7*x + 2*sqrt(b^10*c^14*
x^2 + (b^6*c^8)^(2/3)*b^6*c^8 - sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x)*(b^6*c^8
)^(1/6))/(b^6*c^8)) + 16*(b^6*c^8)^(1/6)*x^4*arctan(-((b^6*c^8)^(1/6)*b^5*c
^7*x - sqrt(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8)*(b^6*c^8)^(1/6))/(b^6*
c^8)) - 24*b*c*x^3 - 8*b*arctan(c*x^3) - 8*a)/x^4
```

giac [A] time = 2.98, size = 161, normalized size = 0.93

$$\frac{1}{16}\,bc^3\left(\frac{\sqrt{3}\,|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{\sqrt{3}\,|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}-\frac{2\,|c|^{\frac{1}{3}}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="giac")
```

```
[Out] 1/16*b*c^3*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c
)^(2/3))/c^2 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/ab
s(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)
^(1/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/c^2 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^2) - 1/4*(3*b*c*x^3 + b*a
rctan(c*x^3) + a)/x^4
```

maple [A] time = 0.10, size = 159, normalized size = 0.91

$$-\frac{a}{4x^4}-\frac{b\arctan\left(c\,x^3\right)}{4x^4}-\frac{bc\arctan\left(\frac{x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}\right)}{4\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\frac{b\,c^3\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{16}-\frac{bc\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x^3)-1/4*b*c/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(
1/6))-1/16*b*c^3*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^
2)^(1/3))-1/8*b*c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))+1/16*b*c^
3*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/8*b
*c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))-3/4*b*c/x
```

maxima [A] time = 0.42, size = 147, normalized size = 0.84

$$\frac{1}{16}\left(\left(c^{2}\left(\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^{2}+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^{2}-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\,\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\frac{2\,\arctan\left(\frac{2\,c^{\frac{2}{3}}x+\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*((c^2*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt
(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/
c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arc
tan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3)) - 12/x)*c - 4*arctan(
c*x^3)/x^4)*b - 1/4*a/x^4
```

mupad [B] time = 0.71, size = 120, normalized size = 0.69

$$-\frac{a}{4\,x^4}+\frac{b\,c^{4/3}\,\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{8}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^5,x)
```

```
[Out] (b*c^(4/3)*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2
)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/8 - a/(
4*x^4) - (b*atan(c*x^3))/(4*x^4) - (3*b*c)/(4*x) - (3^(1/2)*b*c^(4/3)*(atan
((-1)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1
i)/8
```

sympy [A] time = 109.84, size = 320, normalized size = 1.84

$$\begin{cases} -\frac{a}{4x^4}+\frac{3(-1)^{\frac{5}{6}}bc^3\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\log\left(4x^2-4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{16}-\frac{3(-1)^{\frac{5}{6}}bc^3\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\log\left(4x^2+4\sqrt[6]{-1}\,x\sqrt[6]{\frac{1}{c^2}}+4\sqrt[3]{-1}\sqrt[3]{\frac{1}{c^2}}\right)}{16}-\frac{(-1)^{\frac{5}{6}}\sqrt{3}bc^3\left(\right.}{} \\ -\frac{a}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**5,x)
```

```
[Out] Piecewise((-a/(4*x**4) + 3*(-1)**(5/6)*b*c**3*(c**(-2))**(5/6)*log(4*x**2 -
 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/16 - 3*
(-1)**(5/6)*b*c**3*(c**(-2))**(5/6)*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))*
*(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/16 - (-1)**(5/6)*sqrt(3)*b*c**3*(c
**(-2))**(5/6)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/
3)/8 - (-1)**(5/6)*sqrt(3)*b*c**3*(c**(-2))**(5/6)*atan(2*(-1)**(5/6)*sqrt(
3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/8 - (-1)**(1/3)*b*c**2*(c**(-2))**(1
/3)*atan(c*x**3)/4 - 3*b*c/(4*x) - b*atan(c*x**3)/(4*x**4), Ne(c, 0)), (-a/
(4*x**4), True))
```

3.113 $\int x^{11} \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx$

Optimal. Leaf size=124

$$-\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{12c^4}+\frac{abx^3}{6c^3}+\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{bx^9\left(a+b\tan^{-1}\left(cx^3\right)\right)}{18c}+\frac{b^2x^3\tan^{-1}\left(cx^3\right)}{6c^3}+\frac{b^2x^6}{36c^2}-\frac{b^2\log}{}$$

```
[Out] 1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctan(c*x^3)/c^3-1/18*b*x^9*(
a+b*arctan(c*x^3))/c-1/12*(a+b*arctan(c*x^3))^2/c^4+1/12*x^12*(a+b*arctan(c
*x^3))^2-1/9*b^2*ln(c^2*x^6+1)/c^4
```

Rubi [C] time = 1.65, antiderivative size = 731, normalized size of antiderivative = 5.90, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)}{24c^4}-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)}{24c^4}+\frac{abx^3}{12c^3}-\frac{bx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{48c^2}+\frac{1}{288}ib\left(-\frac{3\left(1-i\right.}{c^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(12*c^3) - (((23*I)/288)*b^2*x^3)/c^3 + (b^2*x^6)/(192*c^2) - (((
7*I)/864)*b^2*x^9)/c + (b^2*x^12)/384 - (b^2*(1 - I*c*x^3)^2)/(16*c^4) + (b
^2*(1 - I*c*x^3)^3)/(54*c^4) - (b^2*(1 - I*c*x^3)^4)/(384*c^4) - (b^2*Log[I
 - c*x^3])/(36*c^4) - (b^2*(1 - I*c*x^3)*Log[1 - I*c*x^3])/(24*c^4) - (b^2*
Log[1 - I*c*x^3]^2)/(48*c^4) - (b*x^6*((2*I)*a - b*Log[1 - I*c*x^3]))/(48*c
^2) + ((I/72)*b*x^9*((2*I)*a - b*Log[1 - I*c*x^3]))/c + (b*x^12*((2*I)*a -
b*Log[1 - I*c*x^3]))/96 + (x^12*(2*a + I*b*Log[1 - I*c*x^3])^2)/48 + (I/288
)*b*(2*a + I*b*Log[1 - I*c*x^3])*((48*(1 - I*c*x^3))/c^4 - (36*(1 - I*c*x^3
)^2)/c^4 + (16*(1 - I*c*x^3)^3)/c^4 - (3*(1 - I*c*x^3)^4)/c^4 - (12*Log[1 -
 I*c*x^3])/c^4) + (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/(
24*c^4) + ((I/36)*b^2*x^9*Log[1 + I*c*x^3])/c - (b^2*(1 + I*c*x^3)*Log[1 +
I*c*x^3])/(12*c^4) - (b^2*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3])/(24*c^4) -
 (b*x^12*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/24 + (b^2*Log[1 +
 I*c*x^3]^2)/(48*c^4) - (b^2*x^12*Log[1 + I*c*x^3]^2)/48 + (5*b^2*Log[I + c
*x^3])/(288*c^4) - (b^2*PolyLog[2, (1 - I*c*x^3)/2])/(24*c^4) - (b^2*PolyLo
g[2, (1 + I*c*x^3)/2])/(24*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^{11}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\,dx=\int\left(\frac{1}{4}x^{11}\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^{11}\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+i\right.\right.$$

$$=\frac{1}{4}\int x^{11}\left(2a+ib\log\left(1-icx^3\right)\right)^2dx+\frac{1}{2}b\int x^{11}\left(-2ia+b\log\left(1-icx^3\right)\right)\log$$

$$=\frac{1}{12}\operatorname{Subst}\left(\int x^3(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\operatorname{Subst}\left(\int x^3(-2ia+b\,l\right.$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{24}bx^{12}\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right.$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{24}bx^{12}\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right.$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{288}ib\left(2a+ib\log\left(1-icx^3\right)\right)\left(\frac{48\left(1-icx^3\right)}{c^4}\right.$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{48c^2}+\frac{ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)}{72c}+\frac{1}{96}bx^{12}\left(\right.$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{48c^2}+\frac{ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)}{72c}+\frac{1}{96}bx^{12}\left(\right.$$

$$=\frac{abx^3}{12c^3}-\frac{55ib^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{ib^2x^9}{864c}+\frac{b^2x^{12}}{384}-\frac{b^2\left(1-icx^3\right)^2}{16c^4}+\frac{b^2\left(1-icx^3\right)^3}{54c^4}-$$

$$=\frac{abx^3}{12c^3}-\frac{55ib^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{ib^2x^9}{864c}+\frac{b^2x^{12}}{384}-\frac{b^2\left(1-icx^3\right)^2}{16c^4}+\frac{b^2\left(1-icx^3\right)^3}{54c^4}-$$

Mathematica [A] time = 0.10, size = 121, normalized size = 0.98

$$\frac{cx^3\left(3a^2c^3x^9-2abc^2x^6+6ab+b^2cx^3\right)-2b\tan^{-1}\left(cx^3\right)\left(a\left(3-3c^4x^{12}\right)+bcx^3\left(c^2x^6-3\right)\right)+3b^2\left(c^4x^{12}-1\right)\tan}{36c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (c*x^3*(6*a*b + b^2*c*x^3 - 2*a*b*c^2*x^6 + 3*a^2*c^3*x^9) - 2*b*(b*c*x^3*(
-3 + c^2*x^6) + a*(3 - 3*c^4*x^12))*ArcTan[c*x^3] + 3*b^2*(-1 + c^4*x^12)*A
rcTan[c*x^3]^2 - 4*b^2*Log[1 + c^2*x^6])/(36*c^4)
```

fricas [A] time = 0.47, size = 129, normalized size = 1.04

$$\frac{3\,a^2c^4x^{12}-2\,abc^3x^9+b^2c^2x^6+6\,abcx^3+3\left(b^2c^4x^{12}-b^2\right)\arctan\left(cx^3\right)^2-4\,b^2\log\left(c^2x^6+1\right)+2\left(3\,abc^4x^{12}-\right.}{36\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*a^2*c^4*x^12 - 2*a*b*c^3*x^9 + b^2*c^2*x^6 + 6*a*b*c*x^3 + 3*(b^2*c
^4*x^12 - b^2)*arctan(c*x^3)^2 - 4*b^2*log(c^2*x^6 + 1) + 2*(3*a*b*c^4*x^12
 - b^2*c^3*x^9 + 3*b^2*c*x^3 - 3*a*b)*arctan(c*x^3))/c^4
```

giac [A] time = 0.19, size = 145, normalized size = 1.17

$$\frac{3\,a^2cx^{12}+2\left(3\,cx^{12}\arctan\left(cx^3\right)-\frac{3\,\arctan\left(cx^3\right)}{c^3}-\frac{c^9x^9-3\,c^7x^3}{c^9}\right)ab+\left(3\,cx^{12}\arctan\left(cx^3\right)^2-\frac{2\,c^3x^9\arctan\left(cx^3\right)-c^2x^6-6\,cx^3\arctan\left(cx^3\right)+3\,\arctan\left(cx^3\right)^2+4\,\log\left(c^2x^6+1\right)}{c^3}\right)b^2}{36\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/36*(3*a^2*c*x^12 + 2*(3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9
*x^9 - 3*c^7*x^3)/c^9)*a*b + (3*c*x^12*arctan(c*x^3)^2 - (2*c^3*x^9*arctan(
c*x^3) - c^2*x^6 - 6*c*x^3*arctan(c*x^3) + 3*arctan(c*x^3)^2 + 4*log(c^2*x^
6 + 1))/c^3)*b^2)/c
```

maple [A] time = 0.05, size = 151, normalized size = 1.22

$$\frac{x^{12}a^2}{12}+\frac{b^2x^{12}\arctan\left(c\,x^3\right)^2}{12}-\frac{b^2\arctan\left(c\,x^3\right)x^9}{18c}+\frac{b^2x^3\arctan\left(c\,x^3\right)}{6c^3}-\frac{b^2\arctan\left(c\,x^3\right)^2}{12c^4}+\frac{b^2x^6}{36c^2}-\frac{b^2\ln\left(c^2x^6+1\right)}{9c^4}+\frac{a b x^{12}\arctan\left(c\,x^3\right)}{6}-\frac{a b x^9}{18c}+\frac{a b x^3}{6c^3}-\frac{a b\arctan\left(c\,x^3\right)}{6c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] 1/12*x^12*a^2+1/12*b^2*x^12*arctan(c*x^3)^2-1/18*b^2*arctan(c*x^3)/c*x^9+1/
6*b^2*x^3*arctan(c*x^3)/c^3-1/12*b^2/c^4*arctan(c*x^3)^2+1/36*b^2*x^6/c^2-1
/9*b^2*ln(c^2*x^6+1)/c^4+1/6*a*b*x^12*arctan(c*x^3)-1/18*a*b/c*x^9+1/6*a*b*
x^3/c^3-1/6*a*b/c^4*arctan(c*x^3)
```

maxima [A] time = 0.52, size = 169, normalized size = 1.36

$$\frac{1}{12}\,b^2x^{12}\arctan\left(cx^3\right)^2+\frac{1}{12}\,a^2x^{12}+\frac{1}{18}\left(3\,x^{12}\arctan\left(cx^3\right)-c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\right)ab-\frac{1}{36}\left(2\,c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\arctan\left(cx^3\right)-\frac{c^2x^6+3\,\arctan\left(cx^3\right)^2-3\,\log\left(18\,c^7x^6+18\,c^5\right)-\log\left(c^2x^6+1\right)}{c^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^12*arctan(c*x^3)^2 + 1/12*a^2*x^12 + 1/18*(3*x^12*arctan(c*x^3)
- c*((c^2*x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5))*a*b - 1/36*(2*c*((c^2*x^
9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5)*arctan(c*x^3) - (c^2*x^6 + 3*arctan(c
*x^3)^2 - 3*log(18*c^7*x^6 + 18*c^5) - log(c^2*x^6 + 1))/c^4)*b^2
```

mupad [B] time = 1.14, size = 150, normalized size = 1.21

$$\frac{a^2\,x^{12}}{12}-\frac{b^2\,\mathrm{atan}\left(c\,x^3\right)^2}{12\,c^4}+\frac{b^2\,x^{12}\,\mathrm{atan}\left(c\,x^3\right)^2}{12}-\frac{b^2\,\ln\left(c^2\,x^6+1\right)}{9\,c^4}+\frac{b^2\,x^6}{36\,c^2}+\frac{b^2\,x^3\,\mathrm{atan}\left(c\,x^3\right)}{6\,c^3}-\frac{b^2\,x^9\,\mathrm{atan}\left(c\,x^3\right)}{18\,c}+\frac{a\,b\,x^3}{6\,c^3}-\frac{a\,b\,x^9}{18\,c}-\frac{a\,b\,\mathrm{atan}\left(c\,x^3\right)}{6\,c^4}+\frac{a\,b\,x^{12}\,\mathrm{atan}\left(c\,x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atan(c*x^3))^2,x)
```

```
[Out] (a^2*x^12)/12 - (b^2*atan(c*x^3)^2)/(12*c^4) + (b^2*x^12*atan(c*x^3)^2)/12
- (b^2*log(c^2*x^6 + 1))/(9*c^4) + (b^2*x^6)/(36*c^2) + (b^2*x^3*atan(c*x^3
))/(6*c^3) - (b^2*x^9*atan(c*x^3))/(18*c) + (a*b*x^3)/(6*c^3) - (a*b*x^9)/(
18*c) - (a*b*atan(c*x^3))/(6*c^4) + (a*b*x^12*atan(c*x^3))/6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.114 $\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx$

Optimal. Leaf size=154

$$-\frac{i\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{9c^3}-\frac{2b\log\left(\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{9c^3}+\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{bx^6\left(a+b\tan^{-1}\left(cx^3\right)\right)}{9c}-\frac{ib}{}$$

```
[Out] 1/9*b^2*x^3/c^2-1/9*b^2*arctan(c*x^3)/c^3-1/9*b*x^6*(a+b*arctan(c*x^3))/c-1
/9*I*(a+b*arctan(c*x^3))^2/c^3+1/9*x^9*(a+b*arctan(c*x^3))^2-2/9*b*(a+b*arc
tan(c*x^3))*ln(2/(1+I*c*x^3))/c^3-1/9*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c^3
```

Rubi [B] time = 1.39, antiderivative size = 647, normalized size of antiderivative = 4.20, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)}{18c^3}-\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)}{18c^3}-\frac{iabx^3}{9c^2}+\frac{1}{108}ib\left(\frac{2i\left(1-icx^3\right)^3}{c^3}-\frac{9i\left(1-icx^3\right)^2}{c^3}+\frac{18i\left(\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] ((-I/9)*a*b*x^3)/c^2 + (19*b^2*x^3)/(108*c^2) - (((5*I)/216)*b^2*x^6)/c + (
b^2*x^9)/162 - ((I/24)*b^2*(1 - I*c*x^3)^2)/c^3 + ((I/162)*b^2*(1 - I*c*x^3
)^3)/c^3 + ((I/18)*b^2*Log[I - c*x^3])/c^3 + ((I/18)*b^2*(1 - I*c*x^3)*Log[
1 - I*c*x^3])/c^3 - ((I/36)*b^2*Log[1 - I*c*x^3]^2)/c^3 + ((I/36)*b*x^6*((2
*I)*a - b*Log[1 - I*c*x^3]))/c + (b*x^9*((2*I)*a - b*Log[1 - I*c*x^3]))/54
+ (x^9*(2*a + I*b*Log[1 - I*c*x^3])^2)/36 + (I/108)*b*(2*a + I*b*Log[1 - I*
c*x^3])*(((18*I)*(1 - I*c*x^3))/c^3 - ((9*I)*(1 - I*c*x^3)^2)/c^3 + ((2*I)*
(1 - I*c*x^3)^3)/c^3 - ((6*I)*Log[1 - I*c*x^3])/c^3) - ((I/18)*b*((2*I)*a -
 b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/c^3 + ((I/18)*b^2*x^6*Log[1 + I*
c*x^3])/c - ((I/18)*b^2*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3])/c^3 - (b*x^9
*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/18 - ((I/36)*b^2*Log[1 +
I*c*x^3]^2)/c^3 - (b^2*x^9*Log[1 + I*c*x^3]^2)/36 - ((I/108)*b^2*Log[I + c*
x^3])/c^3 + ((I/18)*b^2*PolyLog[2, (1 - I*c*x^3)/2])/c^3 - ((I/18)*b^2*Poly
Log[2, (1 + I*c*x^3)/2])/c^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx = \int \left(\frac{1}{4}x^8\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^8\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right.\right.$$

$$= \frac{1}{4}\int x^8\left(2a+ib\log\left(1-icx^3\right)\right)^2 dx+\frac{1}{2}b\int x^8\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1\right.$$

$$= \frac{1}{12}\operatorname{Subst}\left(\int x^2(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\operatorname{Subst}\left(\int x^2(-2ia+b\log\right.$$

$$= \frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{18}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-$$

$$= \frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{18}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-$$

$$= \frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{108}ib\left(2a+ib\log\left(1-icx^3\right)\right)\left(\frac{18i\left(1-icx^3\right)}{c^3}\right.$$

$$= -\frac{iabx^3}{9c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{36c}+\frac{1}{54}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{1}{36}x^9$$

$$= -\frac{iabx^3}{9c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{36c}+\frac{1}{54}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{1}{36}x^9$$

$$= -\frac{iabx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{ib^2x^6}{216c}+\frac{b^2x^9}{162}-\frac{ib^2\left(1-icx^3\right)^2}{24c^3}+\frac{ib^2\left(1-icx^3\right)^3}{162c^3}+\frac{ib^2\left(1-\right.}{}$$

$$= -\frac{iabx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{ib^2x^6}{216c}+\frac{b^2x^9}{162}-\frac{ib^2\left(1-icx^3\right)^2}{24c^3}+\frac{ib^2\left(1-icx^3\right)^3}{162c^3}+\frac{ib^2\left(1-\right.}{}$$

Mathematica [A] time = 0.31, size = 141, normalized size = 0.92

$$\frac{a^2c^3x^9-abc^2x^6+ab\log\left(c^2x^6+1\right)-b\tan^{-1}\left(cx^3\right)\left(-2ac^3x^9+bc^2x^6+2b\log\left(1+e^{2i\tan^{-1}\left(cx^3\right)}\right)+b\right)+b^2\left(c^3x\right.}{9c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*c*x^3 - a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(I + c^3*x^9)*ArcTan[c*x^3]^2
- b*ArcTan[c*x^3]*(b + b*c^2*x^6 - 2*a*c^3*x^9 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^3])]) + a*b*Log[1 + c^2*x^6] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
3])])/(9*c^3)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2x^8\arctan\left(cx^3\right)^2+2\,abx^8\arctan\left(cx^3\right)+a^2x^8,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^8*arctan(c*x^3)^2 + 2*a*b*x^8*arctan(c*x^3) + a^2*x^8, x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \arctan\left(cx^3\right) + a\right)^2 x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*x^8, x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a + b \arctan\left(c\, x^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int(x^8*(a+b*arctan(c*x^3))^2,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9} a^2 x^9 + \frac{1}{9} \left(2\,x^9 \arctan\left(cx^3\right) - \left(\frac{x^6}{c^2} - \frac{\log\left(c^2x^6+1\right)}{c^4}\right) c\right) ab + \frac{1}{144} \left(4\,x^9 \arctan\left(cx^3\right)^2 - x^9 \log\left(c^2x^6+1\right)^2 + 144 \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/9*a^2*x^9 + 1/9*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c
)*a*b + 1/144*(4*x^9*arctan(c*x^3)^2 - x^9*log(c^2*x^6 + 1)^2 + 144*integra
te(1/48*(4*c^2*x^14*log(c^2*x^6 + 1) - 8*c*x^11*arctan(c*x^3) + 36*(c^2*x^1
4 + x^8)*arctan(c*x^3)^2 + 3*(c^2*x^14 + x^8)*log(c^2*x^6 + 1)^2)/(c^2*x^6
+ 1), x))*b^2
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 \left(a + b \operatorname{atan}\left(c\, x^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atan(c*x^3))^2,x)
```

```
[Out] int(x^8*(a + b*atan(c*x^3))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.115 $\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=90

$$\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{6c^2}-\frac{abx^3}{3c}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)^2+\frac{b^2\log\left(c^2x^6+1\right)}{6c^2}-\frac{b^2x^3\tan^{-1}\left(cx^3\right)}{3c}$$

```
[Out] -1/3*a*b*x^3/c-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*(a+b*arctan(c*x^3))^2/c^2+1/
6*x^6*(a+b*arctan(c*x^3))^2+1/6*b^2*ln(c^2*x^6+1)/c^2
```

Rubi [C] time = 1.05, antiderivative size = 612, normalized size of antiderivative = 6.80, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)}{12c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)}{12c^2}-\frac{\left(1-icx^3\right)^2\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24c^2}+\frac{\left(1-icx^3\right)(2a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] -(a*b*x^3)/(2*c) + (b^2*x^6)/24 + (b^2*(1 - I*c*x^3)^2)/(48*c^2) + (b^2*(1
+ I*c*x^3)^2)/(48*c^2) - (b^2*Log[I - c*x^3])/(24*c^2) + (b^2*(1 - I*c*x^3)
*Log[1 - I*c*x^3])/(4*c^2) + (b*x^6*((2*I)*a - b*Log[1 - I*c*x^3]))/24 + ((
I/24)*b*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3]))/c^2 + ((1 - I*c*x^3)*
(2*a + I*b*Log[1 - I*c*x^3])^2)/(12*c^2) - ((1 - I*c*x^3)^2*(2*a + I*b*Log[
1 - I*c*x^3])^2)/(24*c^2) - (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*
x^3)/2])/(12*c^2) - (b^2*x^6*Log[1 + I*c*x^3])/24 + (b^2*(1 + I*c*x^3)*Log[
1 + I*c*x^3])/(4*c^2) - (b^2*(1 + I*c*x^3)^2*Log[1 + I*c*x^3])/(24*c^2) + (
b^2*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3])/(12*c^2) - (b*x^6*((2*I)*a - b*L
og[1 - I*c*x^3])*Log[1 + I*c*x^3])/12 - (b^2*(1 + I*c*x^3)*Log[1 + I*c*x^3]
^2)/(12*c^2) + (b^2*(1 + I*c*x^3)^2*Log[1 + I*c*x^3]^2)/(24*c^2) - (b^2*Log
[I + c*x^3])/(24*c^2) + (b^2*PolyLog[2, (1 - I*c*x^3)/2])/(12*c^2) + (b^2*P
olyLog[2, (1 + I*c*x^3)/2])/(12*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx = \int \left(\frac{1}{4}x^5\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^5\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right.\right.$$

$$=\frac{1}{4}\int x^5\left(2a+ib\log\left(1-icx^3\right)\right)^2 dx+\frac{1}{2}b\int x^5\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1\right.$$

$$=\frac{1}{12}\operatorname{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\operatorname{Subst}\left(\int x(-2ia+b\log(\right.$$

$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)+\frac{1}{12}\operatorname{Subst}\left(\int\left(-\frac{i(2a+ib\log(}{c}\right.\right.$$

$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{i\operatorname{Subst}\left(\int(2a+ib\log(1-icx)\right.}{12c}$$

$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{12}b\operatorname{Subst}\left(\int x(-2ia+b\log(\right.$$

$$=-\frac{abx^3}{6c}+\frac{1}{24}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c^2}-$$

$$=-\frac{abx^3}{2c}-\frac{ib^2x^3}{4c}+\frac{b^2\left(1-icx^3\right)^2}{48c^2}+\frac{b^2\left(1+icx^3\right)^2}{48c^2}+\frac{1}{24}bx^6\left(2ia-b\log\left(1-icx^3\right.\right.$$

$$=-\frac{abx^3}{2c}+\frac{b^2x^6}{24}+\frac{b^2\left(1-icx^3\right)^2}{48c^2}+\frac{b^2\left(1+icx^3\right)^2}{48c^2}-\frac{b^2\log\left(i-cx^3\right)}{24c^2}+\frac{b^2\left(1-ic\right.}{}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.94

$$\frac{2b\tan^{-1}\left(cx^3\right)\left(ac^2x^6+a-bcx^3\right)+acx^3\left(acx^3-2b\right)+b^2\log\left(c^2x^6+1\right)+b^2\left(c^2x^6+1\right)\tan^{-1}\left(cx^3\right)^2}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (a*c*x^3*(-2*b + a*c*x^3) + 2*b*(a - b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b
^2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 + b^2*Log[1 + c^2*x^6])/(6*c^2)
```

fricas [A] time = 0.45, size = 91, normalized size = 1.01

$$\frac{a^2c^2x^6-2\,abcx^3+\left(b^2c^2x^6+b^2\right)\arctan\left(cx^3\right)^2+b^2\log\left(c^2x^6+1\right)+2\left(abc^2x^6-b^2cx^3+ab\right)\arctan\left(cx^3\right)}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*c^2*x^6 - 2*a*b*c*x^3 + (b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + b^2*
log(c^2*x^6 + 1) + 2*(a*b*c^2*x^6 - b^2*c*x^3 + a*b)*arctan(c*x^3))/c^2
```

giac [A] time = 1.60, size = 100, normalized size = 1.11

$$\frac{a^2cx^6+\frac{2\left(c^2x^6\arctan\left(cx^3\right)-cx^3+\arctan\left(cx^3\right)\right)ab}{c}+\frac{\left(c^2x^6\arctan\left(cx^3\right)^2-2\,cx^3\arctan\left(cx^3\right)+\arctan\left(cx^3\right)^2+\log\left(c^2x^6+1\right)\right)b^2}{c}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*(a^2*c*x^6 + 2*(c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*a*b/c +
(c^2*x^6*arctan(c*x^3)^2 - 2*c*x^3*arctan(c*x^3) + arctan(c*x^3)^2 + log(c^
2*x^6 + 1))*b^2/c)/c
```

maple [A] time = 0.04, size = 113, normalized size = 1.26

$$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan\left(c\,x^3\right)^2}{6}-\frac{b^2x^3\arctan\left(c\,x^3\right)}{3c}+\frac{b^2\arctan\left(c\,x^3\right)^2}{6c^2}+\frac{b^2\ln\left(c^2x^6+1\right)}{6c^2}+\frac{ab\,x^6\arctan\left(c\,x^3\right)}{3}-\frac{ab\,x^3}{3c}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctan(c*x^3)^2-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*b^2
/c^2*arctan(c*x^3)^2+1/6*b^2*ln(c^2*x^6+1)/c^2+1/3*a*b*x^6*arctan(c*x^3)-1/
3*a*b*x^3/c+1/3*a*b/c^2*arctan(c*x^3)
```

maxima [A] time = 0.51, size = 126, normalized size = 1.40

$$\frac{1}{6}b^2x^6\arctan\left(cx^3\right)^2+\frac{1}{6}a^2x^6+\frac{1}{3}\left(x^6\arctan\left(cx^3\right)-c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\right)ab-\frac{1}{6}\left(2\,c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^2*x^6 + 1/3*(x^6*arctan(c*x^3) - c*(x^3
/c^2 - arctan(c*x^3)/c^3))*a*b - 1/6*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)*arc
tan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*b^2
```

mupad [B] time = 0.72, size = 112, normalized size = 1.24

$$\frac{a^2\,x^6}{6}+\frac{b^2\,\mathrm{atan}\left(c\,x^3\right)^2}{6\,c^2}+\frac{b^2\,x^6\,\mathrm{atan}\left(c\,x^3\right)^2}{6}+\frac{b^2\,\ln\left(c^2\,x^6+1\right)}{6\,c^2}-\frac{b^2\,x^3\,\mathrm{atan}\left(c\,x^3\right)}{3\,c}-\frac{a\,b\,x^3}{3\,c}+\frac{a\,b\,\mathrm{atan}\left(c\,x^3\right)}{3\,c^2}+\frac{a\,b\,x^6\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^3))^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*atan(c*x^3)^2)/(6*c^2) + (b^2*x^6*atan(c*x^3)^2)/6 + (b^
2*log(c^2*x^6 + 1))/(6*c^2) - (b^2*x^3*atan(c*x^3))/(3*c) - (a*b*x^3)/(3*c)
 + (a*b*atan(c*x^3))/(3*c^2) + (a*b*x^6*atan(c*x^3))/3
```

sympy [A] time = 122.14, size = 209, normalized size = 2.32

$$\begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\,\mathrm{atan}\left(cx^3\right)}{3} - \frac{abx^3}{3c} + \frac{ab\,\mathrm{atan}\left(cx^3\right)}{3c^2} + \frac{b^2x^6\,\mathrm{atan}^2\left(cx^3\right)}{6} - \frac{b^2x^3\,\mathrm{atan}\left(cx^3\right)}{3c} - \frac{ib^2\sqrt{\frac{1}{c^2}}\,\mathrm{atan}\left(cx^3\right)}{3c} + \frac{b^2\log\left(x-\sqrt[6]{-1}\sqrt[6]{\frac{1}{c^2}}\right)}{3c^2} + \\ \frac{a^2x^6}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x**3)/3 - a*b*x**3/(3*c) + a*b*ata
n(c*x**3)/(3*c**2) + b**2*x**6*atan(c*x**3)**2/6 - b**2*x**3*atan(c*x**3)/(
3*c) - I*b**2*sqrt(c**(-2))*atan(c*x**3)/(3*c) + b**2*log(x - (-1)**(1/6)*(
c**(-2))**(1/6))/(3*c**2) + b**2*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1
/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(3*c**2) + b**2*atan(c*x**3)**2/(6*c*
*2), Ne(c, 0)), (a**2*x**6/6, True))
```

3.116 $\int x^2 \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=104

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)^2+\frac{i\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{3c}+\frac{2b\log\left(\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{3c}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)}{3c}$$

```
[Out] 1/3*I*(a+b*arctan(c*x^3))^2/c+1/3*x^3*(a+b*arctan(c*x^3))^2+2/3*b*(a+b*arct
an(c*x^3))*ln(2/(1+I*c*x^3))/c+1/3*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c
```

Rubi [B] time = 0.64, antiderivative size = 255, normalized size of antiderivative = 2.45, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)}{6c}+\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)}{6c}-\frac{1}{6}bx^3\log\left(1+icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{i(1-$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] ((I/12)*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)/c + ((I/6)*b*((2*I)*a
 - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/c + ((I/6)*b^2*Log[(1 - I*c*x^
3)/2]*Log[1 + I*c*x^3])/c - (b*x^3*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I
*c*x^3])/6 + ((I/12)*b^2*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/c - ((I/6)*b^2*P
olyLog[2, (1 - I*c*x^3)/2])/c + ((I/6)*b^2*PolyLog[2, (1 + I*c*x^3)/2])/c
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^2dx=\int\left(\frac{1}{4}x^2\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\right.$$
$$=\frac{1}{4}\int x^2\left(2a+ib\log\left(1-icx^3\right)\right)^2dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+\right.$$
$$=\frac{1}{12}\text{Subst}\left(\int(2a+ib\log(1-icx))^2dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int(-2ia+b\log(1-ic\right.$$
$$=-\frac{1}{6}bx^3\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)+\frac{i\,\text{Subst}\left(\int(2a+ib\log(x))^2dx,x,1\right.}{12c}$$
$$=\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}-\frac{1}{6}bx^3\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)$$
$$=-\frac{1}{3}iabx^3-\frac{b^2x^3}{6}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}-\frac{ib^2\left(1+icx^3\right)\log\left(1+ic\right.}{6c}$$
$$=-\frac{1}{3}b^2x^3+\frac{ib^2\left(1-icx^3\right)\log\left(1-icx^3\right)}{6c}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}+\frac{ib}{}$$
$$=-\frac{1}{6}b^2x^3+\frac{ib^2\left(1-icx^3\right)\log\left(1-icx^3\right)}{6c}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}+\frac{ib}{}$$
$$=\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}+\frac{ib\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\frac{1}{2}\left(1+icx^3\right)\right.}{6c}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 1.03

$$\frac{a\left(acx^3-b\log\left(c^2x^6+1\right)\right)+2b\tan^{-1}\left(cx^3\right)\left(acx^3+b\log\left(1+e^{2i\tan^{-1}\left(cx^3\right)}\right)\right)-ib^2\text{Li}_2\left(-e^{2i\tan^{-1}\left(cx^3\right)}\right)+b^2\left(cx^3-i\right.}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*(-I + c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(a*c*x^3 + b*Log[1 +
E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3 - b*Log[1 + c^2*x^6]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^3])])/(3*c)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\arctan\left(cx^3\right)^2+2\,abx^2\arctan\left(cx^3\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c*x^3)^2 + 2*a*b*x^2*arctan(c*x^3) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^3\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*x^2, x)
```

maple [A] time = 0.23, size = 148, normalized size = 1.42

$$\frac{\arctan\left(c\,x^{3}\right)^{2} x^{3} b^{2}}{3}+\frac{2 x^{3} a b \arctan\left(c\,x^{3}\right)}{3}+\frac{x^{3} a^{2}}{3}-\frac{i \arctan\left(c\,x^{3}\right)^{2} b^{2}}{3 c}-\frac{i \operatorname{polylog}\left(2,-\frac{\left(i c\,x^{3}+1\right)^{2}}{c^{2} x^{6}+1}\right) b^{2}}{3 c}+\frac{2 \arctan\left(c\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] 1/3*arctan(c*x^3)^2*x^3*b^2+2/3*x^3*a*b*arctan(c*x^3)+1/3*x^3*a^2-1/3*I/c*a
rctan(c*x^3)^2*b^2-1/3*I/c*polylog(2,-(1+I*c*x^3)^2/(c^2*x^6+1))*b^2+2/3/c*
arctan(c*x^3)*ln((1+I*c*x^3)^2/(c^2*x^6+1)+1)*b^2-1/3/c*a*b*ln(c^2*x^6+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^{2} x^{3}+\frac{1}{48}\left(4\,x^{3} \arctan\left(c x^{3}\right)^{2}-x^{3} \log\left(c^{2} x^{6}+1\right)^{2}+576\,c^{2} \int \frac{x^{8} \arctan\left(c x^{3}\right)^{2}}{16\left(c^{2} x^{6}+1\right)}\,dx+48\,c^{2} \int \frac{x^{8} \log\left(c^{2} x^{6}+1\right.}{16\left(c^{2} x^{6}+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/48*(4*x^3*arctan(c*x^3)^2 - x^3*log(c^2*x^6 + 1)^2 + 576*c^
2*integrate(1/16*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 48*c^2*integrate(1
/16*x^8*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 192*c^2*integrate(1/16*x^8*l
og(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 4*arctan(c*x^3)^3/c - 384*c*integrate(1
/16*x^5*arctan(c*x^3)/(c^2*x^6 + 1), x) + 48*integrate(1/16*x^2*log(c^2*x^6
 + 1)^2/(c^2*x^6 + 1), x))*b^2 + 1/3*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 +
 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2} \left(a+b \operatorname{atan}\left(c\,x^{3}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^3))^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x^3))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a+b \operatorname{atan}\left(c x^{3}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x**3))**2, x)
```

3.117 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{3}ib\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{3}ib\text{Li}_2\left(\frac{2}{icx^3+1}-1\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{2}{3}\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)(a$$

```
[Out] -2/3*(a+b*arctan(c*x^3))^2*arctanh(-1+2/(1+I*c*x^3))-1/3*I*b*(a+b*arctan(c*
x^3))*polylog(2,1-2/(1+I*c*x^3))+1/3*I*b*(a+b*arctan(c*x^3))*polylog(2,-1+2
/(1+I*c*x^3))-1/6*b^2*polylog(3,1-2/(1+I*c*x^3))+1/6*b^2*polylog(3,-1+2/(1+
I*c*x^3))
```

Rubi [A] time = 0.32, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5031, 4850, 4988, 4884, 4994, 6610}

$$-\frac{1}{3}ib\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{3}ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{6}b^2\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/3)*b*(a + b
*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/3)*b*(a + b*ArcTan[c*x
^3])*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^3)
])/6 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^3)])/6
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx, x, x^3\right)$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right.}{1\,\cdot}\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)+\frac{1}{3}(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx\right.}{1+c^2}\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\frac{}{1}\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\frac{}{1}\right.$$

Mathematica [A] time = 0.12, size = 167, normalized size = 1.08

$$\frac{1}{6}\left(b\left(2i\text{Li}_2\left(\frac{cx^3+i}{i-cx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-2i\text{Li}_2\left(\frac{cx^3+i}{cx^3-i}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+b\left(\text{Li}_3\left(\frac{cx^3+i}{i-cx^3}\right)-\text{Li}_3\left(\frac{cx^3+i}{cx^3-i}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] (4*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 + (2*I)/(-I + c*x^3)] + b*((2*I)*(a +
b*ArcTan[c*x^3])*PolyLog[2, (I + c*x^3)/(I - c*x^3)] - (2*I)*(a + b*ArcTan[
c*x^3])*PolyLog[2, (I + c*x^3)/(-I + c*x^3)] + b*(PolyLog[3, (I + c*x^3)/(I
 - c*x^3)] - PolyLog[3, (I + c*x^3)/(-I + c*x^3)])))/6
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(cx^3\right)^2+2\,ab\arctan\left(cx^3\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\arctan\left(cx^3\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^{3}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\log(x)+\frac{1}{16}\int \frac{12\,b^{2}\arctan\left(cx^{3}\right)^{2}+b^{2}\log\left(c^{2}x^{6}+1\right)^{2}+32\,ab\arctan\left(cx^{3}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^3)^2 + b^2*log(c^2*x^6 + 1)^
2 + 32*a*b*arctan(c*x^3))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^{3}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x,x)
```

```
[Out] int((a + b*atan(c*x^3))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx^{3}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**2/x, x)
```

3.118 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=100

$$-\frac{1}{3}ic\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{3x^3}+\frac{2}{3}bc\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{3}ib^2c\text{Li}_2\left(\frac{2}{1-icx^3}\right.$$

```
[Out] -1/3*I*c*(a+b*arctan(c*x^3))^2-1/3*(a+b*arctan(c*x^3))^2/x^3+2/3*b*c*(a+b*a
rctan(c*x^3))*ln(2-2/(1-I*c*x^3))-1/3*I*b^2*c*polylog(2,-1+2/(1-I*c*x^3))
```

Rubi [B] time = 0.66, antiderivative size = 290, normalized size of antiderivative = 2.90, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5035, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$\frac{1}{3}ib^2c\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{3}ib^2c\text{PolyLog}\left(2,icx^3\right)-\frac{1}{6}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)+\frac{1}{6}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)/(12*x^3) +
(I/6)*b*c*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2] + (I/6)*b^2*c
*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3] + (b*((2*I)*a - b*Log[1 - I*c*x^3])*
Log[1 + I*c*x^3])/(6*x^3) + (b^2*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/(12*x^3)
 + (I/3)*b^2*c*PolyLog[2, (-I)*c*x^3] - (I/3)*b^2*c*PolyLog[2, I*c*x^3] - (
I/6)*b^2*c*PolyLog[2, (1 - I*c*x^3)/2] + (I/6)*b^2*c*PolyLog[2, (1 + I*c*x^
3)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

```
&& IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{4x^4}+\frac{b\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{2x^4}-\frac{b^2\log\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{x^4}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{x^4}$$

$$=\frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^2}\,dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-}{x^2}\right.$$

$$=-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{6x^3}+$$

$$=abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\right.}{6x^3}$$

$$=abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\right.}{6x^3}$$

$$=2abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{1}{6}ibc\left(2ia-b\log\left(1-icx^3\right)\right)\text{l}$$

$$=2abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{1}{6}ibc\left(2ia-b\log\left(1-icx^3\right)\right)\text{l}$$

$$=2abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{1}{6}ibc\left(2ia-b\log\left(1-icx^3\right)\right)\text{l}$$

Mathematica [A] time = 0.17, size = 125, normalized size = 1.25

$$\frac{-a\left(a+bcx^3\log\left(c^2x^6+1\right)-2bcx^3\log\left(cx^3\right)\right)+2b\tan^{-1}\left(cx^3\right)\left(-a+bcx^3\log\left(1-e^{2i\tan^{-1}\left(cx^3\right)}\right)\right)-ib^2cx^3\text{Li}_2\left(\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] (b^2*(-1 - I*c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(-a + b*c*x^3*Log[1
 - E^((2*I)*ArcTan[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1
+ c^2*x^6]) - I*b^2*c*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/(3*x^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(cx^3\right)^2+2\,ab\arctan\left(cx^3\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^{3}\right) + a\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x^4, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\, x^{3}\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(c\left(\log\left(c^{2}x^{6}+1\right)-\log\left(x^{6}\right)\right)+\frac{2\,\arctan\left(cx^{3}\right)}{x^{3}}\right)ab+\frac{\frac{1}{4}\left(12\,x^{3}\int -\frac{12\,c^{2}x^{6}\log\left(c^{2}x^{6}+1\right)-56\,cx^{3}\arctan\left(cx^{3}\right)-36\left(c^{2}x^{6}+1\right)\arctan}{4\left(c^{2}x^{10}+x^{4}\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a*b + 1/48*(48
*x^3*integrate(-1/16*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3*arctan(c*x^3) -
12*(c^2*x^6 + 1)*arctan(c*x^3)^2 - (c^2*x^6 + 1)*log(c^2*x^6 + 1)^2)/(c^2*x
^10 + x^4), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2)*b^2/x^3 - 1/3*a^2/
x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\, \mathrm{atan}\left(c\, x^{3}\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x^4,x)
```

```
[Out] int((a + b*atan(c*x^3))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^{3}\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**2/x**4, x)
```

3.119 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{6}c^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{bc\left(a+b\tan^{-1}\left(cx^3\right)\right)}{3x^3}-\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{6x^6}-\frac{1}{6}b^2c^2\log\left(c^2x^6+1\right)+b^2c^2\log(x)$$

```
[Out] -1/3*b*c*(a+b*arctan(c*x^3))/x^3-1/6*c^2*(a+b*arctan(c*x^3))^2-1/6*(a+b*arc
tan(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)
```

Rubi [C] time = 1.12, antiderivative size = 419, normalized size of antiderivative = 4.82, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)+\frac{1}{12}bc^2\log\left(\frac{1}{2}\left(1+icx^3\right)\right)\left(2ia-b\log\left(1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[I - c*x^3])/6 + ((I/12)*b*c*((2*I)*a - b*Log[
1 - I*c*x^3]))/x^3 - (b*c*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3]))/(12*x
^3) - (c^2*(2*a + I*b*Log[1 - I*c*x^3])^2)/24 - (2*a + I*b*Log[1 - I*c*x^3]
)^2/(24*x^6) + (b*c^2*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/
12 + ((I/6)*b^2*c*Log[1 + I*c*x^3])/x^3 - (b^2*c^2*Log[(1 - I*c*x^3)/2]*Log
[1 + I*c*x^3])/12 + (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/(12
*x^6) + (b^2*c^2*Log[1 + I*c*x^3]^2)/24 + (b^2*Log[1 + I*c*x^3]^2)/(24*x^6)
 - (b^2*c^2*Log[I + c*x^3])/12 - (b^2*c^2*PolyLog[2, (1 - I*c*x^3)/2])/12 -
 (b^2*c^2*PolyLog[2, (1 + I*c*x^3)/2])/12
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{4x^7}+\frac{b\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{2x^7}-\frac{b^2\log^2\left(\right.}{4\right.}$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{x^7}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{x^7}\,dx$$

$$=\frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^3}\,dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx}{x^3}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24x^6}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{12x^6}+\frac{b^2\log^2\left(1+\right.}{24x^6}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24x^6}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{12x^6}+\frac{b^2\log^2\left(1+\right.}{24x^6}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24x^6}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{12x^6}+\frac{b^2\log^2\left(1+\right.}{24x^6}$$

$$=-\frac{1}{2}iabc^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{12x^3}-\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)}{12x^3}$$

$$=\frac{1}{4}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{12x^3}-\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)}{12x^3}-$$

$$=\frac{1}{2}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{12x^3}-\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)}{12x^3}-$$

Mathematica [A] time = 0.08, size = 98, normalized size = 1.13

$$-\frac{a^2+2b\tan^{-1}\left(cx^3\right)\left(ac^2x^6+a+bcx^3\right)+2abcx^3-6b^2c^2x^6\log(x)+b^2c^2x^6\log\left(c^2x^6+1\right)+b^2\left(c^2x^6+1\right)\tan^{-1}\left(\right.}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] -1/6*(a^2 + 2*a*b*c*x^3 + 2*b*(a + b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b^2
*(1 + c^2*x^6)*ArcTan[c*x^3]^2 - 6*b^2*c^2*x^6*Log[x] + b^2*c^2*x^6*Log[1 +
 c^2*x^6])/x^6
```

fricas [A] time = 0.46, size = 102, normalized size = 1.17

$$-\frac{b^2c^2x^6\log\left(c^2x^6+1\right)-6\,b^2c^2x^6\log(x)+2\,abcx^3+\left(b^2c^2x^6+b^2\right)\arctan\left(cx^3\right)^2+a^2+2\left(abc^2x^6+b^2cx^3+a\right.}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(b^2*c^2*x^6*log(c^2*x^6 + 1) - 6*b^2*c^2*x^6*log(x) + 2*a*b*c*x^3 + (
b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + a^2 + 2*(a*b*c^2*x^6 + b^2*c*x^3 + a*b
)*arctan(c*x^3))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\arctan\left(cx^3\right)+a\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x^7, x)
```

maple [A] time = 0.04, size = 118, normalized size = 1.36

$$-\frac{a^2}{6x^6}-\frac{b^2\arctan\left(c\,x^3\right)^2}{6x^6}-\frac{b^2c^2\arctan\left(c\,x^3\right)^2}{6}-\frac{b^2c\arctan\left(c\,x^3\right)}{3x^3}-\frac{b^2c^2\ln\left(c^2x^6+1\right)}{6}+b^2c^2\ln(x)-\frac{ab\arctan\left(c\,x^3\right.}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^7,x)
```

```
[Out] -1/6*a^2/x^6-1/6*b^2/x^6*arctan(c*x^3)^2-1/6*b^2*c^2*arctan(c*x^3)^2-1/3*b^
2*c*arctan(c*x^3)/x^3-1/6*b^2*c^2*ln(c^2*x^6+1)+b^2*c^2*ln(x)-1/3*a*b/x^6*a
rctan(c*x^3)-1/3*a*b*c^2*arctan(c*x^3)-1/3*a*b*c/x^3
```

maxima [A] time = 0.54, size = 110, normalized size = 1.26

$$-\frac{1}{3}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)ab+\frac{1}{6}\left(\left(\arctan\left(cx^3\right)^2-\log\left(c^2x^6+1\right)+6\,\log(x)\right)c^2-2\left(c\arctan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a*b + 1/6*((arctan(c
*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*c*
arctan(c*x^3))*b^2 - 1/6*b^2*arctan(c*x^3)^2/x^6 - 1/6*a^2/x^6
```

mupad [B] time = 0.69, size = 152, normalized size = 1.75

$$b^2\,c^2\,\ln(x)-\frac{b^2\,c^2\,\mathrm{atan}\left(c\,x^3\right)^2}{6}-\frac{b^2\,\mathrm{atan}\left(c\,x^3\right)^2}{6\,x^6}-\frac{b^2\,c^2\,\ln\left(c^2\,x^6+1\right)}{6}-\frac{a^2}{6\,x^6}-\frac{b^2\,c\,\mathrm{atan}\left(c\,x^3\right)}{3\,x^3}-\frac{a\,b\,c}{3\,x^3}-\frac{a\,b\,c^2\,\mathrm{atan}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x^7,x)
```

```
[Out] b^2*c^2*log(x) - (b^2*c^2*atan(c*x^3)^2)/6 - (b^2*atan(c*x^3)^2)/(6*x^6) -
(b^2*c^2*log(c^2*x^6 + 1))/6 - a^2/(6*x^6) - (b^2*c*atan(c*x^3))/(3*x^3) -
```

```
(a*b*c)/(3*x^3) - (a*b*c^2*atan((a^2*c*x^3)/(a^2 + 49*b^2) + (49*b^2*c*x^3)
/(a^2 + 49*b^2)))/3 - (a*b*atan(c*x^3))/(3*x^6)
```

sympy [A] time = 173.37, size = 223, normalized size = 2.56

$$\begin{cases} -\frac{a^2}{6x^6} - \frac{abc^2 \operatorname{atan}\left(cx^3\right)}{3} - \frac{abc}{3x^3} - \frac{ab \operatorname{atan}\left(cx^3\right)}{3x^6} + \frac{ib^2c^3\sqrt{\frac{1}{c^2}} \operatorname{atan}\left(cx^3\right)}{3} + b^2c^2\log\left(x\right) - \frac{b^2c^2\log\left(x - \sqrt[6]{-1}\sqrt[6]{\frac{1}{c^2}}\right)}{3} - \frac{b^2c^2\log\left(4x^2 + 4\sqrt[6]{-1}\right.}{3} \\ -\frac{a^2}{6x^6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**7,x)
```

```
[Out] Piecewise((-a**2/(6*x**6) - a*b*c**2*atan(c*x**3)/3 - a*b*c/(3*x**3) - a*b*
atan(c*x**3)/(3*x**6) + I*b**2*c**3*sqrt(c**(-2))*atan(c*x**3)/3 + b**2*c**
2*log(x) - b**2*c**2*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/3 - b**2*c**2*lo
g(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3
))/3 - b**2*c**2*atan(c*x**3)**2/6 - b**2*c*atan(c*x**3)/(3*x**3) - b**2*at
an(c*x**3)**2/(6*x**6), Ne(c, 0)), (-a**2/(6*x**6), True))
```

3.120 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=154

$$\frac{1}{9}ic^3\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{2}{9}bc^3\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{9x^9}-\frac{bc\left(a+b\tan^{-1}\left(c\right.\right.}{9x^6}$$

```
[Out] -1/9*b^2*c^2/x^3-1/9*b^2*c^3*arctan(c*x^3)-1/9*b*c*(a+b*arctan(c*x^3))/x^6+
1/9*I*c^3*(a+b*arctan(c*x^3))^2-1/9*(a+b*arctan(c*x^3))^2/x^9-2/9*b*c^3*(a+
b*arctan(c*x^3))*ln(2-2/(1-I*c*x^3))+1/9*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x^
3))
```

Rubi [B] time = 1.45, antiderivative size = 536, normalized size of antiderivative = 3.48, number of steps used = 59, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2395, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{9}ib^2c^3\text{PolyLog}\left(2,-icx^3\right)+\frac{1}{9}ib^2c^3\text{PolyLog}\left(2,icx^3\right)+\frac{1}{18}ib^2c^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)-\frac{1}{18}ib^2c^3\text{PolyLog}\left(2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -(b^2*c^2)/(9*x^3) - (2*a*b*c^3*Log[x])/3 + (I/18)*b^2*c^3*Log[I - c*x^3] +
 ((I/36)*b*c*((2*I)*a - b*Log[1 - I*c*x^3]))/x^6 + (b*c^2*((2*I)*a - b*Log[
1 - I*c*x^3]))/(18*x^3) - (b*c*(2*a + I*b*Log[1 - I*c*x^3]))/(36*x^6) - ((I
/18)*b*c^2*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3]))/x^3 - (I/36)*c^3*(2*
a + I*b*Log[1 - I*c*x^3])^2 - (2*a + I*b*Log[1 - I*c*x^3])^2/(36*x^9) - (I/
18)*b*c^3*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2] + ((I/18)*b^2
*c*Log[1 + I*c*x^3])/x^6 - (I/18)*b^2*c^3*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*
x^3] + (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/(18*x^9) - (I/36
)*b^2*c^3*Log[1 + I*c*x^3]^2 + (b^2*Log[1 + I*c*x^3]^2)/(36*x^9) - (I/9)*b^
2*c^3*PolyLog[2, (-I)*c*x^3] + (I/9)*b^2*c^3*PolyLog[2, I*c*x^3] + (I/18)*b
^2*c^3*PolyLog[2, (1 - I*c*x^3)/2] - (I/18)*b^2*c^3*PolyLog[2, (1 + I*c*x^3
)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
```

```
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{4x^{10}} + \frac{b\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{2x^{10}} - \frac{b^2\log\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{x^{10}}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{x^{10}}$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^4}\,dx, x, x^3\right) + \frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-}{x^4}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{36x^9} + \frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{18x^9} + \frac{b^2\log^2\left(1\right.}{36x}$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{36x^9} + \frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{18x^9} + \frac{b^2\log^2\left(1\right.}{36x}$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{36x^9} + \frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{18x^9} + \frac{b^2\log^2\left(1\right.}{36x}$$

$$= -\frac{1}{3}abc^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-b\log\left(1-icx^3\right)\right)}{18x^3} - \frac{bc\left(\right.}{}$$

$$= -\frac{1}{3}abc^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-b\log\left(1-icx^3\right)\right)}{18x^3} - \frac{bc\left(\right.}{}$$

$$= -\frac{b^2c^2}{18x^3} - \frac{2}{3}abc^3\log(x) + \frac{1}{6}ib^2c^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-\right.}{}$$

$$= -\frac{b^2c^2}{18x^3} - \frac{2}{3}abc^3\log(x) + \frac{1}{6}ib^2c^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-\right.}{}$$

Mathematica [A] time = 0.41, size = 167, normalized size = 1.08

$$-\frac{a^2+2abc^3x^9\log\left(cx^3\right)+b\tan^{-1}\left(cx^3\right)\left(2a+bc^3x^9+2bc^3x^9\log\left(1-e^{2i\tan^{-1}\left(cx^3\right)}\right)+bcx^3\right)-abc^3x^9\log\left(c^2x^6\right.}{9x^9}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - I*c^3*x^9)*ArcTan[c*x^3]^2 +
 b*ArcTan[c*x^3]*(2*a + b*c*x^3 + b*c^3*x^9 + 2*b*c^3*x^9*Log[1 - E^((2*I)*
ArcTan[c*x^3])]) + 2*a*b*c^3*x^9*Log[c*x^3] - a*b*c^3*x^9*Log[1 + c^2*x^6]
- I*b^2*c^3*x^9*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/x^9
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(cx^3\right)^2+2\,ab\arctan\left(cx^3\right)+a^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^{3}\right) + a\right)^{2}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2/x^10, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\,x^{3}\right)\right)^{2}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^10,x)
[Out] int((a+b*arctan(c*x^3))^2/x^10,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}\left(\left(c^{2}\log\left(c^{2}x^{6}+1\right) - c^{2}\log\left(x^{6}\right) - \frac{1}{x^{6}}\right)c - \frac{2\arctan\left(cx^{3}\right)}{x^{9}}\right)ab + \frac{\frac{1}{4}\left(12\,x^{9}\int -\frac{12\,c^{2}x^{6}\log\left(c^{2}x^{6}+1\right) - 56\,cx^{3}\arctan\left(cx^{3}\right) - 108\left(c^{2}\cdots\right.}{4\left(c^{2}x^{16}\cdots\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="maxima")
[Out] 1/9*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9)
*a*b + 1/144*(144*x^9*integrate(-1/48*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3
*arctan(c*x^3) - 36*(c^2*x^6 + 1)*arctan(c*x^3)^2 - 3*(c^2*x^6 + 1)*log(c^2
*x^6 + 1)^2)/(c^2*x^16 + x^10), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2
)*b^2/x^9 - 1/9*a^2/x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x^{3}\right)\right)^{2}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x^10,x)
[Out] int((a + b*atan(c*x^3))^2/x^10, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**10,x)
[Out] Timed out
```

3.121 $\int x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\,dx$

Optimal. Leaf size=240

$$-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{3c^3}+\frac{ab^2x^3}{3c^2}-\frac{i\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{9c^3}-\frac{b\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1+icx^3}\right)\left(\right.}{}$$

```
[Out] 1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctan(c*x^3)/c^2-1/6*b*(a+b*arctan(c*x^3))^2
/c^3-1/6*b*x^6*(a+b*arctan(c*x^3))^2/c-1/9*I*(a+b*arctan(c*x^3))^3/c^3+1/9*
x^9*(a+b*arctan(c*x^3))^3-1/3*b*(a+b*arctan(c*x^3))^2*ln(2/(1+I*c*x^3))/c^3
-1/6*b^3*ln(c^2*x^6+1)/c^3-1/3*I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I
*c*x^3))/c^3-1/6*b^3*polylog(3,1-2/(1+I*c*x^3))/c^3
```

Rubi [B] time = 7.03, antiderivative size = 1867, normalized size of antiderivative = 7.78, number of steps used = 239, number of rules used = 32, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2411, 43, 2334, 12, 14, 2301, 2439, 2416, 2396, 2433, 2374, 6589, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2410, 2425}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (2*a*b^2*x^3)/(3*c^2) + (((7*I)/216)*b^3*x^3)/c^2 - (23*b^3*x^6)/(432*c) +
(I/324)*b^3*x^9 - (b^3*(1 - I*c*x^3)^2)/(48*c^3) - (b^3*(1 + I*c*x^3)^2)/(2
4*c^3) + (b^3*(1 + I*c*x^3)^3)/(324*c^3) + (7*b^3*Log[I - c*x^3])/(108*c^3)
 - (b^3*(1 - I*c*x^3)*Log[1 - I*c*x^3])/(3*c^3) + (b^3*Log[1 - I*c*x^3]^2)/
(72*c^3) - (b^2*x^6*((2*I)*a - b*Log[1 - I*c*x^3]))/(24*c) - (b^2*(1 - I*c*
x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3]))/(48*c^3) - (I/72)*b*x^9*((2*I)*a - b
*Log[1 - I*c*x^3])^2 - (b*(1 - I*c*x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3])^2)
/(48*c^3) - ((I/16)*b^2*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3]))/c^3 +
 ((I/108)*b^2*(1 - I*c*x^3)^3*(2*a + I*b*Log[1 - I*c*x^3]))/c^3 - (b*(1 - I
*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)/(8*c^3) + (b*(1 - I*c*x^3)^2*(2*a +
 I*b*Log[1 - I*c*x^3])^2)/(16*c^3) - (b*(1 - I*c*x^3)^3*(2*a + I*b*Log[1 -
I*c*x^3])^2)/(72*c^3) - ((I/24)*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^
3)/c^3 + ((I/24)*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3])^3)/c^3 - ((I/
72)*(1 - I*c*x^3)^3*(2*a + I*b*Log[1 - I*c*x^3])^3)/c^3 + (I/216)*b^2*((2*I
)*a - b*Log[1 - I*c*x^3])*(((18*I)*(1 - I*c*x^3))/c^3 - ((9*I)*(1 - I*c*x^3
)^2)/c^3 + ((2*I)*(1 - I*c*x^3)^3)/c^3 - ((6*I)*Log[1 - I*c*x^3])/c^3) + (b
^2*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/(12*c^3) - (b*((2*I
)*a - b*Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/(24*c^3) + (b*(2*a + I*b*
Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/(24*c^3) + (b^3*x^6*Log[1 + I*c*x
^3])/(18*c) - (I/108)*b^3*x^9*Log[1 + I*c*x^3] - (11*b^3*(1 + I*c*x^3)*Log[
1 + I*c*x^3])/(36*c^3) + (b^3*(1 + I*c*x^3)^2*Log[1 + I*c*x^3])/(12*c^3) -
(b^3*(1 + I*c*x^3)^3*Log[1 + I*c*x^3])/(108*c^3) - (b^3*Log[(1 - I*c*x^3)/2
]*Log[1 + I*c*x^3])/(12*c^3) + (b^2*x^6*((2*I)*a - b*Log[1 - I*c*x^3])*Log[
1 + I*c*x^3])/(12*c) + (I/24)*b*x^9*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1
+ I*c*x^3] - (b*(2*a + I*b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3])/(24*c^3) -
 (b^3*Log[1 + I*c*x^3]^2)/(72*c^3) + (I/72)*b^3*x^9*Log[1 + I*c*x^3]^2 + (b
^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/(8*c^3) - (b^3*(1 + I*c*x^3)^2*Log[1 +
 I*c*x^3]^2)/(12*c^3) + (b^3*(1 + I*c*x^3)^3*Log[1 + I*c*x^3]^2)/(72*c^3) -
 (b^3*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3]^2)/(12*c^3) + (I/24)*b^2*x^9*((
2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3]^2 - ((I/24)*b^2*(2*a + I*b*Lo
g[1 - I*c*x^3])*Log[1 + I*c*x^3]^2)/c^3 - (b^3*(1 + I*c*x^3)*Log[1 + I*c*x^
3]^3)/(24*c^3) + (b^3*(1 + I*c*x^3)^2*Log[1 + I*c*x^3]^3)/(24*c^3) - (b^3*(
1 + I*c*x^3)^3*Log[1 + I*c*x^3]^3)/(72*c^3) + (b^3*Log[I + c*x^3])/(24*c^3)
 - (b^3*PolyLog[2, (1 - I*c*x^3)/2])/(12*c^3) + (b^2*((2*I)*a - b*Log[1 - I
```

```
*c*x^3])*PolyLog[2, (1 - I*c*x^3)/2])/(12*c^3) + ((I/12)*b^2*(2*a + I*b*Log
[1 - I*c*x^3])*PolyLog[2, (1 - I*c*x^3)/2])/c^3 - (b^3*PolyLog[2, (1 + I*c*
x^3)/2])/(12*c^3) - (b^3*Log[1 + I*c*x^3]*PolyLog[2, (1 + I*c*x^3)/2])/(6*c
^3) + (b^3*PolyLog[3, (1 - I*c*x^3)/2])/(6*c^3) + (b^3*PolyLog[3, (1 + I*c*
x^3)/2])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
```

```
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 dx = \int \left(\frac{1}{8}x^8\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^8\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)\right.$$

$$= \frac{1}{8}\int x^8\left(2a+ib\log\left(1-icx^3\right)\right)^3 dx+\frac{1}{8}(3ib)\int x^8\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(\right.$$

$$= \frac{1}{24}\,\mathrm{Subst}\left(\int x^2(2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\,\mathrm{Subst}\left(\int x^2(-2ia+b\log\right.$$

$$= \frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx^3\right)\right)1$$

$$= \frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx^3\right)\right)1$$

$$= \frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx^3\right)\right)1$$

$$= -\frac{1}{72}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2-\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c^3}+\frac{i\left(1-i\right.}{}$$

$$= -\frac{1}{72}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2-\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{24c^3}-\frac{b\left(1-\right.}{}$$

$$= \frac{ab^2x^3}{3c^2}+\frac{ib^3x^3}{6c^2}-\frac{b^3\left(1-icx^3\right)^2}{32c^3}-\frac{b^3\left(1+icx^3\right)^2}{32c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\right)^3}{324c^3}$$

$$= \frac{ab^2x^3}{3c^2}-\frac{b^3\left(1-icx^3\right)^2}{32c^3}-\frac{b^3\left(1+icx^3\right)^2}{32c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\right)^3}{324c^3}-\frac{b^3\left(1-\right.}{}$$

$$= \frac{ab^2x^3}{2c^2}+\frac{ib^3x^3}{18c^2}-\frac{b^3\left(1-icx^3\right)^2}{24c^3}-\frac{b^3\left(1+icx^3\right)^2}{24c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\right)^3}{324c^3}$$

$$= \frac{ab^2x^3}{2c^2}+\frac{7ib^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{1}{324}ib^3x^9-\frac{b^3\left(1-icx^3\right)^2}{48c^3}-\frac{b^3\left(1-icx^3\right)^3}{324c^3}-\frac{b^3\left(1+ic\right.}{24c^3}$$

$$= \frac{ab^2x^3}{2c^2}+\frac{7ib^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{1}{324}ib^3x^9-\frac{b^3\left(1-icx^3\right)^2}{48c^3}-\frac{b^3\left(1-icx^3\right)^3}{324c^3}-\frac{b^3\left(1+ic\right.}{24c^3}$$

Mathematica [A] time = 0.56, size = 346, normalized size = 1.44

$$2a^3c^3x^9+6a^2bc^3x^9\tan^{-1}\left(cx^3\right)-3a^2bc^2x^6+3a^2b\log\left(c^2x^6+1\right)+6ab^2c^3x^9\tan^{-1}\left(cx^3\right)^2-6ab^2c^2x^6\tan^{-1}\left(cx^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (6*a*b^2*c*x^3 - 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTan[c*x^3] +
6*b^3*c*x^3*ArcTan[c*x^3] - 6*a*b^2*c^2*x^6*ArcTan[c*x^3] + 6*a^2*b*c^3*x^9
*ArcTan[c*x^3] + (6*I)*a*b^2*ArcTan[c*x^3]^2 - 3*b^3*ArcTan[c*x^3]^2 - 3*b^
3*c^2*x^6*ArcTan[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTan[c*x^3]^2 + (2*I)*b^3*Arc
Tan[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTan[c*x^3]^3 - 12*a*b^2*ArcTan[c*x^3]*Log[1
 + E^((2*I)*ArcTan[c*x^3])] - 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan
[c*x^3])] + 3*a^2*b*Log[1 + c^2*x^6] - 3*b^3*Log[1 + c^2*x^6] + (6*I)*b^2*(
a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] - 3*b^3*PolyLog[3
, -E^((2*I)*ArcTan[c*x^3])])/(18*c^3)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^8\arctan\left(cx^3\right)^3+3\,ab^2x^8\arctan\left(cx^3\right)^2+3\,a^2bx^8\arctan\left(cx^3\right)+a^3x^8,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^8*arctan(c*x^3)^3 + 3*a*b^2*x^8*arctan(c*x^3)^2 + 3*a^2*b*x^
8*arctan(c*x^3) + a^3*x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^3\right)+a\right)^3x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*x^8, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int x^8\left(a+b\arctan\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] int(x^8*(a+b*arctan(c*x^3))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{72}\,b^3x^9\arctan\left(cx^3\right)^3-\frac{1}{96}\,b^3x^9\arctan\left(cx^3\right)\log\left(c^2x^6+1\right)^2+\frac{1}{9}\,a^3x^9+\frac{1}{6}\left(2\,x^9\arctan\left(cx^3\right)-\left(\frac{x^6}{c^2}-\frac{\log\left(c^2x^6+\right.}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/72*b^3*x^9*arctan(c*x^3)^3 - 1/96*b^3*x^9*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 + 1/9*a^3*x^9 + 1/6*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^
4)*c)*a^2*b + integrate(1/32*(4*b^3*c^2*x^14*arctan(c*x^3)*log(c^2*x^6 + 1)
 + 28*(b^3*c^2*x^14 + b^3*x^8)*arctan(c*x^3)^3 + 4*(24*a*b^2*c^2*x^14 - b^3
*c*x^11 + 24*a*b^2*x^8)*arctan(c*x^3)^2 + (b^3*c*x^11 + 3*(b^3*c^2*x^14 + b
^3*x^8)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atan(c*x^3))^3,x)
```

```
[Out] int(x^8*(a + b*atan(c*x^3))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Timed out
```

3.122 $\int x^5 \left(a + b \tan^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=147

$$-\frac{b^2 \log\left(\frac{2}{1+icx^3}\right)\left(a + b\tan^{-1}\left(cx^3\right)\right)}{c^2} + \frac{\left(a + b\tan^{-1}\left(cx^3\right)\right)^3}{6c^2} - \frac{ib\left(a + b\tan^{-1}\left(cx^3\right)\right)^2}{2c^2} - \frac{bx^3\left(a + b\tan^{-1}\left(cx^3\right)\right)^2}{2c} + \frac{1}{6}$$

```
[Out] -1/2*I*b*(a+b*arctan(c*x^3))^2/c^2-1/2*b*x^3*(a+b*arctan(c*x^3))^2/c+1/6*(a
+b*arctan(c*x^3))^3/c^2+1/6*x^6*(a+b*arctan(c*x^3))^3-b^2*(a+b*arctan(c*x^3
))*ln(2/(1+I*c*x^3))/c^2-1/2*I*b^3*polylog(2,1-2/(1+I*c*x^3))/c^2
```

Rubi [B] time = 4.73, antiderivative size = 951, normalized size of antiderivative = 6.47, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2395, 2394, 2393, 2391, 2375, 2317, 2430, 2425}

$$\frac{1}{16}ib^2\left(2ia - b\log\left(1 - icx^3\right)\right)\log^2\left(icx^3 + 1\right)x^6 + \frac{1}{16}ib\left(2ia - b\log\left(1 - icx^3\right)\right)^2\log\left(icx^3 + 1\right)x^6 + \frac{b^2\left(2ia - b\log\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] ((I/32)*b^2*(1 - I*c*x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3]))/c^2 + ((I/32)*b
*(1 - I*c*x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3])^2)/c^2 + (b^2*(1 - I*c*x^3)
^2*(2*a + I*b*Log[1 - I*c*x^3]))/(32*c^2) - ((I/8)*b*(1 - I*c*x^3)*(2*a + I
*b*Log[1 - I*c*x^3])^2)/c^2 + ((I/32)*b*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 -
I*c*x^3])^2)/c^2 + ((1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^3)/(24*c^2)
- ((1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3])^3)/(48*c^2) - ((I/4)*b^2*((
2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/c^2 + ((I/16)*b*((2*I)*a
 - b*Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/c^2 + ((I/16)*b*(2*a + I*b*L
og[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/c^2 - ((I/4)*b^3*Log[(1 - I*c*x^3)
/2]*Log[1 + I*c*x^3])/c^2 + (b^2*x^3*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 +
 I*c*x^3])/(4*c) + (I/16)*b*x^6*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1 + I*
c*x^3] - ((I/16)*b*(2*a + I*b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3])/c^2 - (
(I/8)*b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/c^2 + (I/16)*b^2*x^6*((2*I)*a -
 b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3]^2 - (b^2*(2*a + I*b*Log[1 - I*c*x^3])
*Log[1 + I*c*x^3]^2)/(16*c^2) + ((I/24)*b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^
3)/c^2 - ((I/48)*b^3*(1 + I*c*x^3)^2*Log[1 + I*c*x^3]^3)/c^2 + ((I/4)*b^3*P
olyLog[2, (1 - I*c*x^3)/2])/c^2 - ((I/8)*b^2*((2*I)*a - b*Log[1 - I*c*x^3])
*PolyLog[2, (1 - I*c*x^3)/2])/c^2 - (b^2*(2*a + I*b*Log[1 - I*c*x^3])*PolyL
og[2, (1 - I*c*x^3)/2])/(8*c^2) - ((I/4)*b^3*PolyLog[2, (1 + I*c*x^3)/2])/c
^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
```

```
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\,dx = \int\left(\frac{1}{8}x^5\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^5\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)\right.$$

$$= \frac{1}{8}\int x^5\left(2a+ib\log\left(1-icx^3\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x^5\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(\right.$$

$$= \frac{1}{24}\,\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\,\text{Subst}\left(\int x(-2ia+b\log(\right.$$

$$= \frac{1}{16}ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{16}ib^2x^6\left(2ia-b\log\left(1-icx^3\right)\right)\text{l}$$

$$= \frac{1}{16}ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{16}ib^2x^6\left(2ia-b\log\left(1-icx^3\right)\right)\text{l}$$

$$= \frac{1}{16}ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{16}ib^2x^6\left(2ia-b\log\left(1-icx^3\right)\right)\text{l}$$

$$= \frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c^2}-\frac{\left(1-icx^3\right)^2\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48c^2}+\frac{ib\left(2\right.}{}$$

$$= \frac{ib\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{16c^2}-\frac{ib\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c^2}+\frac{ib}{}$$

$$= \frac{3iab^2x^3}{4c}+\frac{3b^3x^3}{8c}-\frac{ib^3\left(1-icx^3\right)^2}{64c^2}+\frac{ib^3\left(1+icx^3\right)^2}{64c^2}+\frac{ib\left(1-icx^3\right)\left(2ia-b\log\left(1\right.\right.}{16c^2}$$

$$= \frac{3iab^2x^3}{4c}+\frac{3b^3x^3}{4c}-\frac{ib^3\left(1-icx^3\right)^2}{64c^2}+\frac{ib^3\left(1+icx^3\right)^2}{64c^2}-\frac{3ib^3\left(1-icx^3\right)\log\left(1-icx^3\right.}{8c^2}$$

$$= \frac{iab^2x^3}{2c}+\frac{5b^3x^3}{8c}-\frac{3ib^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{8c^2}+\frac{ib^2\left(1-icx^3\right)^2\left(2ia-b\log\left(1-\right.\right.}{32c^2}$$

$$= \frac{iab^2x^3}{2c}+\frac{b^3x^3}{2c}-\frac{ib^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{4c^2}+\frac{ib^2\left(1-icx^3\right)^2\left(2ia-b\log\left(1-ic\right.\right.}{32c^2}$$

Mathematica [A] time = 0.18, size = 170, normalized size = 1.16

$$\frac{a\left(acx^3\left(acx^3-3b\right)+3b^2\log\left(c^2x^6+1\right)\right)+3b^2\tan^{-1}\left(cx^3\right)^2\left(ac^2x^6+a+b\left(-cx^3+i\right)\right)+3b\tan^{-1}\left(cx^3\right)\left(a\left(ac^2x^6\right.\right.}{6c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^6 + b*(I - c*x^3))*ArcTan[c*x^3]^2 + b^3*(1 + c^2*x^6)*
ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a - 2*b*c*x^3 + a*c^2*x^6) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3*(-3*b + a*c*x^3) + 3*b^2*Log
[1 + c^2*x^6]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])])/(6*c^2)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(b^{3}x^{5}\arctan\left(cx^{3}\right)^{3}+3\,ab^{2}x^{5}\arctan\left(cx^{3}\right)^{2}+3\,a^{2}bx^{5}\arctan\left(cx^{3}\right)+a^{3}x^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctan(c*x^3)^3 + 3*a*b^2*x^5*arctan(c*x^3)^2 + 3*a^2*b*x^
5*arctan(c*x^3) + a^3*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^{3}\right)+a\right)^{3}x^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*x^5, x)
```

maple [C] time = 1.78, size = 867, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] 1/2/c^2*a^2*b*arctan(c*x^3)-1/2/c*a^2*b*x^3+1/6*x^6*a^3-1/2*I/c*a*b^2*x^3*l
n(1-I*c*x^3)+1/48*I*b^3*(c^2*x^6+1)/c^2*ln(1+I*c*x^3)^3-1/48*I*b^3/c^2*ln(1
-I*c*x^3)^3+1/8*b^3/c*x^3*ln(1-I*c*x^3)^2+1/4*I*a^2*b*x^6*ln(1-I*c*x^3)+1/8
*I/c^2*b^3*ln(1-I*c*x^3)^2-1/8*a*b^2*x^6*ln(1-I*c*x^3)^2+1/2/c^2*a*b^2*ln(c
^2*x^6+1)-1/8/c^2*a*b^2*ln(1-I*c*x^3)^2-1/16*b^2*(I*x^6*b*ln(1-I*c*x^3)*c^2
+2*a*c^2*x^6-2*b*c*x^3+I*b*ln(1-I*c*x^3)+2*I*b+2*a)/c^2*ln(1+I*c*x^3)^2+3/4
*I/c*b^2*Sum(2/3*(ln(x-_alpha)*ln(1-I*c*x^3)+3*c*(-1/3*ln(x-_alpha)*(ln((Ro
otOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1)-x+_alpha)/RootOf(
_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1))+ln((RootOf(_Z^2+_Z*Ro
otOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(
c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2))+ln(1/2*(2*(I/c)^(1/3)+x-_alpha)/(I/c
)^(1/3)))/c-1/3*(dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,
index=1)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index
=1))+dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_
alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2))+dilog(1
/2*(2*(I/c)^(1/3)+x-_alpha)/(I/c)^(1/3)))/c))*b/c,_alpha=RootOf(c*_Z^3-Root
Of(_Z^2+1,index=1)))-1/48*I*b^3*x^6*ln(1-I*c*x^3)^3+(1/16*I*b^3*(c^2*x^6+1)
/c^2*ln(1-I*c*x^3)^2+1/4*b^2*x^3*(a*c*x^3-b)/c*ln(1-I*c*x^3)-1/4*I*b*(a^2*c
^2*x^6-2*a*b*c*x^3+b^2*ln(1-I*c*x^3)+I*ln(1-I*c*x^3)*a*b)/c^2)*ln(1+I*c*x^3
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ab^{2}x^{6}\arctan\left(cx^{3}\right)^{2}+\frac{1}{6}\,a^{3}x^{6}+\frac{1}{2}\left(x^{6}\arctan\left(cx^{3}\right)-c\left(\frac{x^{3}}{c^{2}}-\frac{\arctan\left(cx^{3}\right)}{c^{3}}\right)\right)a^{2}b-\frac{1}{2}\left(2\,c\left(\frac{x^{3}}{c^{2}}-\frac{\arctan\left(cx^{3}\right)}{c^{3}}\right)\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^3*x^6 + 1/2*(x^6*arctan(c*x^3) - c*(x
^3/c^2 - arctan(c*x^3)/c^3))*a^2*b - 1/2*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)
```

```
*arctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*a*b^2 + 1/
192*(4*x^6*arctan(c*x^3)^3 - 3*x^6*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 192*i
ntegrate(1/64*(12*c^2*x^11*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^8*arctan
(c*x^3)^2 + 56*(c^2*x^11 + x^5)*arctan(c*x^3)^3 + 3*(c*x^8 + 2*(c^2*x^11 +
x^5)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a + b\,\mathrm{atan}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^3))^3,x)
```

```
[Out] int(x^5*(a + b*atan(c*x^3))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + b\,\mathrm{atan}\left(cx^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Integral(x**5*(a + b*atan(c*x**3))**3, x)
```

3.123 $\int x^2 \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=139

$$\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)^3+\frac{i\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{3c}+\frac{b\log\left(\frac{2}{1+icx^3}\right)\left(a+b\text{t}\right.}{c}$$

```
[Out] 1/3*I*(a+b*arctan(c*x^3))^3/c+1/3*x^3*(a+b*arctan(c*x^3))^3+b*(a+b*arctan(c
*x^3))^2*ln(2/(1+I*c*x^3))/c+I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I*c
*x^3))/c+1/2*b^3*polylog(3,1-2/(1+I*c*x^3))/c
```

Rubi [B] time = 2.70, antiderivative size = 545, normalized size of antiderivative = 3.92, number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)\left(2ia-b\log\left(1-icx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(1-icx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(1+icx^3\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (b*(1 - I*c*x^3)*((2*I)*a - b*Log[1 - I*c*x^3])^2)/(8*c) + (b*(1 - I*c*x^3)
*(2*a + I*b*Log[1 - I*c*x^3])^2)/(8*c) + ((I/24)*(1 - I*c*x^3)*(2*a + I*b*L
og[1 - I*c*x^3])^3)/c + (b*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^
3)/2])/(4*c) - (b*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3])/(8*c)
+ (I/8)*b*x^3*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3] + (b^3*Log[
(1 - I*c*x^3)/2]*Log[1 + I*c*x^3]^2)/(4*c) + (b^2*((2*I)*a - b*Log[1 - I*c*
x^3])*Log[1 + I*c*x^3]^2)/(8*c) + (I/8)*b^2*x^3*((2*I)*a - b*Log[1 - I*c*x^
3])*Log[1 + I*c*x^3]^2 + (b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^3)/(24*c) - (b
^2*((2*I)*a - b*Log[1 - I*c*x^3])*PolyLog[2, (1 - I*c*x^3)/2])/(2*c) + (b^3
*Log[1 + I*c*x^3]*PolyLog[2, (1 + I*c*x^3)/2])/(2*c) - (b^3*PolyLog[3, (1 -
 I*c*x^3)/2])/(2*c) - (b^3*PolyLog[3, (1 + I*c*x^3)/2])/(2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^3 dx = \int \left(\frac{1}{8}x^2\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^2\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)\right.$$

$$=\frac{1}{8}\int x^2\left(2a+ib\log\left(1-icx^3\right)\right)^3 dx+\frac{1}{8}(3ib)\int x^2\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(\right.$$

$$=\frac{1}{24}\operatorname{Subst}\left(\int(2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\operatorname{Subst}\left(\int(-2ia+b\log(1-\right.$$

$$=\frac{1}{8}ibx^3\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{8}ib^2x^3\left(2ia-b\log\left(1-icx^3\right)\right)\log$$

$$=\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c}+\frac{1}{8}ibx^3\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx\right.$$

$$=\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c}+\frac{1}{8}ib$$

$$=-\frac{1}{2}ab^2x^3-\frac{1}{4}ib^3x^3+\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i\left(1-icx^3\right)\left(2a+ib\right.}{24c}$$

$$=-\frac{1}{2}ab^2x^3+\frac{b^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{4c}+\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+$$

$$=\frac{1}{4}ib^3x^3+\frac{b^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{4c}+\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+$$

$$=\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+$$

$$=\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+$$

Mathematica [A] time = 0.10, size = 224, normalized size = 1.61

$$2a^3cx^3-3a^2b\log\left(c^2x^6+1\right)+6a^2bcx^3\tan^{-1}\left(cx^3\right)-6ib^2\operatorname{Li}_2\left(-e^{2i\tan^{-1}\left(cx^3\right)}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+6ab^2cx^3\tan^{-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTan[c*x^3] - (6*I)*a*b^2*ArcTan[c*x^3]^2 +
6*a*b^2*c*x^3*ArcTan[c*x^3]^2 - (2*I)*b^3*ArcTan[c*x^3]^3 + 2*b^3*c*x^3*Arc
Tan[c*x^3]^3 + 12*a*b^2*ArcTan[c*x^3]*Log[1 + E^((2*I)*ArcTan[c*x^3])] + 6*
b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 3*a^2*b*Log[1 + c^2*
x^6] - (6*I)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(6*c)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\arctan\left(cx^3\right)^3+3\,ab^2x^2\arctan\left(cx^3\right)^2+3\,a^2bx^2\arctan\left(cx^3\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c*x^3)^3 + 3*a*b^2*x^2*arctan(c*x^3)^2 + 3*a^2*b*x^
2*arctan(c*x^3) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^3\right)+a\right)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*x^2, x)
```

maple [B] time = 0.24, size = 303, normalized size = 2.18

$$\frac{a^3x^3}{3}-\frac{ib^3\arctan\left(c\,x^3\right)^3}{3c}+\frac{b^3x^3\arctan\left(c\,x^3\right)^3}{3}+\frac{b^3\arctan\left(c\,x^3\right)^2\ln\left(\frac{\left(ic\,x^3+1\right)^2}{c^2x^6+1}+1\right)}{c}-\frac{ib^3\arctan\left(c\,x^3\right)\operatorname{polylog}\left(\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] 1/3*a^3*x^3-1/3*I/c*b^3*arctan(c*x^3)^3+1/3*b^3*x^3*arctan(c*x^3)^3+1/c*b^3
*arctan(c*x^3)^2*ln((1+I*c*x^3)^2/(c^2*x^6+1)+1)-I/c*b^3*arctan(c*x^3)*poly
log(2,-(1+I*c*x^3)^2/(c^2*x^6+1))+1/2/c*b^3*polylog(3,-(1+I*c*x^3)^2/(c^2*x
^6+1))-I/c*arctan(c*x^3)^2*a*b^2+x^3*a*b^2*arctan(c*x^3)^2+2/c*ln((1+I*c*x^
3)^2/(c^2*x^6+1)+1)*arctan(c*x^3)*a*b^2-I/c*polylog(2,-(1+I*c*x^3)^2/(c^2*x
^6+1))*a*b^2+x^3*a^2*b*arctan(c*x^3)-1/2/c*a^2*b*ln(c^2*x^6+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}b^3x^3\arctan\left(cx^3\right)^3-\frac{1}{32}b^3x^3\arctan\left(cx^3\right)\log\left(c^2x^6+1\right)^2+\frac{1}{3}a^3x^3+\frac{7\,b^3\arctan\left(cx^3\right)^4}{96\,c}+28\,b^3c^2\int\frac{x^8\arctan}{32\left(c^2x^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan(c*x^3)^3 - 1/32*b^3*x^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 + 1/3*a^3*x^3 + 7/96*b^3*arctan(c*x^3)^4/c + 28*b^3*c^2*integrate(1/32*x^
8*arctan(c*x^3)^3/(c^2*x^6 + 1), x) + 3*b^3*c^2*integrate(1/32*x^8*arctan(c
*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^
8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 12*b^3*c^2*integrate(1/32*x^8*arctan(
```

```
c*x^3)*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 1/3*a*b^2*arctan(c*x^3)^3/c - 1
2*b^3*c*integrate(1/32*x^5*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 3*b^3*c*inte
grate(1/32*x^5*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 3*b^3*integrate(1/32*
x^2*arctan(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 1/2*(2*c*x^3*arcta
n(c*x^3) - log(c^2*x^6 + 1))*a^2*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \operatorname{atan}\left(c\, x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^3))^3,x)
[Out] int(x^2*(a + b*atan(c*x^3))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(cx^3\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3))**3,x)
[Out] Integral(x**2*(a + b*atan(c*x**3))**3, x)
```

3.124 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x}\,dx$

Optimal. Leaf size=232

$$-\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{icx^3+1}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{icx^3+1}-1\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}ib\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)($$

```
[Out] -2/3*(a+b*arctan(c*x^3))^3*arctanh(-1+2/(1+I*c*x^3))-1/2*I*b*(a+b*arctan(c*
x^3))^2*polylog(2,1-2/(1+I*c*x^3))+1/2*I*b*(a+b*arctan(c*x^3))^2*polylog(2,
-1+2/(1+I*c*x^3))-1/2*b^2*(a+b*arctan(c*x^3))*polylog(3,1-2/(1+I*c*x^3))+1/
2*b^2*(a+b*arctan(c*x^3))*polylog(3,-1+2/(1+I*c*x^3))+1/4*I*b^3*polylog(4,1
-2/(1+I*c*x^3))-1/4*I*b^3*polylog(4,-1+2/(1+I*c*x^3))
```

Rubi [A] time = 0.52, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5031, 4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}ibP$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/2)*b*(a + b
*ArcTan[c*x^3])^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/2)*b*(a + b*ArcTan[c
*x^3])^2*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*(a + b*ArcTan[c*x^3])*Poly
Log[3, 1 - 2/(1 + I*c*x^3)])/2 + (b^2*(a + b*ArcTan[c*x^3])*PolyLog[3, -1 +
 2/(1 + I*c*x^3)])/2 + (I/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^3)] - (I/4)*b^
3*PolyLog[4, -1 + 2/(1 + I*c*x^3)]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx,x,x^3\right)$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\text{ta}}{1+c^2}\right.$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)+(bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log}{1+c^2x^2}\right.$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+}\right.$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+}\right.$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+}\right.$$

Mathematica [A] time = 0.22, size = 248, normalized size = 1.07

$$\frac{1}{4}ib\left(2\text{Li}_2\left(\frac{cx^3+i}{i-cx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-2\text{Li}_2\left(\frac{cx^3+i}{cx^3-i}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)^2+b\left(-2i\text{Li}_3\left(\frac{cx^3+i}{i-cx^3}\right)\left(a+b\tan^{-1}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 + (2*I)/(-I + c*x^3)])/3 + (I/4)*b*(2*
(a + b*ArcTan[c*x^3])^2*PolyLog[2, (I + c*x^3)/(I - c*x^3)] - 2*(a + b*ArcT
an[c*x^3])^2*PolyLog[2, (I + c*x^3)/(-I + c*x^3)] + b*((-2*I)*(a + b*ArcTan
[c*x^3])*PolyLog[3, (I + c*x^3)/(I - c*x^3)] + (2*I)*(a + b*ArcTan[c*x^3])*
PolyLog[3, (I + c*x^3)/(-I + c*x^3)] + b*(-PolyLog[4, (I + c*x^3)/(I - c*x^
3)] + PolyLog[4, (I + c*x^3)/(-I + c*x^3)])))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \arctan\left(cx^3\right)^3 + 3\,ab^2 \arctan\left(cx^3\right)^2 + 3\,a^2 b \arctan\left(cx^3\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(cx^3\right) + a\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arctan\left(c\,x^3\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \log(x) + \frac{1}{32} \int \frac{28\, b^3 \arctan\left(cx^3\right)^3 + 3\, b^3 \arctan\left(cx^3\right) \log\left(c^2x^6 + 1\right)^2 + 96\, ab^2 \arctan\left(cx^3\right)^2 + 96\, a^2 b \arcta}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^3)^3 + 3*b^3*arctan(c*x^3)*l
og(c^2*x^6 + 1)^2 + 96*a*b^2*arctan(c*x^3)^2 + 96*a^2*b*arctan(c*x^3))/x, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(c\,x^3\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^3/x,x)
```

```
[Out] int((a + b*atan(c*x^3))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx^3\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**3/x, x)
```

3.125 $\int \frac{(a+b\tan^{-1}(cx^3))^3}{x^4}\,dx$

Optimal. Leaf size=133

$$-ib^2c\text{Li}_2\left(\frac{2}{1-icx^3}-1\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{3}ic\left(a+b\tan^{-1}\left(cx^3\right)\right)^3-\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{3x^3}+bc\log\left(2-\frac{2}{1-icx^3}\right.$$

```
[Out] -1/3*I*c*(a+b*arctan(c*x^3))^3-1/3*(a+b*arctan(c*x^3))^3/x^3+b*c*(a+b*arcta
n(c*x^3))^2*ln(2-2/(1-I*c*x^3))-I*b^2*c*(a+b*arctan(c*x^3))*polylog(2,-1+2/
(1-I*c*x^3))+1/2*b^3*c*polylog(3,-1+2/(1-I*c*x^3))
```

Rubi [F] time = 0.86, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

```
[Out] (b*c*Log[I*c*x^3]*(2*a + I*b*Log[1 - I*c*x^3])^2)/8 - ((1 - I*c*x^3)*(2*a +
 I*b*Log[1 - I*c*x^3])^3)/(24*x^3) - (b^3*c*Log[(-I)*c*x^3]*Log[1 + I*c*x^3
]^2)/8 - ((I/24)*b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^3)/x^3 + (I/4)*b^2*c*(2
*a + I*b*Log[1 - I*c*x^3])*PolyLog[2, 1 - I*c*x^3] - (b^3*c*Log[1 + I*c*x^3
]*PolyLog[2, 1 + I*c*x^3])/4 + (b^3*c*PolyLog[3, 1 - I*c*x^3])/4 + (b^3*c*P
olyLog[3, 1 + I*c*x^3])/4 + (I/8)*b*Defer[Subst][Defer[Int][(((-2*I)*a + b*
Log[1 - I*c*x])^2*Log[1 + I*c*x])/x^2, x], x, x^3] - (I/8)*b^2*Defer[Subst]
[Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])*Log[1 + I*c*x]^2)/x^2, x], x, x^
3]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx=\int\left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{8x^4}+\frac{3ib\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)}{8x^4}-\frac{3ib}{}\right.$$

$$=\frac{1}{8}\int\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{x^4}\,dx+\frac{1}{8}(3ib)\int\frac{\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+i\right.}{x^4}$$

$$=\frac{1}{24}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^2}\,dx,x,x^3\right)+\frac{1}{8}(ib)\,\text{Subst}\left(\int\frac{(-2ia+b\log(1}{}\right.$$

$$=-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3}-\frac{ib^3\left(1+icx^3\right)\log^3\left(1+icx^3\right)}{24x^3}+\frac{1}{8}(ib)\,\text{Subs}$$

$$=\frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3}-\frac{1}{8}$$

$$=\frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3}-\frac{1}{8}$$

$$=\frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3}-\frac{1}{8}$$

$$=\frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3}-\frac{1}{8}$$

Mathematica [A] time = 0.42, size = 240, normalized size = 1.80

$$-\frac{a^3}{3x^3}-\frac{1}{2}a^2bc\log\left(c^2x^6+1\right)-\frac{a^2b\tan^{-1}\left(cx^3\right)}{x^3}+3a^2bc\log(x)+ab^2c\left(\tan^{-1}\left(cx^3\right)\left(\left(-\frac{1}{cx^3}-i\right)\tan^{-1}\left(cx^3\right)+2\log\left(1\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (a^2*b*ArcTan[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log[
1 + c^2*x^6])/2 + a*b^2*c*(ArcTan[c*x^3]*((-I - 1/(c*x^3))*ArcTan[c*x^3] +
2*Log[1 - E^((2*I)*ArcTan[c*x^3])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^3])]
) + (b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^3]^3 - ArcTan[c*x^3]^3/(c*x^3) + 3
*ArcTan[c*x^3]^2*Log[1 - E^((-2*I)*ArcTan[c*x^3])] + (3*I)*ArcTan[c*x^3]*Po
lyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x^3])
])/2))/3
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(cx^3\right)^3+3\,ab^2\arctan\left(cx^3\right)^2+3\,a^2b\arctan\left(cx^3\right)+a^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\arctan\left(cx^3\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x^4, x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\arctan\left(c\,x^3\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\arctan\left(cx^3\right)}{x^3}\right)a^2b-\frac{a^3}{3\,x^3}-\frac{\frac{15}{2}\,b^3\arctan\left(cx^3\right)^3-\frac{21}{8}\,b^3\arctan\left(cx^3\right)\log\left(c^2x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a^2*b - 1/3*a^
3/x^3 - 1/96*(4*b^3*arctan(c*x^3)^3 - 3*b^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 - 96*x^3*integrate(-1/32*(12*b^3*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) -
 28*(b^3*c^2*x^6 + b^3)*arctan(c*x^3)^3 - 12*(8*a*b^2*c^2*x^6 + b^3*c*x^3 +
 8*a*b^2)*arctan(c*x^3)^2 + 3*(b^3*c*x^3 - (b^3*c^2*x^6 + b^3)*arctan(c*x^3
))*log(c^2*x^6 + 1)^2)/(c^2*x^10 + x^4), x))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^3/x^4,x)
```

```
[Out] int((a + b*atan(c*x^3))^3/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**3/x**4, x)
```

3.126 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx$

Optimal. Leaf size=146

$$b^2c^2\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}ibc^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{1}{6}c^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^3-\frac{bc\left(a+b\tan^{-1}\left(\right.\right.}{2x^3}$$

```
[Out] -1/2*I*b*c^2*(a+b*arctan(c*x^3))^2-1/2*b*c*(a+b*arctan(c*x^3))^2/x^3-1/6*c^
2*(a+b*arctan(c*x^3))^3-1/6*(a+b*arctan(c*x^3))^3/x^6+b^2*c^2*(a+b*arctan(c
*x^3))*ln(2-2/(1-I*c*x^3))-1/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^3))
```

Rubi [F] time = 1.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (b*c*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)
/(16*x^3) + (I/16)*b*c^2*Log[I*c*x^3]*(2*a + I*b*Log[1 - I*c*x^3])^2 - (c^2
*(2*a + I*b*Log[1 - I*c*x^3])^3)/48 - (2*a + I*b*Log[1 - I*c*x^3])^3/(48*x^
6) + (b^3*c*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/(16*x^3) + (I/16)*b^3*c^2*Log
[(-I)*c*x^3]*Log[1 + I*c*x^3]^2 - (I/48)*b^3*c^2*Log[1 + I*c*x^3]^3 - ((I/4
8)*b^3*Log[1 + I*c*x^3]^3)/x^6 + (I/8)*b^3*c^2*PolyLog[2, (-I)*c*x^3] - (I/
8)*b^3*c^2*PolyLog[2, I*c*x^3] - (b^2*c^2*(2*a + I*b*Log[1 - I*c*x^3])*Poly
Log[2, 1 - I*c*x^3])/8 + (I/8)*b^3*c^2*Log[1 + I*c*x^3]*PolyLog[2, 1 + I*c*
x^3] + (I/8)*b^3*c^2*PolyLog[3, 1 - I*c*x^3] - (I/8)*b^3*c^2*PolyLog[3, 1 +
 I*c*x^3] + (I/8)*b*Defer[Subst][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])^
2*Log[1 + I*c*x])/x^3, x], x, x^3] - (I/8)*b^2*Defer[Subst][Defer[Int][(((-
2*I)*a + b*Log[1 - I*c*x])*Log[1 + I*c*x]^2)/x^3, x], x, x^3]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{8x^7} + \frac{3ib\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)}{8x^7} - \frac{3ib}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{x^7}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+i\right.}{x^7}$$

$$= \frac{1}{24}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^3}\,dx, x, x^3\right) + \frac{1}{8}(ib)\,\text{Subst}\left(\int \frac{(-2ia+b\log(1}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{8}(ib)\,\text{Subst}\left(\int \frac{(-2ia+b\log(}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{16}(ib)\,\text{Subst}\left(\int \frac{(2a+ib\log(}{x\left(-\frac{i}{c}+\frac{ix}{c}\right)}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{16}(ib)\,\text{Subst}\left(\int \frac{(2a+ib\log(}{\left(-\frac{i}{c}+\frac{ix}{c}\right)^2}\right.$$

$$= -\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} - \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} + \frac{b^3c\left(1+icx\right.}{}$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+i\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+i\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+i\right.$$

Mathematica [A] time = 0.35, size = 196, normalized size = 1.34

$$-\frac{a\left(a\left(a+3bcx^3\right)-6b^2c^2x^6\log\left(\frac{cx^3}{\sqrt{c^2x^6+1}}\right)\right)+3b^2\tan^{-1}\left(cx^3\right)^2\left(ac^2x^6+a+bcx^3\left(1+icx^3\right)\right)+3b\tan^{-1}\left(cx^3\right)\left(\right.}{6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] -1/6*(3*b^2*(a + a*c^2*x^6 + b*c*x^3*(1 + I*c*x^3))*ArcTan[c*x^3]^2 + b^3*(
1 + c^2*x^6)*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a + 2*b*c*x^3 + a*c^2*
x^6) - 2*b^2*c^2*x^6*Log[1 - E^((2*I)*ArcTan[c*x^3])]) + a*(a*(a + 3*b*c*x^
3) - 6*b^2*c^2*x^6*Log[(c*x^3)/Sqrt[1 + c^2*x^6]]) + (3*I)*b^3*c^2*x^6*Poly
Log[2, E^((2*I)*ArcTan[c*x^3])])/x^6
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(cx^3\right)^3+3\,ab^2\arctan\left(cx^3\right)^2+3\,a^2b\arctan\left(cx^3\right)+a^3}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\arctan\left(cx^3\right)+a\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3/x^7, x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x^7,x)
[Out] int((a+b*arctan(c*x^3))^3/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)a^2b+\frac{1}{2}\left(\left(\arctan\left(cx^3\right)^2-\log\left(c^2x^6+1\right)+6\log(x)\right)c^2-2\left(c\arctan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="maxima")
[Out] -1/2*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a^2*b + 1/2*((arctan
(c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*
c*arctan(c*x^3))*a*b^2 - 1/2*a*b^2*arctan(c*x^3)^2/x^6 + 1/192*(192*x^6*int
egrate(-1/64*(12*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^3*arctan(c
*x^3)^2 - 56*(c^2*x^6 + 1)*arctan(c*x^3)^3 + 3*(c*x^3 - 2*(c^2*x^6 + 1)*arc
tan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^13 + x^7), x) - 4*arctan(c*x^3)^3 +
3*arctan(c*x^3)*log(c^2*x^6 + 1)^2)*b^3/x^6 - 1/6*a^3/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^3/x^7,x)
[Out] int((a + b*atan(c*x^3))^3/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x**7,x)
[Out] Integral((a + b*atan(c*x**3))**3/x**7, x)
```

3.127 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 \, dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$$

Mathematica [A] time = 1.98, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\arctan\left(cx^3\right)^3 + 3\,ab^2\arctan\left(cx^3\right)^2 + 3\,a^2b\arctan\left(cx^3\right) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\arctan\left(cx^3\right)+a\right)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3*(d*x)^m, x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx)^{m+1}a^3}{d(m+1)}+\frac{\frac{15}{2}b^3d^mxx^m\arctan\left(cx^3\right)^3-\frac{21}{8}b^3d^mxx^m\arctan\left(cx^3\right)\log\left(c^2x^6+1\right)^2+(m+1)\int\frac{252\,b^3c^2d^mx^6x^m\arctan(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^3)^3 - 3*b
^3*d^m*x*x^m*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 32*(m + 1)*integrate(1/32*(
36*b^3*c^2*d^m*x^6*x^m*arctan(c*x^3)*log(c^2*x^6 + 1) + 28*((b^3*c^2*d^m*m
+ b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3)^3 - 12*(3*b^3*c
*d^m*x^3 - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^6 - 8*a*b^2*d^m*m - 8*a*b^
2*d^m)*x^m*arctan(c*x^3)^2 + 96*((a^2*b*c^2*d^m*m + a^2*b*c^2*d^m)*x^6 + a^
2*b*d^m*m + a^2*b*d^m)*x^m*arctan(c*x^3) + 3*(3*b^3*c*d^m*x^3*x^m + ((b^3*c
^2*d^m*m + b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3))*log(c
^2*x^6 + 1)^2)/((c^2*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^3))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Timed out
```

3.128 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$$

Mathematica [A] time = 1.30, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\arctan\left(cx^3\right)^2+2\,ab\arctan\left(cx^3\right)+a^2\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\arctan\left(cx^3\right)+a\right)^2 (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2*(d*x)^m, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx)^{m+1}a^2}{d(m+1)}+\frac{7b^2d^mxx^m\arctan\left(cx^3\right)^2-\frac{3}{4}b^2d^mxx^m\log\left(c^2x^6+1\right)^2+(m+1)\int\frac{36b^2c^2d^mx^6x^m\log\left(c^2x^6+1\right)+36\left(\left(b^2c^2d^mm+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^3)^2 - b^2
*d^m*x*x^m*log(c^2*x^6 + 1)^2 + 16*(m + 1)*integrate(1/16*(12*b^2*c^2*d^m*x
^6*x^m*log(c^2*x^6 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*d^m*m
 + b^2*d^m)*x^m*arctan(c*x^3)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*
d^m*m + b^2*d^m)*x^m*log(c^2*x^6 + 1)^2 - 8*(3*b^2*c*d^m*x^3 - 4*(a*b*c^2*d
^m*m + a*b*c^2*d^m)*x^6 - 4*a*b*d^m*m - 4*a*b*d^m)*x^m*arctan(c*x^3))/((c^2
*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^3))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.129 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=75

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};-c^2x^6\right)}{d^4(m+1)(m+4)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2/3
+1/6*m],[5/3+1/6*m],-c^2*x^6)/d^4/(1+m)/(4+m)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5033, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};-c^2x^6\right)}{d^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*Hy
pergeometric2F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)])/(d^4*(1 + m)*(4 + m)
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{x^2(dx)^{1+m}}{1+c^2x^6}\,dx}{d(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{(dx)^{3+m}}{1+c^2x^6}\,dx}{d^3(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{3bc(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};-c^2x^6\right)}{d^4(1+m)(4+m)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.87

$$-\frac{x(dx)^m\left(3bcx^3\,{}_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};-c^2x^6\right)-(m+4)\left(a+b\tan^{-1}\left(cx^3\right)\right)\right)}{(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -((x*(d*x)^m*(-((4 + m)*(a + b*ArcTan[c*x^3])) + 3*b*c*x^3*Hypergeometric2F
1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)]))/((1 + m)*(4 + m)))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\arctan\left(cx^3\right)+a\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(cx^3\right)+a\right)(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int(dx)^m\left(a+b\arctan\left(cx^3\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(d^m x x^m\arctan\left(cx^3\right)-3\left(cd^m m+cd^m\right)\int\frac{x^3x^m}{\left(c^2m+c^2\right)x^6+m+1}\,dx\right)b}{m+1}+\frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] (d^m*x*x^m*arctan(c*x^3) - 3*(c*d^m*m + c*d^m)*integrate(x^3*x^m/((c^2*m +
c^2)*x^6 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(d\,x)^m\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^3)),x)
[Out] int((d*x)^m*(a + b*atan(c*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3)),x)
[Out] Timed out
```

3.130 $\int \frac{(dx)^m}{a+b\tan^{-1}(cx^3)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tan^{-1}(cx^3)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx^3)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx^3)}\,dx = \int \frac{(dx)^m}{a+b\tan^{-1}(cx^3)}\,dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx^3)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\arctan(cx^3)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x^3) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \arctan\left(c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^3)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \arctan\left(cx^3\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{atan}\left(c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^3)),x)
[Out] int((d*x)^m/(a + b*atan(c*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3)),x)
[Out] Timed out
```

3.131 $\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\arctan\left(cx^3\right)^2+2\,ab\arctan\left(cx^3\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b\arctan\left(cx^3\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a)^2, x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{m}}{\left(a+b\arctan\left(c\,x^{3}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{2}d^{m}x^{6}+d^{m}\right)x^{m}-\left(b^{2}cx^{2}\arctan\left(cx^{3}\right)+abcx^{2}\right)\int\frac{\left(\left(c^{2}d^{m}m+4\,c^{2}d^{m}\right)x^{6}+d^{m}m-2\,d^{m}\right)x^{m}}{b^{2}cx^{3}\arctan\left(cx^{3}\right)+abcx^{3}}\,dx}{3\left(b^{2}cx^{2}\arctan\left(cx^{3}\right)+abcx^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] -1/3*((c^2*d^m*x^6 + d^m)*x^m - 3*(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)*int
egrate(1/3*((c^2*d^m*m + 4*c^2*d^m)*x^6 + d^m*m - 2*d^m)*x^m/(b^2*c*x^3*arc
tan(c*x^3) + a*b*c*x^3), x))/(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^{m}}{\left(a+b\,\mathrm{atan}\left(c\,x^{3}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atan(c*x^3))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.132 $\int x^3 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right) dx$

Optimal. Leaf size=50

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\tan^{-1}\left(\frac{x}{c}\right)-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3$$

```
[Out] -1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctan(c/x))+1/4*b*c^4*arctan(x/c)
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 302, 203}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^3x+\frac{1}{4}bc^4\tan^{-1}\left(\frac{x}{c}\right)+\frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c/x]),x]
```

```
[Out] -(b*c^3*x)/4 + (b*c*x^3)/12 + (x^4*(a + b*ArcTan[c/x]))/4 + (b*c^4*ArcTan[x
/c])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^2}{1+\frac{c^2}{x^2}}dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^4}{c^2+x^2}dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\left(-c^2+x^2+\frac{c^4}{c^2+x^2}\right)dx\\
&= -\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}\left(bc^5\right)\int\frac{1}{c^2+x^2}dx\\
&= -\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\tan^{-1}\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.10

$$\frac{ax^4}{4}-\frac{1}{4}bc^4\tan^{-1}\left(\frac{c}{x}\right)-\frac{1}{4}bc^3x+\frac{1}{4}bx^4\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x]),x]
```

```
[Out] -1/4*(b*c^3*x) + (b*c*x^3)/12 + (a*x^4)/4 - (b*c^4*ArcTan[c/x])/4 + (b*x^4*
ArcTan[c/x])/4
```

fricas [A] time = 0.40, size = 41, normalized size = 0.82

$$-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}ax^4-\frac{1}{4}\left(bc^4-bx^4\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] -1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/4*(b*c^4 - b*x^4)*arctan(c/x)
```

giac [A] time = 4.01, size = 84, normalized size = 1.68

$$-\frac{\left(\frac{3\,bc^9i\log\left(\frac{ci}{x}-1\right)}{x^4}-\frac{3\,bc^9i\log\left(-\frac{ci}{x}-1\right)}{x^4}-6\,bc^5\arctan\left(\frac{c}{x}\right)-6\,ac^5+\frac{6\,bc^8}{x^3}-\frac{2\,bc^6}{x}\right)x^4}{24\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] -1/24*(3*b*c^9*i*log(c*i/x - 1)/x^4 - 3*b*c^9*i*log(-c*i/x - 1)/x^4 - 6*b*c
^5*arctan(c/x) - 6*a*c^5 + 6*b*c^8/x^3 - 2*b*c^6/x)*x^4/c^5
```

maple [A] time = 0.04, size = 46, normalized size = 0.92

$$\frac{x^4a}{4}+\frac{b\,x^4\arctan\left(\frac{c}{x}\right)}{4}+\frac{bc\,x^3}{12}-\frac{b\,c^3x}{4}+\frac{b\,c^4\arctan\left(\frac{x}{c}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctan(c/x)+1/12*b*c*x^3-1/4*b*c^3*x+1/4*b*c^4*arctan(x
/c)
```

maxima [A] time = 0.42, size = 45, normalized size = 0.90

$$\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c/x) + (3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c
)*b
```

mupad [B] time = 0.41, size = 45, normalized size = 0.90

$$\frac{a\,x^4}{4}-\frac{b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}-\frac{b\,c^3\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c/x)),x)
```

```
[Out] (a*x^4)/4 - (b*c^4*atan(c/x))/4 + (b*x^4*atan(c/x))/4 + (b*c*x^3)/12 - (b*c
^3*x)/4
```

sympy [A] time = 0.50, size = 46, normalized size = 0.92

$$\frac{ax^4}{4} - \frac{bc^4\operatorname{atan}\left(\frac{c}{x}\right)}{4} - \frac{bc^3x}{4} + \frac{bcx^3}{12} + \frac{bx^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x)),x)
```

```
[Out] a*x**4/4 - b*c**4*atan(c/x)/4 - b*c**3*x/4 + b*c*x**3/12 + b*x**4*atan(c/x)
/4
```

3.133 $\int x^2 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right) \, dx$

Optimal. Leaf size=43

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{6}bcx^2$$

```
[Out] 1/6*b*c*x^2+1/3*x^3*(a+b*arctan(c/x))-1/6*b*c^3*ln(c^2+x^2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 266, 43}

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTan[c/x]))/3 - (b*c^3*Log[c^2 + x^2])/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x}{1+\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{c^2+x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\,\text{Subst}\left(\int\frac{x}{c^2+x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\,\text{Subst}\left(\int\left(1-\frac{c^2}{c^2+x}\right)dx,x,x^2\right)\\
&= \frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.12

$$\frac{ax^3}{3}-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{3}bx^3\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTan[c/x])/3 - (b*c^3*Log[c^2 + x^2])/6
```

fricas [A] time = 0.42, size = 40, normalized size = 0.93

$$\frac{1}{3}bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{6}bcx^2+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3*arctan(c/x) - 1/6*b*c^3*log(c^2 + x^2) + 1/6*b*c*x^2 + 1/3*a*x^3
```

giac [A] time = 2.00, size = 69, normalized size = 1.60

$$\frac{\left(2\,bc^4\arctan\left(\frac{c}{x}\right)-\frac{bc^7\log\left(\frac{c^2}{x^2}+1\right)}{x^3}+\frac{2\,bc^7\log\left(\frac{c}{x}\right)}{x^3}+2\,ac^4+\frac{bc^5}{x}\right)x^3}{6\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^4*arctan(c/x) - b*c^7*log(c^2/x^2 + 1)/x^3 + 2*b*c^7*log(c/x)/x^
3 + 2*a*c^4 + b*c^5/x)*x^3/c^4
```

maple [A] time = 0.05, size = 55, normalized size = 1.28

$$\frac{x^3a}{3}+\frac{b\,x^3\arctan\left(\frac{c}{x}\right)}{3}+\frac{bc\,x^2}{6}+\frac{c^3b\ln\left(\frac{c}{x}\right)}{3}-\frac{c^3b\ln\left(1+\frac{c^2}{x^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c/x)+1/6*b*c*x^2+1/3*c^3*b*ln(c/x)-1/6*c^3*b*ln(
1+c^2/x^2)
```

maxima [A] time = 0.43, size = 43, normalized size = 1.00

$$\frac{1}{3}\,ax^3 + \frac{1}{6}\left(2\,x^3\arctan\left(\frac{c}{x}\right) - \left(c^2\log\left(c^2 + x^2\right) - x^2\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*b
```

mupad [B] time = 0.35, size = 40, normalized size = 0.93

$$\frac{a\,x^3}{3} + \frac{b\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{3} - \frac{b\,c^3\,\ln\left(c^2 + x^2\right)}{6} + \frac{b\,c\,x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c/x)),x)
[Out] (a*x^3)/3 + (b*x^3*atan(c/x))/3 - (b*c^3*log(c^2 + x^2))/6 + (b*c*x^2)/6
```

sympy [A] time = 0.37, size = 41, normalized size = 0.95

$$\frac{ax^3}{3} - \frac{bc^3\log\left(c^2 + x^2\right)}{6} + \frac{bcx^2}{6} + \frac{bx^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x)),x)
[Out] a*x**3/3 - b*c**3*log(c**2 + x**2)/6 + b*c*x**2/6 + b*x**3*atan(c/x)/3
```

3.134 $\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tan^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

```
[Out] 1/2*b*c*x+1/2*x^2*(a+b*arctan(c/x))-1/2*b*c^2*arctan(x/c)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5033, 193, 321, 203}

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tan^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTan[c/x]))/2 - (b*c^2*ArcTan[x/c])/2
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx &= \frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{1}{1+\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{x^2}{c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}\left(bc^3\right)\int\frac{1}{c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tan^{-1}\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.13

$$\frac{ax^2}{2}+\frac{1}{2}bc^2\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bx^2\tan^{-1}\left(\frac{c}{x}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c/x]),x]
[Out] (b*c*x)/2 + (a*x^2)/2 + (b*c^2*ArcTan[c/x])/2 + (b*x^2*ArcTan[c/x])/2
```

fricas [A] time = 0.42, size = 31, normalized size = 0.79

$$\frac{1}{2}\,bcx+\frac{1}{2}\,ax^2+\frac{1}{2}\left(bc^2+bx^2\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="fricas")
[Out] 1/2*b*c*x + 1/2*a*x^2 + 1/2*(b*c^2 + b*x^2)*arctan(c/x)
```

giac [B] time = 0.18, size = 74, normalized size = 1.90

$$-\frac{\left(\frac{bc^5 i\log\left(\frac{ci}{x}+1\right)}{x^2}-\frac{bc^5 i\log\left(-\frac{ci}{x}+1\right)}{x^2}-2\,bc^3\arctan\left(\frac{c}{x}\right)-2\,ac^3-\frac{2\,bc^4}{x}\right)x^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="giac")
[Out] -1/4*(b*c^5*i*log(c*i/x + 1)/x^2 - b*c^5*i*log(-c*i/x + 1)/x^2 - 2*b*c^3*ar
ctan(c/x) - 2*a*c^3 - 2*b*c^4/x)*x^2/c^3
```

maple [A] time = 0.04, size = 37, normalized size = 0.95

$$\frac{a\,x^2}{2}+\frac{\arctan\left(\frac{c}{x}\right)b\,x^2}{2}+\frac{xbc}{2}-\frac{b\,c^2\arctan\left(\frac{x}{c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x)),x)
[Out] 1/2*a*x^2+1/2*arctan(c/x)*b*x^2+1/2*x*b*c-1/2*b*c^2*arctan(x/c)
```

maxima [A] time = 0.42, size = 36, normalized size = 0.92

$$\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c/x) - (c*arctan(x/c) - x)*c)*b
```

mupad [B] time = 0.34, size = 36, normalized size = 0.92

$$\frac{a\,x^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c/x)),x)
```

```
[Out] (a*x^2)/2 + (b*c^2*atan(c/x))/2 + (b*x^2*atan(c/x))/2 + (b*c*x)/2
```

sympy [**A**] time = 0.29, size = 36, normalized size = 0.92

$$\frac{ax^2}{2} + \frac{bc^2 \operatorname{atan}\left(\frac{c}{x}\right)}{2} + \frac{bcx}{2} + \frac{bx^2 \operatorname{atan}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x)),x)
```

```
[Out] a*x**2/2 + b*c**2*atan(c/x)/2 + b*c*x/2 + b*x**2*atan(c/x)/2
```

3.135 $\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=27

$$ax+\frac{1}{2}bc\log\left(c^2+x^2\right)+bx\tan^{-1}\left(\frac{c}{x}\right)$$

```
[Out] a*x+b*x*arctan(c/x)+1/2*b*c*ln(c^2+x^2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5027, 263, 260}

$$ax+\frac{1}{2}bc\log\left(c^2+x^2\right)+bx\tan^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c/x],x]
```

```
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx &= ax+b\int \tan^{-1}\left(\frac{c}{x}\right)\,dx\\ &= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x}\,dx\\ &= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{x}{c^2+x^2}\,dx\\ &= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$ax+\frac{1}{2}bc\log\left(c^2+x^2\right)+bx\tan^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c/x],x]
```

```
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

fricas [A] time = 0.42, size = 25, normalized size = 0.93

$$bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}\,bc\log\left(c^2+x^2\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="fricas")
[Out] b*x*arctan(c/x) + 1/2*b*c*log(c^2 + x^2) + a*x
```

giac [A] time = 1.76, size = 46, normalized size = 1.70

$$ax+\frac{\left(c^2\left(\log\left(\frac{c^2}{x^2}+1\right)-\log\left(\frac{c^2}{x^2}\right)\right)+2\,cx\arctan\left(\frac{c}{x}\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="giac")
[Out] a*x + 1/2*(c^2*(log(c^2/x^2 + 1) - log(c^2/x^2)) + 2*c*x*arctan(c/x))*b/c
```

maple [A] time = 0.04, size = 38, normalized size = 1.41

$$ax+bx\arctan\left(\frac{c}{x}\right)-bc\ln\left(\frac{c}{x}\right)+\frac{bc\ln\left(1+\frac{c^2}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c/x),x)
[Out] a*x+b*x*arctan(c/x)-b*c*ln(c/x)+1/2*b*c*ln(1+c^2/x^2)
```

maxima [A] time = 0.32, size = 27, normalized size = 1.00

$$\frac{1}{2}\left(2\,x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="maxima")
[Out] 1/2*(2*x*arctan(c/x) + c*log(c^2 + x^2))*b + a*x
```

mupad [B] time = 0.30, size = 25, normalized size = 0.93

$$a\,x+b\,x\,\mathrm{atan}\left(\frac{c}{x}\right)+\frac{b\,c\,\ln\left(c^2+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c/x),x)
[Out] a*x + b*x*atan(c/x) + (b*c*log(c^2 + x^2))/2
```

sympy [A] time = 0.18, size = 22, normalized size = 0.81

$$ax+b\left(\frac{c\log\left(c^2+x^2\right)}{2}+x\,\mathrm{atan}\left(\frac{c}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c/x),x)
[Out] a*x + b*(c*log(c**2 + x**2)/2 + x*atan(c/x))
```

3.136 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}\,dx$

Optimal. Leaf size=39

$$a\log(x)-\frac{1}{2}ib\text{Li}_2\left(-\frac{ic}{x}\right)+\frac{1}{2}ib\text{Li}_2\left(\frac{ic}{x}\right)$$

```
[Out] a*ln(x)-1/2*I*b*polylog(2,-I*c/x)+1/2*I*b*polylog(2,I*c/x)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5031, 4848, 2391}

$$-\frac{1}{2}ib\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\text{PolyLog}\left(2,\frac{ic}{x}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x,x]
```

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= a\log(x)-\frac{1}{2}(ib)\,\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,\frac{1}{x}\right)+\frac{1}{2}(ib)\,\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,x,\right.\\ &= a\log(x)-\frac{1}{2}ib\text{Li}_2\left(-\frac{ic}{x}\right)+\frac{1}{2}ib\text{Li}_2\left(\frac{ic}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$a\log(x)-\frac{1}{2}ib\text{Li}_2\left(-\frac{ic}{x}\right)+\frac{1}{2}ib\text{Li}_2\left(\frac{ic}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x,x]
```

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \arctan\left(\frac{c}{x}\right) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c/x) + a)/x, x)
```

giac [B] time = 3.96, size = 74, normalized size = 1.90

$$\frac{\left(\frac{bc^{6} i \log\left(\frac{ci}{x}+1\right)}{x^{2}} - \frac{bc^{6} i \log\left(-\frac{ci}{x}+1\right)}{x^{2}} - 2\,bc^{4} \arctan\left(\frac{c}{x}\right) - 2\,ac^{4} - \frac{2\,bc^{5}}{x}\right)x^{2}}{4\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="giac")
```

```
[Out] 1/4*(b*c^6*i*log(c*i/x + 1)/x^2 - b*c^6*i*log(-c*i/x + 1)/x^2 - 2*b*c^4*arc
tan(c/x) - 2*a*c^4 - 2*b*c^5/x)*x^2/c^5
```

maple [B] time = 0.05, size = 94, normalized size = 2.41

$$-a \ln\left(\frac{c}{x}\right) - b \ln\left(\frac{c}{x}\right) \arctan\left(\frac{c}{x}\right) - \frac{ib \ln\left(\frac{c}{x}\right) \ln\left(1+\frac{ic}{x}\right)}{2} + \frac{ib \ln\left(\frac{c}{x}\right) \ln\left(1-\frac{ic}{x}\right)}{2} - \frac{ib\, \text{dilog}\left(1+\frac{ic}{x}\right)}{2} + \frac{ib\, \text{dilog}\left(1-\frac{ic}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x,x)
```

```
[Out] -a*ln(c/x)-b*ln(c/x)*arctan(c/x)-1/2*I*b*ln(c/x)*ln(1+I*c/x)+1/2*I*b*ln(c/x
)*ln(1-I*c/x)-1/2*I*b*dilog(1+I*c/x)+1/2*I*b*dilog(1-I*c/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan\left(c, x\right)}{x}\, dx + a \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(c, x)/x, x) + a*log(x)
```

mupad [B] time = 0.34, size = 32, normalized size = 0.82

$$a\,\ln\left(x\right) + \frac{b\,\left(\mathrm{Li}_2\left(1-\frac{c\,1\mathrm{i}}{x}\right) - \mathrm{Li}_2\left(1+\frac{c\,1\mathrm{i}}{x}\right)\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x,x)
```

```
[Out] (b*(dilog(1 - (c*1i)/x) - dilog((c*1i)/x + 1))*1i)/2 + a*log(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{atan}\left(\frac{c}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x,x)
```

```
[Out] Integral((a + b*atan(c/x))/x, x)
```

3.137 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=34

$$\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}$$

```
[Out] (-a-b*arctan(c/x))/x+1/2*b*ln(1+c^2/x^2)/c
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5033, 260}

$$\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c/x])/x) + (b*Log[1 + c^2/x^2])/(2*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx&=-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}-(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^3}\,dx\\&=-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.09

$$-\frac{a}{x}+\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{b\tan^{-1}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c/x])/x + (b*Log[1 + c^2/x^2])/(2*c)
```

fricas [A] time = 0.43, size = 41, normalized size = 1.21

$$-\frac{2\,bc\arctan\left(\frac{c}{x}\right)-bx\log\left(c^2+x^2\right)+2\,bx\log(x)+2\,ac}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="fricas")
[Out] -1/2*(2*b*c*arctan(c/x) - b*x*log(c^2 + x^2) + 2*b*x*log(x) + 2*a*c)/(c*x)
```

giac [A] time = 0.17, size = 39, normalized size = 1.15

$$-\frac{\frac{2\,bc\arctan\left(\frac{c}{x}\right)}{x} - b\log\left(\frac{c^2}{x^2}+1\right) + \frac{2\,ac}{x}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="giac")
[Out] -1/2*(2*b*c*arctan(c/x)/x - b*log(c^2/x^2 + 1) + 2*a*c/x)/c
```

maple [A] time = 0.02, size = 36, normalized size = 1.06

$$-\frac{a}{x} - \frac{b\arctan\left(\frac{c}{x}\right)}{x} + \frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^2,x)
[Out] -a/x-b/x*arctan(c/x)+1/2*b*ln(1+c^2/x^2)/c
```

maxima [A] time = 0.32, size = 38, normalized size = 1.12

$$-\frac{b\left(\frac{2\,c\arctan\left(\frac{c}{x}\right)}{x} - \log\left(\frac{c^2}{x^2}+1\right)\right)}{2\,c} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="maxima")
[Out] -1/2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a/x
```

mupad [B] time = 0.34, size = 43, normalized size = 1.26

$$\frac{\frac{b\,\ln\left(c^2+x^2\right)}{2} - b\,\ln(x)}{c} - \frac{a\,c + b\,c\,\mathrm{atan}\left(\frac{c}{x}\right)}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x^2,x)
[Out] ((b*log(c^2 + x^2))/2 - b*log(x))/c - (a*c + b*c*atan(c/x))/(c*x)
```

sympy [A] time = 0.65, size = 36, normalized size = 1.06

$$\begin{cases} -\frac{a}{x} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{x} - \frac{b\log(x)}{c} + \frac{b\log\left(c^2+x^2\right)}{2c} & \text{for}\ c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**2,x)
[Out] Piecewise((-a/x - b*atan(c/x)/x - b*log(x)/c + b*log(c**2 + x**2)/(2*c), Ne
(c, 0)), (-a/x, True))
```

3.138 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=43

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2}+\frac{b}{2cx}$$

```
[Out] 1/2*b/c/x+1/2*(-a-b*arctan(c/x))/x^2+1/2*b*arctan(x/c)/c^2
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 325, 203}

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2}+\frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^3,x]
```

```
[Out] b/(2*c*x) - (a + b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^4}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{x^2\left(c^2+x^2\right)}\,dx$$
$$= \frac{b}{2cx} - \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b\int \frac{1}{c^2+x^2}\,dx}{2c}$$
$$= \frac{b}{2cx} - \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.12

$$-\frac{a}{2x^2} + \frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2} - \frac{b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^3,x]
[Out] -1/2*a/x^2 + b/(2*c*x) - (b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

fricas [A] time = 0.44, size = 37, normalized size = 0.86

$$-\frac{ac^2 - bcx + \left(bc^2 + bx^2\right)\arctan\left(\frac{c}{x}\right)}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="fricas")
[Out] -1/2*(a*c^2 - b*c*x + (b*c^2 + b*x^2)*arctan(c/x))/(c^2*x^2)
```

giac [A] time = 1.80, size = 67, normalized size = 1.56

$$-\frac{\frac{2\,bc^2i\arctan\left(\frac{c}{x}\right)}{x^2} + \frac{2\,ac^2i}{x^2} - \frac{2\,bci}{x} - b\log\left(\frac{ci}{x}-1\right) + b\log\left(-\frac{ci}{x}-1\right)}{4\,c^2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="giac")
[Out] -1/4*(2*b*c^2*i*arctan(c/x)/x^2 + 2*a*c^2*i/x^2 - 2*b*c*i/x - b*log(c*i/x -
 1) + b*log(-c*i/x - 1))/(c^2*i)
```

maple [A] time = 0.03, size = 41, normalized size = 0.95

$$-\frac{a}{2x^2} - \frac{b\arctan\left(\frac{c}{x}\right)}{2x^2} + \frac{b}{2cx} + \frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^3,x)
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c/x)+1/2*b/c/x+1/2*b*arctan(x/c)/c^2
```

maxima [A] time = 0.42, size = 42, normalized size = 0.98

$$\frac{1}{2}\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*b - 1/2*a/x^2
```

mupad [B] time = 0.38, size = 50, normalized size = 1.16

$$\frac{b\,c\,\mathrm{atan}\left(\frac{x}{\sqrt{c^2}}\right)}{2\,\left(c^2\right)^{3/2}}-\frac{\frac{a\,c^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}-\frac{b\,c\,x}{2}}{c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x^3,x)
```

```
[Out] (b*c*atan(x/(c^2)^(1/2)))/(2*(c^2)^(3/2)) - ((a*c^2)/2 + (b*c^2*atan(c/x))/
2 - (b*c*x)/2)/(c^2*x^2)
```

sympy [A] time = 0.85, size = 44, normalized size = 1.02

$$\begin{cases} -\frac{a}{2x^2}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2x^2}+\frac{b}{2cx}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2c^2} & \text{for}\ c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atan(c/x)/(2*x**2) + b/(2*c*x) - b*atan(c/x)/(2*
c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

3.139 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^4} dx$

Optimal. Leaf size=55

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}+\frac{b}{6cx^2}$$

```
[Out] 1/6*b/c/x^2+1/3*(-a-b*arctan(c/x))/x^3+1/3*b*ln(x)/c^3-1/6*b*ln(c^2+x^2)/c^
3
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 266, 44}

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}+\frac{b\log(x)}{3c^3}+\frac{b}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^4,x]
```

```
[Out] b/(6*c*x^2) - (a + b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c^2
 + x^2])/(6*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^5}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{x^3\left(c^2+x^2\right)}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(c^2+x\right)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \left(\frac{1}{c^2x^2}-\frac{1}{c^4x}+\frac{1}{c^4\left(c^2+x\right)}\right)dx,x,x^2\right)\\ &= \frac{b}{6cx^2}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.09

$$-\frac{a}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}-\frac{b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b/(6*c*x^2) - (b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (
b*Log[c^2 + x^2])/(6*c^3)
```

fricas [A] time = 0.43, size = 55, normalized size = 1.00

$$-\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)+bx^3\log\left(c^2+x^2\right)-2\,bx^3\log(x)+2\,ac^3-bc^2x}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*c^3*arctan(c/x) + b*x^3*log(c^2 + x^2) - 2*b*x^3*log(x) + 2*a*c^3
 - b*c^2*x)/(c^3*x^3)
```

giac [A] time = 0.20, size = 51, normalized size = 0.93

$$-\frac{\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)}{x^3}+b\log\left(\frac{c^2}{x^2}+1\right)+\frac{2\,ac^3}{x^3}-\frac{bc^2}{x^2}}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*c^3*arctan(c/x)/x^3 + b*log(c^2/x^2 + 1) + 2*a*c^3/x^3 - b*c^2/x^
2)/c^3
```

maple [A] time = 0.03, size = 45, normalized size = 0.82

$$-\frac{a}{3x^3}-\frac{b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6c\,x^2}-\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c/x)+1/6*b/c/x^2-1/6/c^3*b*ln(1+c^2/x^2)
```

maxima [A] time = 0.31, size = 54, normalized size = 0.98

$$-\frac{1}{6}\left(c\left(\frac{\log\left(c^2+x^2\right)}{c^4}-\frac{\log\left(x^2\right)}{c^4}-\frac{1}{c^2x^2}\right)+\frac{2\arctan\left(\frac{c}{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(c^2 + x^2)/c^4 - log(x^2)/c^4 - 1/(c^2*x^2)) + 2*arctan(c/x)/x
^3)*b - 1/3*a/x^3
```

mupad [B] time = 0.37, size = 56, normalized size = 1.02

$$\frac{\frac{b\,x^3\,\ln(x)}{3}+\frac{b\,c^2\,x}{6}-\frac{b\,x^3\,\ln\left(c^2+x^2\right)}{6}}{c^3\,x^3}-\frac{\frac{a}{3}+\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x^4,x)
```

```
[Out] ((b*x^3*log(x))/3 + (b*c^2*x)/6 - (b*x^3*log(c^2 + x^2))/6)/(c^3*x^3) - (a/
3 + (b*atan(c/x))/3)/x^3
```

sympy [A] time = 1.09, size = 60, normalized size = 1.09

$$\begin{cases} -\frac{a}{3x^3}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6cx^2}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3} & \text{for}\ c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*atan(c/x)/(3*x**3) + b/(6*c*x**2) + b*log(x)/(3*
c**3) - b*log(c**2 + x**2)/(6*c**3), Ne(c, 0)), (-a/(3*x**3), True))
```

3.140 $\int x^3 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=122

$$-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{2}{3}b^2c^4\log(x$$

```
[Out] 1/12*b^2*c^2*x^2-1/2*b*c^3*x*(a+b*arccot(x/c))+1/6*b*c*x^3*(a+b*arccot(x/c)
)-1/4*c^4*(a+b*arccot(x/c))^2+1/4*x^4*(a+b*arccot(x/c))^2-1/3*b^2*c^4*ln(1+
c^2/x^2)-2/3*b^2*c^4*ln(x)
```

Rubi [C] time = 1.79, antiderivative size = 862, normalized size of antiderivative = 7.07, number of steps used = 88, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{16}\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2c^4+\frac{1}{16}b^2\log^2\left(\frac{ic}{x}+1\right)c^4-\frac{11}{48}b^2\log\left(i-\frac{c}{x}\right)c^4-\frac{5}{48}b^2\log\left(\frac{c}{x}+i\right)c^4-\frac{5}{48}b^2\log(c-ix)c^4$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] -(a*b*c^3*x)/4 - (I/8)*a*b*c^2*x^2 + (b^2*c^2*x^2)/12 + (a*b*c*x^3)/12 - (1
1*b^2*c^4*Log[I - c/x])/48 - (I/8)*b^2*c^3*x*Log[1 - (I*c)/x] + (b^2*c^2*x^
2*Log[1 - (I*c)/x])/16 + (I/24)*b^2*c*x^3*Log[1 - (I*c)/x] - (b*c^3*(1 - (I
*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]))/8 + (I/16)*b*c^2*x^2*(2*a + I*b*Log[
1 - (I*c)/x]) + (b*c*x^3*(2*a + I*b*Log[1 - (I*c)/x]))/24 - (c^4*(2*a + I*b
*Log[1 - (I*c)/x])^2)/16 + (x^4*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 + (I/4)*
b^2*c^3*x*Log[1 + (I*c)/x] - (I/12)*b^2*c*x^3*Log[1 + (I*c)/x] - (I/4)*a*b*
x^4*Log[1 + (I*c)/x] + (b^2*x^4*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/8 + (b^2
*c^4*Log[1 + (I*c)/x]^2)/16 - (b^2*x^4*Log[1 + (I*c)/x]^2)/16 - (5*b^2*c^4*
Log[I + c/x])/48 + (I/4)*a*b*c^4*Log[c - I*x] - (5*b^2*c^4*Log[c - I*x])/48
 - (b^2*c^4*Log[1 - (I*c)/x]*Log[c - I*x])/8 - (5*b^2*c^4*Log[c + I*x])/48
- (b^2*c^4*Log[1 + (I*c)/x]*Log[c + I*x])/8 + (b^2*c^4*Log[(c - I*x)/(2*c)]
*Log[c + I*x])/8 + (b^2*c^4*Log[c - I*x]*Log[(c + I*x)/(2*c)])/8 - (I/4)*a*
b*c^4*Log[x] - (11*b^2*c^4*Log[x])/24 - (b^2*c^4*Log[c + I*x]*Log[((-I)*x)/
c])/8 - (b^2*c^4*Log[c - I*x]*Log[(I*x)/c])/8 + (b^2*c^4*PolyLog[2, (c - I*
x)/(2*c)])/8 + (b^2*c^4*PolyLog[2, (c + I*x)/(2*c)])/8 + (b^2*c^4*PolyLog[2
, ((-I)*c)/x])/8 + (b^2*c^4*PolyLog[2, (I*c)/x])/8 - (b^2*c^4*PolyLog[2, 1
- (I*x)/c])/8 - (b^2*c^4*PolyLog[2, 1 + (I*x)/c])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
```

```
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \left(a + b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(\frac{1}{4}x^3 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 + \frac{1}{2}bx^3 \left(-2ia + b\log\left(1 - \frac{ic}{x}\right)\right)\log\left(1 + \frac{ic}{x}\right) - \frac{1}{4}\right.$$

$$= \frac{1}{4}\int x^3 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 dx + \frac{1}{2}b\int x^3 \left(-2ia + b\log\left(1 - \frac{ic}{x}\right)\right)\log\left(1 + \frac{ic}{x}\right.$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{(2a + ib\log(1 - icx))^2}{x^5}dx, x, \frac{1}{x}\right)\right) + \frac{1}{2}b\int \left(-2iax^3\log\left(1 + \frac{ic}{x}\right)\right.$$

$$= \frac{1}{16}x^4 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 - \frac{1}{16}b^2x^4\log^2\left(1 + \frac{ic}{x}\right) - (iab)\int x^3\log\left(1 + \frac{ic}{x}\right) d$$

$$= \frac{1}{16}x^4 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 - \frac{1}{4}iabx^4\log\left(1 + \frac{ic}{x}\right) + \frac{1}{8}b^2x^4\log\left(1 - \frac{ic}{x}\right)\log\left(1\right.$$

$$= \frac{1}{16}x^4 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 - \frac{1}{4}iabx^4\log\left(1 + \frac{ic}{x}\right) + \frac{1}{8}b^2x^4\log\left(1 - \frac{ic}{x}\right)\log\left(1\right.$$

$$= \frac{1}{24}bcx^3 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right) + \frac{1}{16}x^4 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 + \frac{1}{8}ib^2c^3x\log\left(1 +\right.$$

$$= -\frac{1}{4}abc^3x - \frac{1}{8}iabc^2x^2 + \frac{1}{12}abcx^3 + \frac{1}{16}ibc^2x^2 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right) + \frac{1}{24}bcx^3 \left(2a\right.$$

$$= -\frac{1}{4}abc^3x - \frac{1}{16}ib^2c^3x - \frac{1}{8}iabc^2x^2 + \frac{1}{24}b^2c^2x^2 + \frac{1}{12}abcx^3 - \frac{11}{48}b^2c^4\log\left(i - \frac{c}{x}\right) -$$

$$= -\frac{1}{4}abc^3x - \frac{1}{8}iabc^2x^2 + \frac{1}{24}b^2c^2x^2 + \frac{1}{12}abcx^3 - \frac{11}{48}b^2c^4\log\left(i - \frac{c}{x}\right) - \frac{5}{48}b^2c^4\log$$

$$= -\frac{1}{4}abc^3x - \frac{1}{8}iabc^2x^2 + \frac{1}{24}b^2c^2x^2 + \frac{1}{12}abcx^3 - \frac{11}{48}b^2c^4\log\left(i - \frac{c}{x}\right) - \frac{5}{48}b^2c^4\log$$

$$= -\frac{1}{4}abc^3x - \frac{1}{8}iabc^2x^2 + \frac{1}{12}b^2c^2x^2 + \frac{1}{12}abcx^3 - \frac{11}{48}b^2c^4\log\left(i - \frac{c}{x}\right) - \frac{5}{48}b^2c^4\log$$

$$= -\frac{1}{4}abc^3x - \frac{1}{8}iabc^2x^2 + \frac{1}{12}b^2c^2x^2 + \frac{1}{12}abcx^3 - \frac{11}{48}b^2c^4\log\left(i - \frac{c}{x}\right) - \frac{5}{48}b^2c^4\log$$

$$= -\frac{1}{4}abc^3x - \frac{1}{8}iabc^2x^2 + \frac{1}{12}b^2c^2x^2 + \frac{1}{12}abcx^3 - \frac{11}{48}b^2c^4\log\left(i - \frac{c}{x}\right) - \frac{5}{48}b^2c^4\log$$

Mathematica [A] time = 0.10, size = 111, normalized size = 0.91

$$\frac{1}{12}\left(x\left(3a^2x^3 + 2abc\left(x^2 - 3c^2\right) + b^2c^2x\right) + 2b\tan^{-1}\left(\frac{c}{x}\right)\left(3a\left(x^4 - c^4\right) + bcx\left(x^2 - 3c^2\right)\right) + 3b^2\left(x^4 - c^4\right)\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (x*(b^2*c^2*x + 3*a^2*x^3 + 2*a*b*c*(-3*c^2 + x^2)) + 2*b*(b*c*x*(-3*c^2 +
x^2) + 3*a*(-c^4 + x^4))*ArcTan[c/x] + 3*b^2*(-c^4 + x^4)*ArcTan[c/x]^2 - 4
*b^2*c^4*Log[c^2 + x^2])/12
```

fricas [A] time = 0.43, size = 125, normalized size = 1.02

$$\frac{1}{2}\,abc^{4}\arctan\left(\frac{x}{c}\right)-\frac{1}{3}\,b^{2}c^{4}\log\left(c^{2}+x^{2}\right)-\frac{1}{2}\,abc^{3}x+\frac{1}{12}\,b^{2}c^{2}x^{2}+\frac{1}{6}\,abcx^{3}+\frac{1}{4}\,a^{2}x^{4}-\frac{1}{4}\,\left(b^{2}c^{4}-b^{2}x^{4}\right)\arctan\left(\frac{c}{x}\right)^{2}-\frac{1}{6}\,(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*c^4*arctan(x/c) - 1/3*b^2*c^4*log(c^2 + x^2) - 1/2*a*b*c^3*x + 1/12
*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/4*(b^2*c^4 - b^2*x^4)*arctan
(c/x)^2 - 1/6*(3*b^2*c^3*x - b^2*c*x^3 - 3*a*b*x^4)*arctan(c/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^{2}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2*x^3, x)
```

maple [A] time = 0.06, size = 157, normalized size = 1.29

$$\frac{a^{2}x^{4}}{4}+\frac{b^{2}x^{4}\arctan\left(\frac{c}{x}\right)^{2}}{4}+\frac{c\,b^{2}\arctan\left(\frac{c}{x}\right)x^{3}}{6}-\frac{c^{3}b^{2}\arctan\left(\frac{c}{x}\right)x}{2}-\frac{c^{4}b^{2}\arctan\left(\frac{c}{x}\right)^{2}}{4}+\frac{b^{2}c^{2}x^{2}}{12}+\frac{2c^{4}b^{2}\ln\left(\frac{c}{x}\right)}{3}-\frac{b^{2}c^{4}\ln\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctan(c/x)^2+1/6*c*b^2*arctan(c/x)*x^3-1/2*c^3*b^2
*arctan(c/x)*x-1/4*c^4*b^2*arctan(c/x)^2+1/12*b^2*c^2*x^2+2/3*c^4*b^2*ln(c/
x)-1/3*b^2*c^4*ln(1+c^2/x^2)+1/2*a*b*x^4*arctan(c/x)+1/6*a*b*c*x^3-1/2*c^3*
a*b*x+1/2*c^4*a*b*arctan(x/c)
```

maxima [A] time = 0.43, size = 134, normalized size = 1.10

$$\frac{1}{4}\,b^{2}x^{4}\arctan\left(\frac{c}{x}\right)^{2}+\frac{1}{4}\,a^{2}x^{4}+\frac{1}{6}\,\left(3\,x^{4}\arctan\left(\frac{c}{x}\right)+\left(3\,c^{3}\arctan\left(\frac{x}{c}\right)-3\,c^{2}x+x^{3}\right)c\right)ab+\frac{1}{12}\,\left(\left(3\,c^{2}\arctan\left(\frac{x}{c}\right)^{2}-4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c/x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c/x) + (3*c^3*a
rctan(x/c) - 3*c^2*x + x^3)*c)*a*b + 1/12*((3*c^2*arctan(x/c)^2 - 4*c^2*log
(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(c/x
))*b^2
```

mupad [B] time = 0.47, size = 140, normalized size = 1.15

$$\frac{a^{2}\,x^{4}}{4}-\frac{b^{2}\,c^{4}\,\mathrm{atan}\left(\frac{c}{x}\right)^{2}}{4}-\frac{b^{2}\,c^{4}\,\ln\left(c^{2}+x^{2}\right)}{3}+\frac{b^{2}\,x^{4}\,\mathrm{atan}\left(\frac{c}{x}\right)^{2}}{4}+\frac{b^{2}\,c^{2}\,x^{2}}{12}+\frac{b^{2}\,c\,x^{3}\,\mathrm{atan}\left(\frac{c}{x}\right)}{6}-\frac{b^{2}\,c^{3}\,x\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,c\,x^{3}}{6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c/x))^2,x)
```

```
[Out] (a^2*x^4)/4 - (b^2*c^4*atan(c/x)^2)/4 - (b^2*c^4*log(c^2 + x^2))/3 + (b^2*x
^4*atan(c/x)^2)/4 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atan(c/x))/6 - (b^2*c^3*x
```

```
*atan(c/x))/2 + (a*b*c*x^3)/6 - (a*b*c^3*x)/2 - (a*b*c^4*atan(c/x))/2 + (a*
b*x^4*atan(c/x))/2
```

sympy [A] time = 0.81, size = 144, normalized size = 1.18

$$\frac{a^2x^4}{4} - \frac{abc^4\operatorname{atan}\left(\frac{c}{x}\right)}{2} - \frac{abc^3x}{2} + \frac{abcx^3}{6} + \frac{abx^4\operatorname{atan}\left(\frac{c}{x}\right)}{2} - \frac{b^2c^4\log\left(c^2+x^2\right)}{3} - \frac{b^2c^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4} - \frac{b^2c^3x\operatorname{atan}\left(\frac{c}{x}\right)}{2} + \frac{b^2c^2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**4*atan(c/x)/2 - a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**4
*atan(c/x)/2 - b**2*c**4*log(c**2 + x**2)/3 - b**2*c**4*atan(c/x)**2/4 - b*
*2*c**3*x*atan(c/x)/2 + b**2*c**2*x**2/12 + b**2*c*x**3*atan(c/x)/6 + b**2*
x**4*atan(c/x)**2/4
```

3.141 $\int x^2 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=152

$$-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right.\right.$$

```
[Out] 1/3*b^2*c^2*x+1/3*b^2*c^3*arccot(x/c)+1/3*b*c*x^2*(a+b*arccot(x/c))-1/3*I*c
^3*(a+b*arccot(x/c))^2+1/3*x^3*(a+b*arccot(x/c))^2+2/3*b*c^3*(a+b*arccot(x/
c))*ln(2-2/(1-I*c/x))-1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c/x))
```

Rubi [B] time = 1.49, antiderivative size = 787, normalized size of antiderivative = 5.18, number of steps used = 73, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,\frac{c-ix}{2c}\right)-\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,\frac{c+ix}{2c}\right)+\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,-\frac{ic}{x}\right)-\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,\frac{ic}{x}\right)-\frac{1}{6}ib^2c^3$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (-I/3)*a*b*c^2*x + (b^2*c^2*x)/3 + (a*b*c*x^2)/6 - (I/4)*b^2*c^3*Log[I - c/
x] + (b^2*c^2*x*Log[1 - (I*c)/x])/6 + (I/12)*b^2*c*x^2*Log[1 - (I*c)/x] + (
I/6)*b*c^2*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]) + (b*c*x^2*(2*a + I
*b*Log[1 - (I*c)/x]))/12 + (I/12)*c^3*(2*a + I*b*Log[1 - (I*c)/x])^2 + (x^3
*(2*a + I*b*Log[1 - (I*c)/x])^2)/12 - (I/6)*b^2*c*x^2*Log[1 + (I*c)/x] - (I
/3)*a*b*x^3*Log[1 + (I*c)/x] + (b^2*x^3*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/
6 + (I/12)*b^2*c^3*Log[1 + (I*c)/x]^2 - (b^2*x^3*Log[1 + (I*c)/x]^2)/12 + (
I/12)*b^2*c^3*Log[I + c/x] - (a*b*c^3*Log[c - I*x])/3 + (I/12)*b^2*c^3*Log[
c - I*x] - (I/6)*b^2*c^3*Log[1 - (I*c)/x]*Log[c - I*x] - (I/12)*b^2*c^3*Log
[c + I*x] + (I/6)*b^2*c^3*Log[1 + (I*c)/x]*Log[c + I*x] - (I/6)*b^2*c^3*Log
[(c - I*x)/(2*c)]*Log[c + I*x] + (I/6)*b^2*c^3*Log[c - I*x]*Log[(c + I*x)/(
2*c)] - (a*b*c^3*Log[x])/3 + (I/6)*b^2*c^3*Log[c + I*x]*Log[((-I)*x)/c] - (
I/6)*b^2*c^3*Log[c - I*x]*Log[(I*x)/c] + (I/6)*b^2*c^3*PolyLog[2, (c - I*x)
/(2*c)] - (I/6)*b^2*c^3*PolyLog[2, (c + I*x)/(2*c)] + (I/6)*b^2*c^3*PolyLog
[2, ((-I)*c)/x] - (I/6)*b^2*c^3*PolyLog[2, (I*c)/x] - (I/6)*b^2*c^3*PolyLog
[2, 1 - (I*x)/c] + (I/6)*b^2*c^3*PolyLog[2, 1 + (I*x)/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2dx=\int\left(\frac{1}{4}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{4}\right.$$

$$=\frac{1}{4}\int x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right.$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax^2\log\left(1+\frac{ic}{x}\right)\right.$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{12}b^2x^3\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x^2\log\left(1+\frac{ic}{x}\right)d$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{6}b^2x^3\log\left(1-\frac{ic}{x}\right)\log\left(1\right.$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{6}b^2x^3\log\left(1-\frac{ic}{x}\right)\log\left(1\right.$$

$$=\frac{1}{12}bcx^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{6}b^2c^2x\log\left(1+\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}abcx^2+\frac{1}{6}ibc^2\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{12}bcx^2\left(2a+ib\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b$$

Mathematica [A] time = 0.36, size = 152, normalized size = 1.00

$$\frac{1}{3}\left(a^2x^3+2abc^3\log\left(\frac{c}{x}\right)-abc^3\log\left(\frac{c^2}{x^2}+1\right)+b\tan^{-1}\left(\frac{c}{x}\right)\left(2ax^3+2bc^3\log\left(1-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+bc\left(c^2+x^2\right)\right)+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*((-I)*c^3 + x^3)*ArcTan[c/x]^2 + b*A
rcTan[c/x]*(2*a*x^3 + b*c*(c^2 + x^2) + 2*b*c^3*Log[1 - E^((2*I)*ArcTan[c/x
])]) - a*b*c^3*Log[1 + c^2/x^2] + 2*a*b*c^3*Log[c/x] - I*b^2*c^3*PolyLog[2,
 E^((2*I)*ArcTan[c/x])])/3
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\arctan\left(\frac{c}{x}\right)^2 + 2\,abx^2\arctan\left(\frac{c}{x}\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c/x)^2 + 2*a*b*x^2*arctan(c/x) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(\frac{c}{x}\right) + a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2*x^2, x)
```

maple [B] time = 0.13, size = 445, normalized size = 2.93

$$\frac{x^3a^2}{3}+\frac{b^2x^3\arctan\left(\frac{c}{x}\right)^2}{3}+\frac{c\,b^2\arctan\left(\frac{c}{x}\right)x^2}{3}+\frac{2c^3b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}-\frac{c^3b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{ic^3b^2\ln\left(\frac{c}{x}-i\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctan(c/x)^2+1/3*c*b^2*arctan(c/x)*x^2+2/3*c^3*b^2
*ln(c/x)*arctan(c/x)-1/3*c^3*b^2*arctan(c/x)*ln(1+c^2/x^2)-1/6*I*c^3*b^2*ln
(c/x-I)*ln(1+c^2/x^2)+1/12*I*c^3*b^2*ln(c/x-I)^2+1/3*I*c^3*b^2*dilog(1+I*c/
x)-1/12*I*c^3*b^2*ln(I+c/x)^2-1/6*I*c^3*b^2*ln(I+c/x)*ln(1/2*I*(c/x-I))-1/3
*I*c^3*b^2*dilog(1-I*c/x)-1/6*I*c^3*b^2*dilog(1/2*I*(c/x-I))+1/6*I*c^3*b^2*
ln(c/x-I)*ln(-1/2*I*(I+c/x))+1/3*b^2*c^2*x-1/3*c^3*b^2*arctan(x/c)+1/6*I*c^
3*b^2*ln(I+c/x)*ln(1+c^2/x^2)+1/3*I*c^3*b^2*ln(c/x)*ln(1+I*c/x)+1/6*I*c^3*b
^2*dilog(-1/2*I*(I+c/x))-1/3*I*c^3*b^2*ln(c/x)*ln(1-I*c/x)+2/3*a*b*x^3*arct
an(c/x)+1/3*a*b*c*x^2+2/3*c^3*a*b*ln(c/x)-1/3*c^3*a*b*ln(1+c^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2x^3+\frac{1}{3}\left(2\,x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)ab+\frac{1}{48}\left(4\,x^3\arctan\left(c,x\right)^2-x^3\log\left(c^2+x^2\right)^2+48\int\frac{36\,c^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a*b +
1/48*(4*x^3*arctan2(c, x)^2 - x^3*log(c^2 + x^2)^2 + 48*integrate(1/48*(36*
c^2*x^2*arctan2(c, x)^2 + 36*x^4*arctan2(c, x)^2 + 8*c*x^3*arctan2(c, x) +
4*x^4*log(c^2 + x^2) + 3*(c^2*x^2 + x^4)*log(c^2 + x^2)^2)/(c^2 + x^2), x))
*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c/x))^2,x)
```

```
[Out] int(x^2*(a + b*atan(c/x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c/x))**2, x)
```

3.142 $\int x \left(a+b \tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=82

$$\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2c^2\log\left(\frac{c^2}{x^2}+1\right)+b^2c^2\log(x)$$

```
[Out] b*c*x*(a+b*arccot(x/c))+1/2*c^2*(a+b*arccot(x/c))^2+1/2*x^2*(a+b*arccot(x/c
))^2+1/2*b^2*c^2*ln(1+c^2/x^2)+b^2*c^2*ln(x)
```

Rubi [C] time = 1.13, antiderivative size = 663, normalized size of antiderivative = 8.09, number of steps used = 58, number of rules used = 32, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.286, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 193, 43, 6742, 30, 2557, 12, 2466, 2448, 263, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c-ix}{2c}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c+ix}{2c}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,-\frac{ic}{x}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{ic}{x}\right)+\frac{1}{4}b^2c^2\text{P}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (a*b*c*x)/2 + (b^2*c^2*Log[I - c/x])/4 + (I/4)*b^2*c*x*Log[1 - (I*c)/x] + (
b*c*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]))/4 + (c^2*(2*a + I*b*Log[1
 - (I*c)/x])^2)/8 + (x^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/8 - (I/2)*b^2*c*x*
Log[1 + (I*c)/x] - (I/2)*a*b*x^2*Log[1 + (I*c)/x] + (b^2*x^2*Log[1 - (I*c)/
x]*Log[1 + (I*c)/x])/4 - (b^2*c^2*Log[1 + (I*c)/x]^2)/8 - (b^2*x^2*Log[1 +
(I*c)/x]^2)/8 - (I/2)*a*b*c^2*Log[c - I*x] + (b^2*c^2*Log[c - I*x])/4 + (b^
2*c^2*Log[1 - (I*c)/x]*Log[c - I*x])/4 + (b^2*c^2*Log[c + I*x])/4 + (b^2*c^
2*Log[1 + (I*c)/x]*Log[c + I*x])/4 - (b^2*c^2*Log[(c - I*x)/(2*c)]*Log[c +
I*x])/4 - (b^2*c^2*Log[c - I*x]*Log[(c + I*x)/(2*c)])/4 + (I/2)*a*b*c^2*Log
[x] + (b^2*c^2*Log[x])/2 + (b^2*c^2*Log[c + I*x]*Log[((-I)*x)/c])/4 + (b^2*
c^2*Log[c - I*x]*Log[(I*x)/c])/4 - (b^2*c^2*PolyLog[2, (c - I*x)/(2*c)])/4
- (b^2*c^2*PolyLog[2, (c + I*x)/(2*c)])/4 - (b^2*c^2*PolyLog[2, ((-I)*c)/x]
)/4 - (b^2*c^2*PolyLog[2, (I*c)/x])/4 + (b^2*c^2*PolyLog[2, 1 - (I*x)/c])/4
 + (b^2*c^2*PolyLog[2, 1 + (I*x)/c])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int\left(\frac{1}{4}x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{4}b^2x\log$$
$$=\frac{1}{4}\int x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2 dx+\frac{1}{2}b\int x\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right) dx$$
$$=-\left(\frac{1}{4}\mathrm{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax\log\left(1+\frac{ic}{x}\right)+bx\log\right.$$
$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{8}b^2x^2\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x\log\left(1+\frac{ic}{x}\right)dx+\frac{1}{2}b$$
$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{2}iabx^2\log\left(1+\frac{ic}{x}\right)+\frac{1}{4}b^2x^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)$$
$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{2}iabx^2\log\left(1+\frac{ic}{x}\right)+\frac{1}{4}b^2x^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)$$
$$=\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{4}ib^2cx\log\left(1\right.$$
$$=\frac{1}{2}abcx+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{8}c^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{8}x^2\left(\right.$$
$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$
$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$
$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$
$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$
$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$
$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$

Mathematica [A] time = 0.06, size = 73, normalized size = 0.89

$$\frac{1}{2}\left(2b\tan^{-1}\left(\frac{c}{x}\right)\left(a\left(c^2+x^2\right)+bcx\right)+ax(ax+2bc)+b^2c^2\log\left(c^2+x^2\right)+b^2\left(c^2+x^2\right)\tan^{-1}\left(\frac{c}{x}\right)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (a*x*(2*b*c + a*x) + 2*b*(b*c*x + a*(c^2 + x^2))*ArcTan[c/x] + b^2*(c^2 + x
^2)*ArcTan[c/x]^2 + b^2*c^2*Log[c^2 + x^2])/2
```

fricas [A] time = 0.42, size = 88, normalized size = 1.07

$$-abc^2\arctan\left(\frac{x}{c}\right)+\frac{1}{2}b^2c^2\log\left(c^2+x^2\right)+abcx+\frac{1}{2}a^2x^2+\frac{1}{2}\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2+\left(b^2cx+abx^2\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] -a*b*c^2*arctan(x/c) + 1/2*b^2*c^2*log(c^2 + x^2) + a*b*c*x + 1/2*a^2*x^2 +
 1/2*(b^2*c^2 + b^2*x^2)*arctan(c/x)^2 + (b^2*c*x + a*b*x^2)*arctan(c/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2*x, x)
```

maple [A] time = 0.05, size = 116, normalized size = 1.41

$$\frac{a^2x^2}{2}+\frac{b^2x^2\arctan\left(\frac{c}{x}\right)^2}{2}+c\,b^2\arctan\left(\frac{c}{x}\right)x+\frac{c^2b^2\arctan\left(\frac{c}{x}\right)^2}{2}-c^2b^2\ln\left(\frac{c}{x}\right)+\frac{b^2c^2\ln\left(1+\frac{c^2}{x^2}\right)}{2}+ab\,x^2\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*b^2*x^2*arctan(c/x)^2+c*b^2*arctan(c/x)*x+1/2*c^2*b^2*arcta
n(c/x)^2-c^2*b^2*ln(c/x)+1/2*b^2*c^2*ln(1+c^2/x^2)+a*b*x^2*arctan(c/x)+a*b*
c*x-c^2*a*b*arctan(x/c)
```

maxima [A] time = 0.43, size = 104, normalized size = 1.27

$$\frac{1}{2}b^2x^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}a^2x^2+\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)ab-\frac{1}{2}\left(\left(\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)\right)c^2+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c/x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c/x) - (c*arctan(x/c)
 - x)*c)*a*b - 1/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arctan(x/c)
 - x)*c*arctan(c/x))*b^2
```

mupad [B] time = 0.40, size = 98, normalized size = 1.20

$$\frac{a^2\,x^2}{2}+\frac{b^2\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,c^2\,\ln\left(c^2+x^2\right)}{2}+\frac{b^2\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+a\,b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)+a\,b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)+b^2\,c\,x\,\mathrm{atan}\left(\frac{c}{x}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c/x))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*c^2*atan(c/x)^2)/2 + (b^2*c^2*log(c^2 + x^2))/2 + (b^2*x
^2*atan(c/x)^2)/2 + a*b*c^2*atan(c/x) + a*b*x^2*atan(c/x) + b^2*c*x*atan(c/
x) + a*b*c*x
```

sympy [A] time = 0.44, size = 97, normalized size = 1.18

$$\frac{a^2x^2}{2}+abc^2\operatorname{atan}\left(\frac{c}{x}\right)+abcx+abx^2\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2c^2\log\left(c^2+x^2\right)}{2}+\frac{b^2c^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}+b^2cx\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2x^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**2/2 + a*b*c**2*atan(c/x) + a*b*c*x + a*b*x**2*atan(c/x) + b**2*c**2
*log(c**2 + x**2)/2 + b**2*c**2*atan(c/x)**2/2 + b**2*c*x*atan(c/x) + b**2*
x**2*atan(c/x)**2/2
```

3.143 $\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=83

$$ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+ib^2c\text{Li}_2\left(1-\frac{2c}{c+ix}\right)$$

```
[Out] I*c*(a+b*arccot(x/c))^2+x*(a+b*arccot(x/c))^2-2*b*c*(a+b*arccot(x/c))*ln(2*
c/(c+I*x))+I*b^2*c*polylog(2,1-2*c/(c+I*x))
```

Rubi [B] time = 0.44, antiderivative size = 478, normalized size of antiderivative = 5.76, number of steps used = 31, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {5029, 2448, 263, 31, 2449, 2391, 2556, 12, 2462, 260, 2416, 2394, 2393, 2315}

$$-\frac{1}{2}ib^2c\text{PolyLog}\left(2,\frac{c-ix}{2c}\right)+\frac{1}{2}ib^2c\text{PolyLog}\left(2,\frac{c+ix}{2c}\right)-\frac{1}{2}ib^2c\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib^2c\text{PolyLog}\left(2,\frac{ic}{x}\right)+\frac{1}{2}ib^2c$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^2,x]
```

```
[Out] a^2*x + I*a*b*x*Log[1 - (I*c)/x] + (b^2*(I*c - x)*Log[1 - (I*c)/x]^2)/4 - I
*a*b*x*Log[1 + (I*c)/x] + (b^2*x*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/2 - (b^
2*(I*c + x)*Log[1 + (I*c)/x]^2)/4 - (I/2)*b^2*c*Log[1 + (I*c)/x]*Log[-c - I
*x] + a*b*c*Log[c - I*x] + (I/2)*b^2*c*Log[-c - I*x]*Log[(c - I*x)/(2*c)] +
 (I/2)*b^2*c*Log[1 - (I*c)/x]*Log[-c + I*x] + a*b*c*Log[c + I*x] - (I/2)*b^
2*c*Log[-c + I*x]*Log[(c + I*x)/(2*c)] - (I/2)*b^2*c*Log[-c - I*x]*Log[((-I
)*x)/c] + (I/2)*b^2*c*Log[-c + I*x]*Log[(I*x)/c] - (I/2)*b^2*c*PolyLog[2, (
c - I*x)/(2*c)] + (I/2)*b^2*c*PolyLog[2, (c + I*x)/(2*c)] - (I/2)*b^2*c*Pol
yLog[2, ((-I)*c)/x] + (I/2)*b^2*c*PolyLog[2, (I*c)/x] + (I/2)*b^2*c*PolyLog
[2, 1 - (I*x)/c] - (I/2)*b^2*c*PolyLog[2, 1 + (I*x)/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 5029

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x],
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rubi steps

$$\int \left(a + b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a^2 + iab\log\left(1 - \frac{ic}{x}\right) - \frac{1}{4}b^2\log^2\left(1 - \frac{ic}{x}\right) - iab\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2\log\left(1 - \frac{ic}{x}\right)\right.$$

$$= a^2x + (iab)\int \log\left(1 - \frac{ic}{x}\right)dx - (iab)\int \log\left(1 + \frac{ic}{x}\right)dx - \frac{1}{4}b^2\int \log^2\left(1 - \frac{ic}{x}\right)dx$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

$$= a^2x + iabx\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}b^2(ic - x)\log^2\left(1 - \frac{ic}{x}\right) - iabx\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}b^2x\log$$

Mathematica [A] time = 0.11, size = 105, normalized size = 1.27

$$a\left(ax + bc\log\left(\frac{c^2}{x^2} + 1\right) - 2bc\log\left(\frac{c}{x}\right)\right) + 2b\tan^{-1}\left(\frac{c}{x}\right)\left(ax - bc\log\left(1 - e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\right) + ib^2c\operatorname{Li}_2\left(e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right) + b^2(x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^2,x]
[Out] b^2*(I*c + x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*x - b*c*Log[1 - E^((2*I)*A
rcTan[c/x])]) + a*(a*x + b*c*Log[1 + c^2/x^2] - 2*b*c*Log[c/x]) + I*b^2*c*P
olyLog[2, E^((2*I)*ArcTan[c/x])]
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\arctan\left(\frac{c}{x}\right)^2 + 2\,ab\arctan\left(\frac{c}{x}\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="fricas")
[Out] integral(b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\arctan\left(\frac{c}{x}\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2, x)
```

maple [B] time = 0.10, size = 357, normalized size = 4.30

$$a^2x+b^2x\arctan\left(\frac{c}{x}\right)^2-2cb^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+cb^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)+icb^2\,\mathrm{dilog}\left(1-\frac{ic}{x}\right)+icb^2\ln\left(\frac{c}{x}\right)\ln\left(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2,x)
```

```
[Out] a^2*x+b^2*x*arctan(c/x)^2-2*c*b^2*ln(c/x)*arctan(c/x)+c*b^2*arctan(c/x)*ln(
1+c^2/x^2)-1/2*I*c*b^2*dilog(-1/2*I*(I+c/x))+I*c*b^2*ln(c/x)*ln(1-I*c/x)+1/
2*I*c*b^2*dilog(1/2*I*(c/x-I))+1/2*I*c*b^2*ln(I+c/x)*ln(1/2*I*(c/x-I))-1/2*
I*c*b^2*ln(c/x-I)*ln(-1/2*I*(I+c/x))-1/2*I*c*b^2*ln(I+c/x)*ln(1+c^2/x^2)+1/
2*I*c*b^2*ln(c/x-I)*ln(1+c^2/x^2)-I*c*b^2*ln(c/x)*ln(1+I*c/x)+1/4*I*c*b^2*l
n(I+c/x)^2-I*c*b^2*dilog(1+I*c/x)-1/4*I*c*b^2*ln(c/x-I)^2+I*c*b^2*dilog(1-I
*c/x)+2*a*b*x*arctan(c/x)-2*c*a*b*ln(c/x)+c*a*b*ln(1+c^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)ab+\frac{1}{16}\left(12\,c\arctan\left(\frac{c}{x}\right)^2\arctan\left(\frac{x}{c}\right)+4\left(\frac{3\,\arctan\left(\frac{c}{x}\right)\arctan\left(\frac{x}{c}\right)^2}{c}+\frac{\arctan\left(\right.}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] (2*x*arctan(c/x) + c*log(c^2 + x^2))*a*b + 1/16*(12*c*arctan(c/x)^2*arctan(
x/c) + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*c^2 + 4*x*arctan
2(c, x)^2 + 16*c^2*integrate(1/16*log(c^2 + x^2)^2/(c^2 + x^2), x) - x*log(
c^2 + x^2)^2 + 128*c*integrate(1/16*x*arctan(c/x)/(c^2 + x^2), x) + 192*int
egrate(1/16*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 16*integrate(1/16*x^2*log(c
^2 + x^2)^2/(c^2 + x^2), x) + 64*integrate(1/16*x^2*log(c^2 + x^2)/(c^2 + x
^2), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2,x)
```

```
[Out] int((a + b*atan(c/x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2,x)
```

```
[Out] Integral((a + b*atan(c/x))**2, x)
```

3.144 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=148

$$ib\text{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-ib\text{Li}_2\left(\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right.\right.$$

```
[Out] 2*(a+b*arccot(x/c))^2*arctanh(-1+2/(1+I*c/x))+I*b*(a+b*arccot(x/c))*polylog
(2,1-2/(1+I*c/x))-I*b*(a+b*arccot(x/c))*polylog(2,-1+2/(1+I*c/x))+1/2*b^2*p
olylog(3,1-2/(1+I*c/x))-1/2*b^2*polylog(3,-1+2/(1+I*c/x))
```

Rubi [A] time = 0.29, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5031, 4850, 4988, 4884, 4994, 6610}

$$ib\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-ib\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcCot[x/c])^2*ArcTanh[1 - 2/(1 + (I*c)/x)] + I*b*(a + b*ArcCot[x
/c])*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - I*b*(a + b*ArcCot[x/c])*PolyLog[2, -
1 + 2/(1 + (I*c)/x)] + (b^2*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/2 - (b^2*PolyL
og[3, -1 + 2/(1 + (I*c)/x)])/2
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x}\right)$$
$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh^{-1}}{1+c^2x^2}\right.$$
$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)-(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+i}\right.}{1+c^2x^2}\right.$$
$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right)-ib\left(\right.$$
$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right)-ib\left(\right.$$

Mathematica [A] time = 0.09, size = 148, normalized size = 1.00

$$\frac{1}{2}b\left(2i\text{Li}_2\left(\frac{c+ix}{c-ix}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-2i\text{Li}_2\left(\frac{x-ic}{ic+x}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+b\left(\text{Li}_3\left(\frac{c+ix}{c-ix}\right)-\text{Li}_3\left(\frac{x-ic}{ic+x}\right)\right)\right)-2\tanh$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcTan[c/x])^2*ArcTanh[(c + I*x)/(c - I*x)] + (b*((2*I)*(a + b*Ar
cTan[c/x])*PolyLog[2, (c + I*x)/(c - I*x)] - (2*I)*(a + b*ArcTan[c/x])*Poly
Log[2, ((-I)*c + x)/(I*c + x)] + b*(PolyLog[3, (c + I*x)/(c - I*x)] - PolyL
og[3, ((-I)*c + x)/(I*c + x)])))/2
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(\frac{c}{x}\right)^2+2\,ab\arctan\left(\frac{c}{x}\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2/x, x)
```

maple [C] time = 0.34, size = 1249, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x,x)
```

```
[Out] -1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1
))^3*arctan(c/x)^2-I*a*b*ln(c/x)*ln(1+I*c/x)+I*a*b*ln(c/x)*ln(1-I*c/x)-1/2*
I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+c^2/x^2
)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arcta
n(c/x)^2+1/2*b^2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-2*b^2*polylog(3,-(1+I*
c/x)/(1+c^2/x^2)^(1/2))-2*b^2*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-a^2*ln
(c/x)-1/2*I*b^2*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2
)+1))^3*arctan(c/x)^2+1/2*I*b^2*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c
/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-b^2*ln(c/x)*arctan(c/x)^2+1/2*I*b^2*P
i*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((
1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-
1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^2+1/2*I*b^2*Pi*csgn(I*((1+I*c/x
)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/
x^2)+1))^2*arctan(c/x)^2+1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((
1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1
+c^2/x^2)+1))^2*arctan(c/x)^2-I*b^2*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c
^2/x^2))+2*I*b^2*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*I*b^2
*arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-1/2*I*b^2*Pi*arctan(c/
x)^2-I*a*b*dilog(1+I*c/x)+I*a*b*dilog(1-I*c/x)-2*a*b*ln(c/x)*arctan(c/x)+b^
2*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-b^2*arctan(c/x)^2*ln(1+(1+I*c
/x)/(1+c^2/x^2)^(1/2))-b^2*arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \log(x) + \frac{1}{16} \int \frac{12\, b^2 \arctan\left(c, x\right)^2 + b^2 \log\left(c^2 + x^2\right)^2 + 32\, ab \arctan\left(c, x\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan2(c, x)^2 + b^2*log(c^2 + x^2)^2
+ 32*a*b*arctan2(c, x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2/x,x)
```

```
[Out] int((a + b*atan(c/x))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c/x))**2/x, x)
```

3.145 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=96

$$-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x}-\frac{2b\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{ib^2\text{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{c}$$

```
[Out] -I*(a+b*arccot(x/c))^2/c-(a+b*arccot(x/c))^2/x-2*b*(a+b*arccot(x/c))*ln(2/(
1+I*c/x))/c-I*b^2*polylog(2,1-2/(1+I*c/x))/c
```

Rubi [B] time = 0.53, antiderivative size = 259, normalized size of antiderivative = 2.70, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{ib^2\text{PolyLog}\left(2,-\frac{-x+ic}{2x}\right)}{2c}-\frac{ib^2\text{PolyLog}\left(2,\frac{x+ic}{2x}\right)}{2c}+\frac{b\log\left(1+\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)}{2x}-\frac{ib\log\left(\frac{x+ic}{2x}\right)\left(2ia-b\log\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] ((-I/4)*(1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^2)/c + (b*((2*I)*a - b*L
og[1 - (I*c)/x])*Log[1 + (I*c)/x])/(2*x) - ((I/4)*b^2*(1 + (I*c)/x)*Log[1 +
 (I*c)/x]^2)/c - ((I/2)*b^2*Log[1 + (I*c)/x]*Log[-(I*c - x)/(2*x)])/c - ((I
/2)*b*((2*I)*a - b*Log[1 - (I*c)/x])*Log[(I*c + x)/(2*x)])/c + ((I/2)*b^2*P
olyLog[2, -(I*c - x)/(2*x)])/c - ((I/2)*b^2*PolyLog[2, (I*c + x)/(2*x)])/c
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4x^2}+\frac{b\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x^2}-\frac{b^2\log^2\left(1+\frac{ic}{x}\right.}{4x^2}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{x^2}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{x^2}\,dx-\frac{1}{4}$$

$$=-\left(\frac{1}{4}\,\text{Subst}\left(\int (2a+ib\log(1-icx))^2\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{2}b\,\text{Subst}\left(\int (-2ia+b\log(1-ic\right.$$

$$=\frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x}-\frac{i\,\text{Subst}\left(\int (2a+ib\log(x))^2\,dx,x,1-\frac{ic}{x}\right)}{4c}-$$

$$=-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x}-\frac{ib^2\left(1+\right.}{}$$

$$=\frac{iab}{x}+\frac{b^2}{2x}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{ib^2\left(1+\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{2c}+\frac{b\left(2ia-b\right.}{}$$

$$=\frac{b^2}{x}-\frac{ib^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{ib^2\left(1+\frac{ic}{x}\right)\log\left(\right.}{2c}$$

$$=\frac{b^2}{2x}-\frac{ib^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{b\left(2ia-b\log\left(1\right.\right.}{}$$

$$=-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x}-\frac{ib^2\left(1+\right.}{}$$

Mathematica [A] time = 0.12, size = 107, normalized size = 1.11

$$-\frac{a\left(ac+2bx\log\left(\frac{1}{\sqrt{\frac{c^2}{x^2}+1}}\right)\right)+2b\tan^{-1}\left(\frac{c}{x}\right)\left(ac+bx\log\left(1+e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\right)-ib^2x\text{Li}_2\left(-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+b^2(c-ix)\,\text{t}}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] -((b^2*(c - I*x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*c + b*x*Log[1 + E^((2*I
)*ArcTan[c/x])]) + a*(a*c + 2*b*x*Log[1/Sqrt[1 + c^2/x^2]]) - I*b^2*x*PolyL
og[2, -E^((2*I)*ArcTan[c/x])])/(c*x))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(\frac{c}{x}\right)^2+2\,ab\arctan\left(\frac{c}{x}\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(\frac{c}{x}\right) + a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2/x^2, x)
```

maple [A] time = 0.21, size = 147, normalized size = 1.53

$$-\frac{a^2}{x}+\frac{i\arctan\left(\frac{c}{x}\right)^2 b^2}{c}-\frac{\arctan\left(\frac{c}{x}\right)^2 b^2}{x}+\frac{i\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2}{c}-\frac{2\arctan\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)b^2}{c}-\frac{2ab\arctan\left(\frac{c}{x}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x^2,x)
```

```
[Out] -a^2/x+I/c*arctan(c/x)^2*b^2-arctan(c/x)^2/x*b^2+I/c*polylog(2,-(1+I*c/x)^2
/(1+c^2/x^2))*b^2-2/c*arctan(c/x)*ln((1+I*c/x)^2/(1+c^2/x^2)+1)*b^2-2/x*a*b
*arctan(c/x)+1/c*a*b*ln(1+c^2/x^2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2/x^2,x)
```

```
[Out] int((a + b*atan(c/x))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c/x))**2/x**2, x)
```

3.146 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}+\frac{ab}{cx}-\frac{b^2\log\left(\frac{c^2}{x^2}+1\right)}{2c^2}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}$$

```
[Out] a*b/c/x+b^2*arccot(x/c)/c/x-1/2*(a+b*arccot(x/c))^2/c^2-1/2*(a+b*arccot(x/c
))^2/x^2-1/2*b^2*ln(1+c^2/x^2)/c^2
```

Rubi [C] time = 1.29, antiderivative size = 836, normalized size of antiderivative = 9.95, number of steps used = 66, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2393, 2391, 2315}

$$-\frac{\left(1-\frac{ic}{x}\right)^2b^2}{16c^2}-\frac{\left(\frac{ic}{x}+1\right)^2b^2}{16c^2}-\frac{\left(\frac{ic}{x}+1\right)^2\log^2\left(\frac{ic}{x}+1\right)b^2}{8c^2}+\frac{\left(\frac{ic}{x}+1\right)\log^2\left(\frac{ic}{x}+1\right)b^2}{4c^2}+\frac{\log\left(i-\frac{c}{x}\right)b^2}{8c^2}-\frac{3\left(1-\frac{ic}{x}\right)\log\left(\right.}{4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] -(b^2*(1 - (I*c)/x)^2)/(16*c^2) - (b^2*(1 + (I*c)/x)^2)/(16*c^2) - ((I/4)*a
*b)/x^2 - b^2/(8*x^2) + (3*a*b)/(2*c*x) + ((I/2)*a*b*Log[I - c/x])/c^2 + (b
^2*Log[I - c/x])/(8*c^2) - (3*b^2*(1 - (I*c)/x)*Log[1 - (I*c)/x])/(4*c^2) +
 (b^2*Log[1 - (I*c)/x])/(8*x^2) - ((I/8)*b*(1 - (I*c)/x)^2*(2*a + I*b*Log[1
 - (I*c)/x]))/c^2 - ((1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^2)/(4*c^2)
+ ((1 - (I*c)/x)^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/(8*c^2) - (3*b^2*(1 + (I
*c)/x)*Log[1 + (I*c)/x])/(4*c^2) + (b^2*(1 + (I*c)/x)^2*Log[1 + (I*c)/x])/(
8*c^2) + ((I/2)*a*b*Log[1 + (I*c)/x])/x^2 + (b^2*Log[1 + (I*c)/x])/(8*x^2)
- (b^2*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/(4*x^2) + (b^2*(1 + (I*c)/x)*Log[
1 + (I*c)/x]^2)/(4*c^2) - (b^2*(1 + (I*c)/x)^2*Log[1 + (I*c)/x]^2)/(8*c^2)
+ (b^2*Log[I + c/x])/(8*c^2) - (b^2*Log[1 - (I*c)/x]*Log[c - I*x])/(4*c^2)
- (b^2*Log[1 + (I*c)/x]*Log[c + I*x])/(4*c^2) + (b^2*Log[(c - I*x)/(2*c)]*L
og[c + I*x])/(4*c^2) + (b^2*Log[c - I*x]*Log[(c + I*x)/(2*c)])/(4*c^2) - (b
^2*Log[c + I*x]*Log[((-I)*x)/c])/(4*c^2) - (b^2*Log[c - I*x]*Log[(I*x)/c])/
(4*c^2) + (b^2*PolyLog[2, (c - I*x)/(2*c)])/(4*c^2) + (b^2*PolyLog[2, (c +
I*x)/(2*c)])/(4*c^2) + (b^2*PolyLog[2, ((-I)*c)/x])/(4*c^2) + (b^2*PolyLog[
2, (I*c)/x])/(4*c^2) - (b^2*PolyLog[2, 1 - (I*x)/c])/(4*c^2) - (b^2*PolyLog
[2, 1 + (I*x)/c])/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4x^3} + \frac{b\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x^3} - \frac{b^2\log^2\left(1+\frac{i}{x}\right.}{4x^3}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{x^3}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{x^3}\,dx - \frac{1}{4}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{2}b\int \left(-\frac{2ia\log\left(1+\frac{ic}{x}\right)}{x^3} + \frac{b\log}{\right.}\right.$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \left(-\frac{i(2a+ib\log(1-icx))^2}{c} + \frac{i(1-icx)(2a+ib\log(1-icx))^2}{c}\right)dx, x\right.\right.$$

$$= -\frac{b^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + (iab)\,\text{Subst}\left(\int x\log(1+icx)\,dx, x, \frac{1}{x}\right) - \frac{1}{2}b^2\int \frac{c\log}{2(}$$

$$= \frac{iab\log\left(1+\frac{ic}{x}\right)}{2x^2} - \frac{b^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} - \frac{\text{Subst}\left(\int(2a+ib\log(x))^2\,dx, x, 1-\right.}{4c^2}$$

$$= -\frac{\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c^2} + \frac{\left(1-\frac{ic}{x}\right)^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c^2} + \frac{iab\log\left(1+\frac{i}{}\right.}{2x^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{ib^2}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{ib\left(1-\frac{ic}{x}\right)^2\left(2a\right.}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{b^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right.}{2c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{b^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right.}{2c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{3b^2\left(1-\frac{ic}{x}\right)\log\left(1-\right.}{4c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\frac{c}{x}\right)}{8c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\frac{c}{x}\right)}{8c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\frac{c}{x}\right)}{8c^2}$$

Mathematica [A] time = 0.08, size = 99, normalized size = 1.18

$$-\frac{a^2c^2 - 2abx^2\tan^{-1}\left(\frac{x}{c}\right) - 2abcx + 2bc\tan^{-1}\left(\frac{c}{x}\right)(ac-bx) + b^2x^2\log\left(c^2+x^2\right) + b^2\left(c^2+x^2\right)\tan^{-1}\left(\frac{c}{x}\right)^2 - 2b^2}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] -1/2*(a^2*c^2 - 2*a*b*c*x + 2*b*c*(a*c - b*x)*ArcTan[c/x] + b^2*(c^2 + x^2)
*ArcTan[c/x]^2 - 2*a*b*x^2*ArcTan[x/c] - 2*b^2*x^2*Log[x] + b^2*x^2*Log[c^2
 + x^2])/(c^2*x^2)
```

fricas [A] time = 0.47, size = 109, normalized size = 1.30

$$\frac{2\,abx^2\arctan\left(\frac{x}{c}\right)-b^2x^2\log\left(c^2+x^2\right)+2\,b^2x^2\log(x)-a^2c^2+2\,abcx-\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2-2\left(abc^2-b^2c\right.}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x^2*arctan(x/c) - b^2*x^2*log(c^2 + x^2) + 2*b^2*x^2*log(x) - a^
2*c^2 + 2*a*b*c*x - (b^2*c^2 + b^2*x^2)*arctan(c/x)^2 - 2*(a*b*c^2 - b^2*c*
x)*arctan(c/x))/(c^2*x^2)
```

giac [A] time = 0.23, size = 140, normalized size = 1.67

$$-\frac{b^2\arctan\left(\frac{c}{x}\right)^2+\frac{b^2c^2\arctan\left(\frac{c}{x}\right)^2}{x^2}+abi\log\left(\frac{ci}{x}-1\right)-abi\log\left(-\frac{ci}{x}-1\right)+\frac{2\,abc^2\arctan\left(\frac{c}{x}\right)}{x^2}-\frac{2\,b^2c\arctan\left(\frac{c}{x}\right)}{x}+b^2\log\left(\frac{ci}{x}-\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(c/x)^2 + b^2*c^2*arctan(c/x)^2/x^2 + a*b*i*log(c*i/x - 1)
- a*b*i*log(-c*i/x - 1) + 2*a*b*c^2*arctan(c/x)/x^2 - 2*b^2*c*arctan(c/x)/x
 + b^2*log(c*i/x - 1) + b^2*log(-c*i/x - 1) + a^2*c^2/x^2 - 2*a*b*c/x)/c^2
```

maple [A] time = 0.05, size = 110, normalized size = 1.31

$$-\frac{a^2}{2x^2}-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}+\frac{b^2\arctan\left(\frac{c}{x}\right)}{cx}-\frac{b^2\ln\left(1+\frac{c^2}{x^2}\right)}{2c^2}-\frac{ab\arctan\left(\frac{c}{x}\right)}{x^2}+\frac{ab\arctan\left(\frac{x}{c}\right)}{c^2}+\frac{ab}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctan(c/x)^2-1/2/c^2*b^2*arctan(c/x)^2+1/c*b^2*ar
ctan(c/x)/x-1/2*b^2*ln(1+c^2/x^2)/c^2-a*b/x^2*arctan(c/x)+1/c^2*a*b*arctan(
x/c)+a*b/c/x
```

maxima [A] time = 0.43, size = 120, normalized size = 1.43

$$\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)ab+\frac{1}{2}\left(2\,c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)\arctan\left(\frac{c}{x}\right)+\frac{\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)+2}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="maxima")
```

```
[Out] (c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a*b + 1/2*(2*c*(arctan(
x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2) + 2*log
(x))/c^2)*b^2 - 1/2*b^2*arctan(c/x)^2/x^2 - 1/2*a^2/x^2
```

mupad [B] time = 2.75, size = 143, normalized size = 1.70

$$\frac{b^2\ln(x)-\frac{b^2\ln(x+c\,1\mathrm{i})}{2}-\frac{b^2\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\ln\left(\frac{1}{-x+c\,1\mathrm{i}}\right)}{2}+\frac{a\,b\ln(x+c\,1\mathrm{i})\,1\mathrm{i}}{2}-\frac{a\,b\ln(-x+c\,1\mathrm{i})\,1\mathrm{i}}{2}}{c^2}-\frac{\frac{a^2c^2}{2}-x\left(c\,\mathrm{atan}\left(\frac{c}{x}\right)b^2+a\,c\,b\right)+}{c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2/x^3,x)
```

```
[Out] (b^2*log(x) - (b^2*log(c*1i + x))/2 - (b^2*atan(c/x)^2)/2 + (b^2*log(1/(c*1
i - x)))/2 + (a*b*log(c*1i + x)*1i)/2 - (a*b*log(c*1i - x)*1i)/2)/c^2 - ((a
^2*c^2)/2 - x*(b^2*c*atan(c/x) + a*b*c) + (b^2*c^2*atan(c/x)^2)/2 + a*b*c^2
*atan(c/x))/(c^2*x^2)
```

sympy [A] time = 1.04, size = 117, normalized size = 1.39

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{x^2} + \frac{ab}{cx} - \frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{c^2} - \frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2x^2} + \frac{b^2\operatorname{atan}\left(\frac{c}{x}\right)}{cx} + \frac{b^2\log(x)}{c^2} - \frac{b^2\log\left(c^2+x^2\right)}{2c^2} - \frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2c^2} & \text{for}\ c \neq 0 \\ -\frac{a^2}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atan(c/x)/x**2 + a*b/(c*x) - a*b*atan(c/x)/
c**2 - b**2*atan(c/x)**2/(2*x**2) + b**2*atan(c/x)/(c*x) + b**2*log(x)/c**2
 - b**2*log(c**2 + x**2)/(2*c**2) - b**2*atan(c/x)**2/(2*c**2), Ne(c, 0)),
(-a**2/(2*x**2), True))
```

3.147 $\int x^3 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=214

$$2b^2c^4\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right.$$

```
[Out] 1/4*b^3*c^3*x+1/4*b^3*c^4*arccot(x/c)+1/4*b^2*c^2*x^2*(a+b*arccot(x/c))-I*b
*c^4*(a+b*arccot(x/c))^2-3/4*b*c^3*x*(a+b*arccot(x/c))^2+1/4*b*c*x^3*(a+b*a
rccot(x/c))^2-1/4*c^4*(a+b*arccot(x/c))^3+1/4*x^4*(a+b*arccot(x/c))^3+2*b^2
*c^4*(a+b*arccot(x/c))*ln(2-2/(1-I*c/x))-I*b^3*c^4*polylog(2,-1+2/(1-I*c/x)
)
```

Rubi [F] time = 4.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (-3*a^2*b*c^3*x)/8 - ((5*I)/16)*a*b^2*c^3*x + (b^3*c^3*x)/16 - ((3*I)/16)*a
^2*b*c^2*x^2 + (3*a*b^2*c^2*x^2)/16 + (a^2*b*c*x^3)/8 - (11*a*b^2*c^4*Log[I
 - c/x])/16 - (I/32)*b^3*c^4*Log[I - c/x] - ((3*I)/8)*a*b^2*c^3*x*Log[1 - (
I*c)/x] + (3*a*b^2*c^2*x^2*Log[1 - (I*c)/x])/16 + (I/8)*a*b^2*c*x^3*Log[1 -
 (I*c)/x] + ((5*I)/32)*b^2*c^3*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x])
 + (b^2*c^2*x^2*(2*a + I*b*Log[1 - (I*c)/x]))/32 + ((5*I)/64)*b*c^4*(2*a +
I*b*Log[1 - (I*c)/x])^2 - (3*b*c^3*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)
/x])^2)/32 + ((3*I)/64)*b*c^2*x^2*(2*a + I*b*Log[1 - (I*c)/x])^2 + (b*c*x^3
*(2*a + I*b*Log[1 - (I*c)/x])^2)/32 - (c^4*(2*a + I*b*Log[1 - (I*c)/x])^3)/
32 + (x^4*(2*a + I*b*Log[1 - (I*c)/x])^3)/32 + ((3*I)/4)*a*b^2*c^3*x*Log[1
+ (I*c)/x] - (5*b^3*c^3*(1 + (I*c)/x)*x*Log[1 + (I*c)/x])/32 - (I/32)*b^3*c
^2*x^2*Log[1 + (I*c)/x] - (I/4)*a*b^2*c*x^3*Log[1 + (I*c)/x] - ((3*I)/8)*a^
2*b*x^4*Log[1 + (I*c)/x] + (3*a*b^2*x^4*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/
8 + (3*a*b^2*c^4*Log[1 + (I*c)/x]^2)/16 + ((5*I)/64)*b^3*c^4*Log[1 + (I*c)/
x]^2 + (3*b^3*c^3*(1 + (I*c)/x)*x*Log[1 + (I*c)/x]^2)/32 + ((3*I)/64)*b^3*c
^2*x^2*Log[1 + (I*c)/x]^2 - (b^3*c*x^3*Log[1 + (I*c)/x]^2)/32 - (3*a*b^2*x^
4*Log[1 + (I*c)/x]^2)/16 - (I/32)*b^3*c^4*Log[1 + (I*c)/x]^3 + (I/32)*b^3*x
^4*Log[1 + (I*c)/x]^3 + (I/32)*b^3*c^4*Log[I + c/x] + ((3*I)/8)*a^2*b*c^4*L
og[c - I*x] - (5*a*b^2*c^4*Log[c - I*x])/16 - (3*a*b^2*c^4*Log[1 - (I*c)/x]
*Log[c - I*x])/8 - (5*a*b^2*c^4*Log[c + I*x])/16 - (3*a*b^2*c^4*Log[1 + (I*
c)/x]*Log[c + I*x])/8 + (3*a*b^2*c^4*Log[(c - I*x)/(2*c)]*Log[c + I*x])/8 +
 (3*a*b^2*c^4*Log[c - I*x]*Log[(c + I*x)/(2*c)])/8 + ((3*I)/32)*b^3*c^4*Log
[1 + (I*c)/x]^2*Log[((-I)*c)/x] + ((3*I)/32)*b*c^4*(2*a + I*b*Log[1 - (I*c)
/x])^2*Log[(I*c)/x] - (11*a*b^2*c^4*Log[x])/8 - (3*a*b^2*c^4*Log[c + I*x]*L
og[((-I)*x)/c])/8 - (3*a*b^2*c^4*Log[c - I*x]*Log[(I*x)/c])/8 - (3*b^2*c^4*
(2*a + I*b*Log[1 - (I*c)/x])*PolyLog[2, 1 - (I*c)/x])/16 + ((3*I)/16)*b^3*c
^4*Log[1 + (I*c)/x]*PolyLog[2, 1 + (I*c)/x] + (3*a*b^2*c^4*PolyLog[2, (c -
I*x)/(2*c)])/8 + (3*a*b^2*c^4*PolyLog[2, (c + I*x)/(2*c)])/8 + (3*a*b^2*c^4
*PolyLog[2, ((-I)*c)/x])/8 + ((11*I)/32)*b^3*c^4*PolyLog[2, ((-I)*c)/x] - (
(11*I)/32)*b^3*c^4*PolyLog[2, (I*c)/x] - (3*a*b^2*c^4*PolyLog[2, 1 - (I*x)/
c])/8 - (3*a*b^2*c^4*PolyLog[2, 1 + (I*x)/c])/8 + ((3*I)/16)*b^3*c^4*PolyLo
g[3, 1 - (I*c)/x] - ((3*I)/16)*b^3*c^4*PolyLog[3, 1 + (I*c)/x] + ((3*I)/8)*
b^3*Defer[Int][x^3*Log[1 - (I*c)/x]^2*Log[1 + (I*c)/x], x] - ((3*I)/8)*b^3*
Defer[Int][x^3*Log[1 - (I*c)/x]*Log[1 + (I*c)/x]^2, x]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int\left(\frac{1}{8}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{3}{8}ibx^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)-\right.$$
$$=\frac{1}{8}\int x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3 dx+\frac{1}{8}(3ib)\int x^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1\right.$$
$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^5}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3ib)\int\left(-4a^2x^3\log\left(1+\right.\right.$$
$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{1}{32}ib^3x^4\log^3\left(1+\frac{ic}{x}\right)-\frac{1}{2}\left(3ia^2b\right)\int x^3\log\left(1+\right.$$
$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1+\frac{ic}{x}\right)+\frac{3}{8}ab^2x^4\log\left(1-\frac{ic}{x}\right)\log\left(\right.$$
$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1+\frac{ic}{x}\right)+\frac{3}{8}ab^2x^4\log\left(1-\frac{ic}{x}\right)\log\left(\right.$$
$$=\frac{1}{32}bcx^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1\right.$$
$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3+\frac{3}{64}ibc^2x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{32}bcx$$
$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}iab^2c^3x\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\left(1\right.$$
$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}iab^2c^3x\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\left(1\right.$$
$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab$$
$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab$$
$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab$$
$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab$$

Mathematica [A] time = 0.65, size = 253, normalized size = 1.18

$$\frac{1}{4}\left(a^3x^4+b\tan^{-1}\left(\frac{c}{x}\right)\left(3a^2\left(x^4-c^4\right)+2abcx\left(x^2-3c^2\right)+8b^2c^4\log\left(1-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+b^2c^2\left(c^2+x^2\right)\right)-3a^2bc^3\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (a*b^2*c^4 - 3*a^2*b*c^3*x + b^3*c^3*x + a*b^2*c^2*x^2 + a^2*b*c*x^3 + a^3*
x^4 + b^2*(b*c*((-4*I)*c^3 - 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*ArcTan[c/x]
^2 + b^3*(-c^4 + x^4)*ArcTan[c/x]^3 + b*ArcTan[c/x]*(2*a*b*c*x*(-3*c^2 + x^
2) + b^2*c^2*(c^2 + x^2) + 3*a^2*(-c^4 + x^4) + 8*b^2*c^4*Log[1 - E^((2*I)*
ArcTan[c/x])]) + 8*a*b^2*c^4*Log[c/(Sqrt[1 + c^2/x^2]*x)] - (4*I)*b^3*c^4*P
olyLog[2, E^((2*I)*ArcTan[c/x])])/4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\arctan\left(\frac{c}{x}\right)^3+3\,ab^2x^3\arctan\left(\frac{c}{x}\right)^2+3\,a^2bx^3\arctan\left(\frac{c}{x}\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c/x)^3 + 3*a*b^2*x^3*arctan(c/x)^2 + 3*a^2*b*x^3*ar
ctan(c/x) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x^3, x)
```

maple [B] time = 0.13, size = 608, normalized size = 2.84

$$-\frac{c^4b^3\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{c^4b^3\arctan\left(\frac{x}{c}\right)}{4}+\frac{b^3x^4\arctan\left(\frac{c}{x}\right)^3}{4}+\frac{b^3c^3x}{4}+2c^4a\,b^2\ln\left(\frac{c}{x}\right)-\frac{3c^4a\,b^2\arctan\left(\frac{c}{x}\right)^2}{4}+\frac{3a\,b^2x^4\,\text{arct}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x))^3,x)
```

```
[Out] -1/4*c^4*b^3*arctan(c/x)^3-1/4*c^4*b^3*arctan(x/c)+1/4*b^3*x^4*arctan(c/x)^
3+1/4*b^3*c^3*x+2*c^4*a*b^2*ln(c/x)-3/4*c^4*a*b^2*arctan(c/x)^2+1/4*I*c^4*b
^3*ln(c/x-I)^2-1/2*I*c^4*b^3*dilog(1/2*I*(c/x-I))+3/4*a*b^2*x^4*arctan(c/x)
^2+3/4*a^2*b*x^4*arctan(c/x)-3/4*c^3*x*a^2*b+I*c^4*b^3*dilog(1+I*c/x)+1/4*c
*b^3*arctan(c/x)^2*x^3+1/4*c^2*b^3*arctan(c/x)*x^2-1/4*I*c^4*b^3*ln(I+c/x)^
2+1/2*I*c^4*b^3*dilog(-1/2*I*(I+c/x))+1/4*c^2*x^2*a*b^2-c^4*b^3*arctan(c/x)
*ln(1+c^2/x^2)+2*c^4*b^3*arctan(c/x)*ln(c/x)+3/4*c^4*a^2*b*arctan(x/c)-I*c^
4*b^3*dilog(1-I*c/x)+1/2*I*c^4*b^3*ln(I+c/x)*ln(1+c^2/x^2)+1/2*I*c^4*b^3*ln
(c/x-I)*ln(-1/2*I*(I+c/x))+1/2*c*a*b^2*arctan(c/x)*x^3-3/2*c^3*a*b^2*x*arct
an(c/x)+I*c^4*b^3*ln(c/x)*ln(1+I*c/x)-1/2*I*c^4*b^3*ln(c/x-I)*ln(1+c^2/x^2)
-1/2*I*c^4*b^3*ln(I+c/x)*ln(1/2*I*(c/x-I))+1/4*x^4*a^3-I*c^4*b^3*ln(c/x)*ln
(1-I*c/x)-c^4*a*b^2*ln(1+c^2/x^2)-3/4*c^3*b^3*arctan(c/x)^2*x+1/4*c*a^2*b*x
^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}\,ab^2x^4\arctan\left(\frac{c}{x}\right)^2+\frac{1}{4}\,a^3x^4+\frac{1}{4}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)a^2b+\frac{1}{4}\left(\left(3\,c^2\arctan\left(\frac{x}{c}\right)^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c/x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c/x) + (3*c^3
*arctan(x/c) - 3*c^2*x + x^3)*c)*a^2*b + 1/4*((3*c^2*arctan(x/c)^2 - 4*c^2*
log(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(
c/x))*a*b^2 - 1/64*(12*c^4*arctan(c/x)^2*arctan(x/c) + 8*c^4*arctan2(c, x)^
3 - 8*x^4*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^
3/c)*c^5 + 12*c^3*x*arctan2(c, x)^2 - 4*c*x^3*arctan2(c, x)^2 + 192*c^5*int
egrate(1/64*log(c^2 + x^2)^2/(c^2 + x^2), x) + 1536*c^4*integrate(1/64*x*ar
ctan(c/x)/(c^2 + x^2), x) + 768*c^3*integrate(1/64*x^2*log(c^2 + x^2)/(c^2
+ x^2), x) - 2048*c^2*integrate(1/64*x^3*arctan(c/x)^3/(c^2 + x^2), x) - 51
2*c^2*integrate(1/64*x^3*arctan(c/x)/(c^2 + x^2), x) - (3*c^3*x - c*x^3)*lo
g(c^2 + x^2)^2 - 768*c*integrate(1/64*x^4*arctan(c/x)^2/(c^2 + x^2), x) - 1
92*c*integrate(1/64*x^4*log(c^2 + x^2)^2/(c^2 + x^2), x) - 256*c*integrate(
1/64*x^4*log(c^2 + x^2)/(c^2 + x^2), x) - 2048*integrate(1/64*x^5*arctan(c/
x)^3/(c^2 + x^2), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c/x))^3,x)
[Out] int(x^3*(a + b*atan(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x))**3,x)
[Out] Integral(x**3*(a + b*atan(c/x))**3, x)
```

3.148 $\int x^2 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=229

$$-ib^2c^3\text{Li}_2\left(\frac{2}{1-\frac{ic}{x}}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right.$$

```
[Out] b^2*c^2*x*(a+b*arccot(x/c))+1/2*b*c^3*(a+b*arccot(x/c))^2+1/2*b*c*x^2*(a+b*
arccot(x/c))^2-1/3*I*c^3*(a+b*arccot(x/c))^3+1/3*x^3*(a+b*arccot(x/c))^3+b*
c^3*(a+b*arccot(x/c))^2*ln(2-2/(1-I*c/x))+1/2*b^3*c^3*ln(1+c^2/x^2)+b^3*c^3
*ln(x)-I*b^2*c^3*(a+b*arccot(x/c))*polylog(2,-1+2/(1-I*c/x))+1/2*b^3*c^3*po
lylog(3,-1+2/(1-I*c/x))
```

Rubi [F] time = 3.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (-I/2)*a^2*b*c^2*x + (3*a*b^2*c^2*x)/4 + (a^2*b*c*x^2)/4 - ((3*I)/4)*a*b^2*
c^3*Log[I - c/x] + (a*b^2*c^2*x*Log[1 - (I*c)/x])/2 + (I/4)*a*b^2*c*x^2*Log
[1 - (I*c)/x] + (b^2*c^2*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]))/8 +
(b*c^3*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 + (I/8)*b*c^2*(1 - (I*c)/x)*x*(2*
a + I*b*Log[1 - (I*c)/x])^2 + (b*c*x^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 +
 (I/24)*c^3*(2*a + I*b*Log[1 - (I*c)/x])^3 + (x^3*(2*a + I*b*Log[1 - (I*c)/
x])^3)/24 - (I/8)*b^3*c^2*(1 + (I*c)/x)*x*Log[1 + (I*c)/x] - (I/2)*a*b^2*c*
x^2*Log[1 + (I*c)/x] - (I/2)*a^2*b*x^3*Log[1 + (I*c)/x] + (a*b^2*x^3*Log[1
- (I*c)/x]*Log[1 + (I*c)/x])/2 + (I/4)*a*b^2*c^3*Log[1 + (I*c)/x]^2 - (b^3*
c^3*Log[1 + (I*c)/x]^2)/16 + (I/8)*b^3*c^2*(1 + (I*c)/x)*x*Log[1 + (I*c)/x]
^2 - (b^3*c*x^2*Log[1 + (I*c)/x]^2)/16 - (a*b^2*x^3*Log[1 + (I*c)/x]^2)/4 +
 (b^3*c^3*Log[1 + (I*c)/x]^3)/24 + (I/24)*b^3*x^3*Log[1 + (I*c)/x]^3 - (a^2
*b*c^3*Log[c - I*x])/2 + (I/4)*a*b^2*c^3*Log[c - I*x] - (I/2)*a*b^2*c^3*Log
[1 - (I*c)/x]*Log[c - I*x] - (I/4)*a*b^2*c^3*Log[c + I*x] + (I/2)*a*b^2*c^3
*Log[1 + (I*c)/x]*Log[c + I*x] - (I/2)*a*b^2*c^3*Log[(c - I*x)/(2*c)]*Log[c
 + I*x] + (I/2)*a*b^2*c^3*Log[c - I*x]*Log[(c + I*x)/(2*c)] - (b^3*c^3*Log[
1 + (I*c)/x]^2*Log[((-I)*c)/x])/8 + (b*c^3*(2*a + I*b*Log[1 - (I*c)/x])^2*L
og[(I*c)/x])/8 + (b^3*c^3*Log[x])/4 + (I/2)*a*b^2*c^3*Log[c + I*x]*Log[((-I
)*x)/c] - (I/2)*a*b^2*c^3*Log[c - I*x]*Log[(I*x)/c] + (I/4)*b^2*c^3*(2*a +
I*b*Log[1 - (I*c)/x])*PolyLog[2, 1 - (I*c)/x] - (b^3*c^3*Log[1 + (I*c)/x]*P
olyLog[2, 1 + (I*c)/x])/4 + (I/2)*a*b^2*c^3*PolyLog[2, (c - I*x)/(2*c)] - (
I/2)*a*b^2*c^3*PolyLog[2, (c + I*x)/(2*c)] + (I/2)*a*b^2*c^3*PolyLog[2, ((-
I)*c)/x] - (3*b^3*c^3*PolyLog[2, ((-I)*c)/x])/8 - (3*b^3*c^3*PolyLog[2, (I*
c)/x])/8 - (I/2)*a*b^2*c^3*PolyLog[2, 1 - (I*x)/c] + (I/2)*a*b^2*c^3*PolyLo
g[2, 1 + (I*x)/c] + (b^3*c^3*PolyLog[3, 1 - (I*c)/x])/4 + (b^3*c^3*PolyLog[
3, 1 + (I*c)/x])/4 + ((3*I)/8)*b^3*Defer[Int][x^2*Log[1 - (I*c)/x]^2*Log[1
+ (I*c)/x], x] - ((3*I)/8)*b^3*Defer[Int][x^2*Log[1 - (I*c)/x]*Log[1 + (I*c
)/x]^2, x]
```

Rubi steps

$$\int x^2 \left(a + b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x^2 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^3 + \frac{3}{8}ibx^2 \left(-2ia + b\log\left(1 - \frac{ic}{x}\right)\right)^2 \log\left(1 + \frac{ic}{x}\right) -\right.$$

$$= \frac{1}{8}\int x^2 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^3 dx + \frac{1}{8}(3ib)\int x^2 \left(-2ia + b\log\left(1 - \frac{ic}{x}\right)\right)^2 \log\left(1\right.$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{(2a + ib\log(1 - icx))^3}{x^4} dx, x, \frac{1}{x}\right)\right) + \frac{1}{8}(3ib)\int \left(-4a^2x^2\log\left(1 + \right.\right.$$

$$= \frac{1}{24}x^3 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^3 + \frac{1}{24}ib^3x^3\log^3\left(1 + \frac{ic}{x}\right) - \frac{1}{2}\left(3ia^2b\right)\int x^2\log\left(1 + \right.$$

$$= \frac{1}{24}x^3 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^3 - \frac{1}{2}ia^2bx^3\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}ab^2x^3\log\left(1 - \frac{ic}{x}\right)\log\left(\right.$$

$$= \frac{1}{24}x^3 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^3 - \frac{1}{2}ia^2bx^3\log\left(1 + \frac{ic}{x}\right) + \frac{1}{2}ab^2x^3\log\left(1 - \frac{ic}{x}\right)\log\left(\right.$$

$$= \frac{1}{16}bcx^2 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 + \frac{1}{24}x^3 \left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^3 - \frac{1}{2}ia^2bx^3\log\left(1\right.$$

$$= -\frac{1}{2}ia^2bc^2x + \frac{1}{4}a^2bcx^2 + \frac{1}{8}ibc^2\left(1 - \frac{ic}{x}\right)x\left(2a + ib\log\left(1 - \frac{ic}{x}\right)\right)^2 + \frac{1}{16}bcx^2\left(2a + \right.$$

$$= -\frac{1}{2}ia^2bc^2x + \frac{1}{4}a^2bcx^2 + \frac{1}{2}ab^2c^2x\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}iab^2cx^2\log\left(1 - \frac{ic}{x}\right) + \frac{1}{8}b^2c^2\left(\right.$$

$$= -\frac{1}{2}ia^2bc^2x + \frac{1}{4}a^2bcx^2 + \frac{1}{2}ab^2c^2x\log\left(1 - \frac{ic}{x}\right) + \frac{1}{4}iab^2cx^2\log\left(1 - \frac{ic}{x}\right) + \frac{1}{8}b^2c^2\left(\right.$$

$$= -\frac{1}{2}ia^2bc^2x + \frac{1}{4}ab^2c^2x + \frac{1}{4}a^2bcx^2 - \frac{3}{4}iab^2c^3\log\left(i - \frac{c}{x}\right) + \frac{1}{2}ab^2c^2x\log\left(1 - \frac{ic}{x}\right)$$

$$= -\frac{1}{2}ia^2bc^2x + \frac{3}{4}ab^2c^2x + \frac{1}{4}a^2bcx^2 - \frac{3}{4}iab^2c^3\log\left(i - \frac{c}{x}\right) + \frac{1}{2}ab^2c^2x\log\left(1 - \frac{ic}{x}\right)$$

$$= -\frac{1}{2}ia^2bc^2x + \frac{3}{4}ab^2c^2x + \frac{1}{4}a^2bcx^2 - \frac{3}{4}iab^2c^3\log\left(i - \frac{c}{x}\right) + \frac{1}{2}ab^2c^2x\log\left(1 - \frac{ic}{x}\right)$$

$$= -\frac{1}{2}ia^2bc^2x + \frac{3}{4}ab^2c^2x + \frac{1}{4}a^2bcx^2 - \frac{3}{4}iab^2c^3\log\left(i - \frac{c}{x}\right) + \frac{1}{2}ab^2c^2x\log\left(1 - \frac{ic}{x}\right)$$

Mathematica [A] time = 0.73, size = 330, normalized size = 1.44

$$\frac{a^3x^3}{3} - \frac{1}{2}a^2bc^3\log\left(c^2 + x^2\right) + a^2bx^3\tan^{-1}\left(\frac{c}{x}\right) + \frac{1}{2}a^2bcx^2 + ab^2\left(-ic^3\text{Li}_2\left(e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right) + \left(x^3 - ic^3\right)\tan^{-1}\left(\frac{c}{x}\right)^2 + c\,\text{t}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (a^2*b*c*x^2)/2 + (a^3*x^3)/3 + a^2*b*x^3*ArcTan[c/x] - (a^2*b*c^3*Log[c^2
+ x^2])/2 + a*b^2*(c^2*x + ((-I)*c^3 + x^3)*ArcTan[c/x]^2 + c*ArcTan[c/x]*(
```

```
c^2 + x^2 + 2*c^2*Log[1 - E^((2*I)*ArcTan[c/x])]) - I*c^3*PolyLog[2, E^((2*
I)*ArcTan[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTan[c/x] + 12*c^3*Arc
Tan[c/x]^2 + 12*c*x^2*ArcTan[c/x]^2 + (8*I)*c^3*ArcTan[c/x]^3 + 8*x^3*ArcTa
n[c/x]^3 + 24*c^3*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c/x])] - 24*c^3*Lo
g[c/(Sqrt[1 + c^2/x^2]*x)] + (24*I)*c^3*ArcTan[c/x]*PolyLog[2, E^((-2*I)*Ar
cTan[c/x])] + 12*c^3*PolyLog[3, E^((-2*I)*ArcTan[c/x])]))/24
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\arctan\left(\frac{c}{x}\right)^3 + 3\,ab^2x^2\arctan\left(\frac{c}{x}\right)^2 + 3\,a^2bx^2\arctan\left(\frac{c}{x}\right) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c/x)^3 + 3*a*b^2*x^2*arctan(c/x)^2 + 3*a^2*b*x^2*ar
ctan(c/x) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(\frac{c}{x}\right) + a\right)^3 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x^2, x)
```

maple [C] time = 1.38, size = 6441, normalized size = 28.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}\,b^3x^3\arctan\left(c,x\right)^3 - \frac{1}{32}\,b^3x^3\arctan\left(c,x\right)\log\left(c^2+x^2\right)^2 + \frac{1}{3}\,a^3x^3 + \frac{1}{2}\left(2\,x^3\arctan\left(\frac{c}{x}\right) - \left(c^2\log\left(c^2+x^2\right) - x^2\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan2(c, x)^3 - 1/32*b^3*x^3*arctan2(c, x)*log(c^2 + x^2)^2
+ 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a^2*
b + integrate(1/32*(4*b^3*c*x^3*arctan2(c, x)^2 + 4*b^3*x^4*arctan2(c, x)*l
og(c^2 + x^2) + 4*(7*b^3*arctan2(c, x)^3 + 24*a*b^2*arctan2(c, x)^2)*x^4 +
4*(7*b^3*c^2*arctan2(c, x)^3 + 24*a*b^2*c^2*arctan2(c, x)^2)*x^2 + (3*b^3*c
^2*x^2*arctan2(c, x) + 3*b^3*x^4*arctan2(c, x) - b^3*c*x^3)*log(c^2 + x^2)^
2)/(c^2 + x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c/x))^3,x)
```

```
[Out] int(x^2*(a + b*atan(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atan(c/x))**3, x)
```

3.149 $\int x \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal. Leaf size=145

$$-3b^2c^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right.\right.$$

```
[Out] 3/2*I*b*c^2*(a+b*arccot(x/c))^2+3/2*b*c*x*(a+b*arccot(x/c))^2+1/2*c^2*(a+b*
arccot(x/c))^3+1/2*x^2*(a+b*arccot(x/c))^3-3*b^2*c^2*(a+b*arccot(x/c))*ln(2
-2/(1-I*c/x))+3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c/x))
```

Rubi [F] time = 2.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x)/4 + (3*a*b^2*c^2*Log[I - c/x])/4 + ((3*I)/4)*a*b^2*c*x*Log[1
 - (I*c)/x] + (3*b*c*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 + (c
^2*(2*a + I*b*Log[1 - (I*c)/x])^3)/16 + (x^2*(2*a + I*b*Log[1 - (I*c)/x])^3
)/16 - ((3*I)/2)*a*b^2*c*x*Log[1 + (I*c)/x] - ((3*I)/4)*a^2*b*x^2*Log[1 + (
I*c)/x] + (3*a*b^2*x^2*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/4 - (3*a*b^2*c^2*
Log[1 + (I*c)/x]^2)/8 - (3*b^3*c*(1 + (I*c)/x)*x*Log[1 + (I*c)/x]^2)/16 - (
3*a*b^2*x^2*Log[1 + (I*c)/x]^2)/8 + (I/16)*b^3*c^2*Log[1 + (I*c)/x]^3 + (I/
16)*b^3*x^2*Log[1 + (I*c)/x]^3 - ((3*I)/4)*a^2*b*c^2*Log[c - I*x] + (3*a*b^
2*c^2*Log[c - I*x])/4 + (3*a*b^2*c^2*Log[1 - (I*c)/x]*Log[c - I*x])/4 + (3*
a*b^2*c^2*Log[c + I*x])/4 + (3*a*b^2*c^2*Log[1 + (I*c)/x]*Log[c + I*x])/4 -
 (3*a*b^2*c^2*Log[(c - I*x)/(2*c)]*Log[c + I*x])/4 - (3*a*b^2*c^2*Log[c - I
*x]*Log[(c + I*x)/(2*c)])/4 - ((3*I)/16)*b^3*c^2*Log[1 + (I*c)/x]^2*Log[((-
I)*c)/x] - ((3*I)/16)*b*c^2*(2*a + I*b*Log[1 - (I*c)/x])^2*Log[(I*c)/x] + (
3*a*b^2*c^2*Log[x])/2 + (3*a*b^2*c^2*Log[c + I*x]*Log[((-I)*x)/c])/4 + (3*a
*b^2*c^2*Log[c - I*x]*Log[(I*x)/c])/4 + (3*b^2*c^2*(2*a + I*b*Log[1 - (I*c)
/x])*PolyLog[2, 1 - (I*c)/x])/8 - ((3*I)/8)*b^3*c^2*Log[1 + (I*c)/x]*PolyLo
g[2, 1 + (I*c)/x] - (3*a*b^2*c^2*PolyLog[2, (c - I*x)/(2*c)])/4 - (3*a*b^2*
c^2*PolyLog[2, (c + I*x)/(2*c)])/4 - (3*a*b^2*c^2*PolyLog[2, ((-I)*c)/x])/4
 - ((3*I)/8)*b^3*c^2*PolyLog[2, ((-I)*c)/x] + ((3*I)/8)*b^3*c^2*PolyLog[2,
(I*c)/x] + (3*a*b^2*c^2*PolyLog[2, 1 - (I*x)/c])/4 + (3*a*b^2*c^2*PolyLog[2
, 1 + (I*x)/c])/4 - ((3*I)/8)*b^3*c^2*PolyLog[3, 1 - (I*c)/x] + ((3*I)/8)*b
^3*c^2*PolyLog[3, 1 + (I*c)/x] + ((3*I)/8)*b^3*Defer[Int][x*Log[1 - (I*c)/x
]^2*Log[1 + (I*c)/x], x] - ((3*I)/8)*b^3*Defer[Int][x*Log[1 - (I*c)/x]*Log[
1 + (I*c)/x]^2, x]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3dx = \int \Big(\frac{1}{8}x\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^3+\frac{3}{8}ibx\Big(-2ia+b\log\Big(1-\frac{ic}{x}\Big)\Big)^2\log\Big(1+\frac{ic}{x}\Big)-\frac{3}{8}i$$

$$=\frac{1}{8}\int x\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^3dx+\frac{1}{8}(3ib)\int x\Big(-2ia+b\log\Big(1-\frac{ic}{x}\Big)\Big)^2\log\Big(1+$$

$$=-\Big(\frac{1}{8}\text{Subst}\Big(\int\frac{(2a+ib\log(1-icx))^3}{x^3}dx,x,\frac{1}{x}\Big)\Big)+\frac{1}{8}(3ib)\int\Big(-4a^2x\log\Big(1+\frac{ic}{x}$$

$$=\frac{1}{16}x^2\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^3+\frac{1}{16}ib^3x^2\log^3\Big(1+\frac{ic}{x}\Big)-\frac{1}{2}\big(3ia^2b\big)\int x\log\Big(1+\frac{ic}{x}$$

$$=\frac{1}{16}x^2\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^3-\frac{3}{4}ia^2bx^2\log\Big(1+\frac{ic}{x}\Big)+\frac{3}{4}ab^2x^2\log\Big(1-\frac{ic}{x}\Big)\log\Big(1$$

$$=\frac{1}{16}x^2\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^3-\frac{3}{4}ia^2bx^2\log\Big(1+\frac{ic}{x}\Big)+\frac{3}{4}ab^2x^2\log\Big(1-\frac{ic}{x}\Big)\log\Big(1$$

$$=\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^2+\frac{1}{16}x^2\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^3-\frac{3}{4}ia^2bx^2$$

$$=\frac{3}{4}a^2bcx+\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^2+\frac{1}{16}x^2\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^3$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}iab^2cx\log\Big(1-\frac{ic}{x}\Big)+\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^2+\frac{1}{16}c^2$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}iab^2cx\log\Big(1-\frac{ic}{x}\Big)+\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log\Big(1-\frac{ic}{x}\Big)\Big)^2+\frac{1}{16}c^2$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\Big(i-\frac{c}{x}\Big)+\frac{3}{4}iab^2cx\log\Big(1-\frac{ic}{x}\Big)+\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\Big(i-\frac{c}{x}\Big)+\frac{3}{4}iab^2cx\log\Big(1-\frac{ic}{x}\Big)+\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\Big(i-\frac{c}{x}\Big)+\frac{3}{4}iab^2cx\log\Big(1-\frac{ic}{x}\Big)+\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\Big(i-\frac{c}{x}\Big)+\frac{3}{4}iab^2cx\log\Big(1-\frac{ic}{x}\Big)+\frac{3}{16}bc\Big(1-\frac{ic}{x}\Big)x\Big(2a+ib\log$$

Mathematica [A] time = 0.29, size = 174, normalized size = 1.20

$$\frac{1}{2}\Bigg(a\Bigg(ax(ax+3bc)-6b^2c^2\log\Bigg(\frac{c}{x\sqrt{\frac{c^2}{x^2}+1}}\Bigg)\Bigg)+3b^2\tan^{-1}\Big(\frac{c}{x}\Big)^2\big(a\big(c^2+x^2\big)+bc(x+ic)\big)+3b\tan^{-1}\Big(\frac{c}{x}\Big)\Big(a\Big(a\Big($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (3*b^2*(b*c*(I*c + x) + a*(c^2 + x^2))*ArcTan[c/x]^2 + b^3*(c^2 + x^2)*ArcT
an[c/x]^3 + 3*b*ArcTan[c/x]*(a*(2*b*c*x + a*(c^2 + x^2)) - 2*b^2*c^2*Log[1
```

```
- E^((2*I)*ArcTan[c/x])]) + a*(a*x*(3*b*c + a*x) - 6*b^2*c^2*Log[c/(Sqrt[1
+ c^2/x^2]*x)]) + (3*I)*b^3*c^2*PolyLog[2, E^((2*I)*ArcTan[c/x])])/2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\arctan\left(\frac{c}{x}\right)^3 + 3\,ab^2x\arctan\left(\frac{c}{x}\right)^2 + 3\,a^2bx\arctan\left(\frac{c}{x}\right) + a^3x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c/x)^3 + 3*a*b^2*x*arctan(c/x)^2 + 3*a^2*b*x*arctan(c
/x) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\arctan\left(\frac{c}{x}\right) + a\right)^3 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x, x)
```

maple [B] time = 0.12, size = 507, normalized size = 3.50

$$\frac{c^2b^3\arctan\left(\frac{c}{x}\right)^3}{2} + \frac{b^3x^2\arctan\left(\frac{c}{x}\right)^3}{2} + \frac{x^2a^3}{2} - \frac{3ic^2b^3\operatorname{dilog}\left(1+\frac{ic}{x}\right)}{2} - \frac{3ic^2b^3\ln\left(\frac{c}{x}-i\right)^2}{8} + \frac{3ic^2b^3\ln\left(i+\frac{c}{x}\right)^2}{8} + \frac{3ic^2b^3\,\mathrm{d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x))^3,x)
```

```
[Out] 1/2*c^2*b^3*arctan(c/x)^3+1/2*b^3*x^2*arctan(c/x)^3+1/2*x^2*a^3-3/4*I*c^2*b
^3*ln(c/x-I)*ln(-1/2*I*(I+c/x))+3/2*I*c^2*b^3*ln(c/x)*ln(1-I*c/x)+3/4*I*c^2
*b^3*ln(c/x-I)*ln(1+c^2/x^2)+3/4*I*c^2*b^3*ln(I+c/x)*ln(1/2*I*(c/x-I))+3*c*
a*b^2*x*arctan(c/x)-3/2*I*c^2*b^3*ln(c/x)*ln(1+I*c/x)-3/4*I*c^2*b^3*ln(I+c/
x)*ln(1+c^2/x^2)+3/2*c^2*b^3*arctan(c/x)*ln(1+c^2/x^2)-3*c^2*b^3*arctan(c/x
)*ln(c/x)-3*c^2*a*b^2*ln(c/x)+3/2*c^2*a*b^2*ln(1+c^2/x^2)+3/2*I*c^2*b^3*dil
og(1-I*c/x)-3/8*I*c^2*b^3*ln(c/x-I)^2+3/4*I*c^2*b^3*dilog(1/2*I*(c/x-I))+3/
8*I*c^2*b^3*ln(I+c/x)^2-3/4*I*c^2*b^3*dilog(-1/2*I*(I+c/x))-3/2*I*c^2*b^3*d
ilog(1+I*c/x)+3/2*c*x*a^2*b+3/2*a^2*b*x^2*arctan(c/x)+3/2*a*b^2*x^2*arctan(
c/x)^2-3/2*c^2*a^2*b*arctan(x/c)+3/2*c^2*a*b^2*arctan(c/x)^2+3/2*c*b^3*arct
an(c/x)^2*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}\,ab^2x^2\arctan\left(\frac{c}{x}\right)^2 + \frac{1}{2}\,a^3x^2 + \frac{3}{2}\left(x^2\arctan\left(\frac{c}{x}\right) - \left(c\arctan\left(\frac{x}{c}\right) - x\right)c\right)a^2b - \frac{3}{2}\left(\left(\arctan\left(\frac{x}{c}\right)^2 - \log\left(c^2+x^2\right)\right)c^2 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctan(c/x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c/x) - (c*arcta
n(x/c) - x)*c)*a^2*b - 3/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arc
tan(x/c) - x)*c*arctan(c/x))*a*b^2 + 1/32*(12*c^2*arctan(c/x)^2*arctan(x/c)
 + 8*c^2*arctan2(c, x)^3 + 8*x^2*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(
x/c)^2/c + arctan(x/c)^3/c)*c^3 + 12*c*x*arctan2(c, x)^2 + 96*c^3*integrate
```

```
(1/32*log(c^2 + x^2)^2/(c^2 + x^2), x) - 3*c*x*log(c^2 + x^2)^2 + 512*c^2*i
ntegrate(1/32*x*arctan(c/x)^3/(c^2 + x^2), x) + 768*c^2*integrate(1/32*x*ar
ctan(c/x)/(c^2 + x^2), x) + 384*c*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x
^2), x) + 96*c*integrate(1/32*x^2*log(c^2 + x^2)^2/(c^2 + x^2), x) + 384*c*
integrate(1/32*x^2*log(c^2 + x^2)/(c^2 + x^2), x) + 512*integrate(1/32*x^3*
arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c/x))^3,x)
```

```
[Out] int(x*(a + b*atan(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c/x))**3, x)
```

3.150 $\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=119

$$3ib^2c\text{Li}_2\left(1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\right.$$

```
[Out] I*c*(a+b*arccot(x/c))^3+x*(a+b*arccot(x/c))^3-3*b*c*(a+b*arccot(x/c))^2*ln(
2*c/(c+I*x))+3*I*b^2*c*(a+b*arccot(x/c))*polylog(2,1-2*c/(c+I*x))-3/2*b^3*c
*polylog(3,1-2*c/(c+I*x))
```

Rubi [F] time = 0.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] a^3*x + ((3*I)/2)*a^2*b*x*Log[1 - (I*c)/x] + (3*a*b^2*(I*c - x)*Log[1 - (I*
c)/x]^2)/4 + (I/8)*b^3*(I*c - x)*Log[1 - (I*c)/x]^3 - ((3*I)/2)*a^2*b*x*Log
[1 + (I*c)/x] + (3*a*b^2*x*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/2 - (3*a*b^2*
(I*c + x)*Log[1 + (I*c)/x]^2)/4 + (I/8)*b^3*(I*c + x)*Log[1 + (I*c)/x]^3 -
((3*I)/2)*a*b^2*c*Log[1 + (I*c)/x]*Log[-c - I*x] + (3*a^2*b*c*Log[c - I*x])
/2 + ((3*I)/2)*a*b^2*c*Log[-c - I*x]*Log[(c - I*x)/(2*c)] + ((3*I)/2)*a*b^2
*c*Log[1 - (I*c)/x]*Log[-c + I*x] + (3*a^2*b*c*Log[c + I*x])/2 - ((3*I)/2)*
a*b^2*c*Log[-c + I*x]*Log[(c + I*x)/(2*c)] + (3*b^3*c*Log[1 + (I*c)/x]^2*Lo
g[((-I)*c)/x])/8 + (3*b^3*c*Log[1 - (I*c)/x]^2*Log[(I*c)/x])/8 - ((3*I)/2)*
a*b^2*c*Log[-c - I*x]*Log[((-I)*x)/c] + ((3*I)/2)*a*b^2*c*Log[-c + I*x]*Log
[(I*x)/c] + (3*b^3*c*Log[1 - (I*c)/x]*PolyLog[2, 1 - (I*c)/x])/4 + (3*b^3*c
*Log[1 + (I*c)/x]*PolyLog[2, 1 + (I*c)/x])/4 - ((3*I)/2)*a*b^2*c*PolyLog[2,
 (c - I*x)/(2*c)] + ((3*I)/2)*a*b^2*c*PolyLog[2, (c + I*x)/(2*c)] - ((3*I)/
2)*a*b^2*c*PolyLog[2, ((-I)*c)/x] + ((3*I)/2)*a*b^2*c*PolyLog[2, (I*c)/x] +
 ((3*I)/2)*a*b^2*c*PolyLog[2, 1 - (I*x)/c] - ((3*I)/2)*a*b^2*c*PolyLog[2, 1
 + (I*x)/c] - (3*b^3*c*PolyLog[3, 1 - (I*c)/x])/4 - (3*b^3*c*PolyLog[3, 1 +
 (I*c)/x])/4 + ((3*I)/8)*b^3*Defer[Int][Log[1 - (I*c)/x]^2*Log[1 + (I*c)/x]
, x] - ((3*I)/8)*b^3*Defer[Int][Log[1 - (I*c)/x]*Log[1 + (I*c)/x]^2, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3\,dx = \int \left(a^3+\frac{3}{2}ia^2b\log\left(1-\frac{ic}{x}\right)-\frac{3}{4}ab^2\log^2\left(1-\frac{ic}{x}\right)-\frac{1}{8}ib^3\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2b\log$$
$$= a^3x+\frac{1}{2}\left(3ia^2b\right)\int\log\left(1-\frac{ic}{x}\right)dx-\frac{1}{2}\left(3ia^2b\right)\int\log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}\left(3ab^2\right)\int\text{lo}$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right.$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)$$
$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)$$

Mathematica [A] time = 0.27, size = 215, normalized size = 1.81

$$a^3x+\frac{3}{2}a^2bc\log\left(c^2+x^2\right)+3a^2bx\tan^{-1}\left(\frac{c}{x}\right)-3ab^2\left(-ic\text{Li}_2\left(e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)-\left((x+ic)\tan^{-1}\left(\frac{c}{x}\right)^2\right)+2c\tan^{-1}\left(\frac{c}{x}\right)\text{lo}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c/x] + (3*a^2*b*c*Log[c^2 + x^2])/2 - 3*a*b^2*(-((
I*c + x)*ArcTan[c/x]^2) + 2*c*ArcTan[c/x]*Log[1 - E^((2*I)*ArcTan[c/x])] -
I*c*PolyLog[2, E^((2*I)*ArcTan[c/x])]) - (b^3*((-I)*c*Pi^3 + (8*I)*c*ArcTan
[c/x]^3 - 8*x*ArcTan[c/x]^3 + 24*c*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c
/x])] + (24*I)*c*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] + 12*c*Poly
Log[3, E^((-2*I)*ArcTan[c/x])]))/8
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\arctan\left(\frac{c}{x}\right)^3+3\,ab^2\arctan\left(\frac{c}{x}\right)^2+3\,a^2b\arctan\left(\frac{c}{x}\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3, x)
```

maple [C] time = 0.35, size = 2363, normalized size = 19.86

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3,x)
```

```
[Out] 3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*csgn(I*(
1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/
x^2)+1)^2)-3/2*I*c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*
c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c
^2/x^2)+1))*arctan(c/x)^2+3*a^2*b*x*arctan(c/x)+3*a*b^2*x*arctan(c/x)^2-3*c
*b^3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2))+3/2*c*b^3*arctan(c/x)^2*
ln(1+c^2/x^2)-3*c*b^3*ln(c/x)*arctan(c/x)^2-3*c*b^3*arctan(c/x)^2*ln(1-(1+I
*c/x)/(1+c^2/x^2)^(1/2))-3*c*b^3*arctan(c/x)^2*ln(1+(1+I*c/x)/(1+c^2/x^2)^(
1/2))+3*c*b^3*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)+3/2*c*a^2*b*ln(1+
c^2/x^2)-3*c*b^3*ln(2)*arctan(c/x)^2+I*c*b^3*arctan(c/x)^3-3*c*a^2*b*ln(c/x
)-3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*csgn(I
*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2-3/4*I*c*b^3*arcta
n(c/x)^2*Pi*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/
((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2-3/2*I*c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/
x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I
*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^2+3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*(
1+I*c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))+3/2*I*c*b^3*P
i*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+
I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-3/4*I*
c*b^3*arctan(c/x)^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1))^2*csgn(I*((1+I*c
/x)^2/(1+c^2/x^2)+1)^2)+3/2*I*c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*
csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/
x)^2+3/2*I*c*b^3*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-3/2*I*c*b^3*ar
ctan(c/x)^2*Pi*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))*csgn(I*(1+I*c/x)^2/(1+c^
2/x^2))^2+3/2*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1))*
csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2+3/2*I*c*a*b^2*dilog(1/2*I*(c/x-I))+
6*I*c*b^3*arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*I*c*a*b^2*d
ilog(1+I*c/x)+3*I*c*a*b^2*dilog(1-I*c/x)+3*c*a*b^2*arctan(c/x)*ln(1+c^2/x^2
)-6*c*a*b^2*ln(c/x)*arctan(c/x)-3/2*I*c*b^3*Pi*arctan(c/x)^2-3/4*I*c*a*b^2*
ln(c/x-I)^2-3/2*I*c*a*b^2*dilog(-1/2*I*(I+c/x))+3/4*I*c*a*b^2*ln(I+c/x)^2+6
*I*c*b^3*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-3/2*I*c*a*b^2*l
n(I+c/x)*ln(1+c^2/x^2)-3/2*I*c*b^3*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+
I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2+3/2*I*c*b^3*Pi*csgn(((1+I*c/x)^2/(
1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2+3/2*I*c*a*b^2*ln
(I+c/x)*ln(1/2*I*(c/x-I))-3*I*c*a*b^2*ln(c/x)*ln(1+I*c/x)+3*I*c*a*b^2*ln(c/
x)*ln(1-I*c/x)-3/2*I*c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)
^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2-3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*((1+
I*c/x)^2/(1+c^2/x^2)+1)^2)^3+3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*(1+I*c/x)^
2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^3+3/4*I*c*b^3*arctan(c/x)^2*Pi
*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^3+3/2*I*c*a*b^2*ln(c/x-I)*ln(1+c^2/x^2)-6*
c*b^3*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*c*b^3*polylog(3,-(1+I*c/x)/(
```

```
1+c^2/x^2)^(1/2))+b^3*x*arctan(c/x)^3-3/2*I*c*a*b^2*ln(c/x-I)*ln(-1/2*I*(I+
c/x))+x*a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{7}{8}\,b^3c\arctan\left(\frac{c}{x}\right)^3\arctan\left(\frac{x}{c}\right)+3\,ab^2c\arctan\left(\frac{c}{x}\right)^2\arctan\left(\frac{x}{c}\right)+\frac{1}{8}\,b^3x\arctan\left(c,x\right)^3-\frac{3}{32}\,b^3x\arctan\left(c,x\right)\log\left(c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c*arctan(c/x)^3*arctan(x/c) + 3*a*b^2*c*arctan(c/x)^2*arctan(x/c) +
 1/8*b^3*x*arctan2(c, x)^3 - 3/32*b^3*x*arctan2(c, x)*log(c^2 + x^2)^2 + (3
*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2*c^2 + 7/32*(6*arctan(
c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arctan(x/c)^4/c)*b
^3*c^2 + 3*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2 + x^2),
 x) + 12*b^3*c*integrate(1/32*x*arctan(c/x)^2/(c^2 + x^2), x) - 3*b^3*c*int
egrate(1/32*x*log(c^2 + x^2)^2/(c^2 + x^2), x) + 3/2*(2*x*arctan(c/x) + c*l
og(c^2 + x^2))*a^2*b + a^3*x + 28*b^3*integrate(1/32*x^2*arctan(c/x)^3/(c^2
 + x^2), x) + 3*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^2/(c^2 +
x^2), x) + 96*a*b^2*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 12*b
^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)/(c^2 + x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3,x)
```

```
[Out] int((a + b*atan(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atan}{\left(\frac{c}{x} \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3,x)
```

```
[Out] Integral((a + b*atan(c/x))**3, x)
```

3.151 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=230

$$\frac{3}{2}b^2\mathrm{Li}_3\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}b^2\mathrm{Li}_3\left(\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ib\mathrm{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right.\right.$$

```
[Out] 2*(a+b*arccot(x/c))^3*arctanh(-1+2/(1+I*c/x))+3/2*I*b*(a+b*arccot(x/c))^2*p
olylog(2,1-2/(1+I*c/x))-3/2*I*b*(a+b*arccot(x/c))^2*polylog(2,-1+2/(1+I*c/x
))+3/2*b^2*(a+b*arccot(x/c))*polylog(3,1-2/(1+I*c/x))-3/2*b^2*(a+b*arccot(x
/c))*polylog(3,-1+2/(1+I*c/x))-3/4*I*b^3*polylog(4,1-2/(1+I*c/x))+3/4*I*b^3
*polylog(4,-1+2/(1+I*c/x))
```

Rubi [A] time = 0.49, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5031, 4850, 4988, 4884, 4994, 4998, 6610}

$$\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ib\text{PolyLog}\left(2,1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcCot[x/c])^3*ArcTanh[1 - 2/(1 + (I*c)/x)] + ((3*I)/2)*b*(a + b*
ArcCot[x/c])^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - ((3*I)/2)*b*(a + b*ArcCot[
x/c])^2*PolyLog[2, -1 + 2/(1 + (I*c)/x)] + (3*b^2*(a + b*ArcCot[x/c])*PolyL
og[3, 1 - 2/(1 + (I*c)/x)])/2 - (3*b^2*(a + b*ArcCot[x/c])*PolyLog[3, -1 +
2/(1 + (I*c)/x)])/2 - ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + (I*c)/x)] + ((3*I
)/4)*b^3*PolyLog[4, -1 + 2/(1 + (I*c)/x)]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
```

```
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+(6bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tan}{1+c^2x^2}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)-(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log}{1+c^2x^2}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right.$$

Mathematica [A] time = 0.22, size = 219, normalized size = 0.95

$$\frac{3}{4}ib\left(2\text{Li}_2\left(\frac{c+ix}{c-ix}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2-2\text{Li}_2\left(\frac{x-ic}{ic+x}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2+b\left(-2i\text{Li}_3\left(\frac{c+ix}{c-ix}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcTan[c/x])^3*ArcTanh[(c + I*x)/(c - I*x)] + ((3*I)/4)*b*(2*(a +
 b*ArcTan[c/x])^2*PolyLog[2, (c + I*x)/(c - I*x)] - 2*(a + b*ArcTan[c/x])^2
```

```
*PolyLog[2, ((-I)*c + x)/(I*c + x)] + b*((-2*I)*(a + b*ArcTan[c/x])*PolyLog
[3, (c + I*x)/(c - I*x)] + (2*I)*(a + b*ArcTan[c/x])*PolyLog[3, ((-I)*c + x
)/(I*c + x)] + b*(-PolyLog[4, (c + I*x)/(c - I*x)] + PolyLog[4, ((-I)*c + x
)/(I*c + x)])))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(\frac{c}{x}\right)^3+3\,ab^2\arctan\left(\frac{c}{x}\right)^2+3\,a^2b\arctan\left(\frac{c}{x}\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3/x, x)
```

maple [C] time = 0.24, size = 2542, normalized size = 11.05

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x,x)
```

```
[Out] -3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+
c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1)
)*arctan(c/x)^2-3/2*I*a^2*b*dilog(1+I*c/x)+3/2*I*a^2*b*dilog(1-I*c/x)-1/2*I
*b^3*Pi*arctan(c/x)^3+3*I*b^3*arctan(c/x)^2*polylog(2,-(1+I*c/x)/(1+c^2/x^2
)^(1/2))-3/2*I*b^3*arctan(c/x)^2*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))+3*I*b^
3*arctan(c/x)^2*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*a*b^2*ln(c/x)*arct
an(c/x)^2+3*a*b^2*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-3*a*b^2*arcta
n(c/x)^2*ln(1+(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*a*b^2*arctan(c/x)^2*ln(1-(1+I*
c/x)/(1+c^2/x^2)^(1/2))-3*a^2*b*ln(c/x)*arctan(c/x)-a^3*ln(c/x)-1/2*I*b^3*P
i*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*c
sgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^
3+3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1
+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2+3/2*I*a*b^2*Pi*cs
gn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*
c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2+6*I*a*b^2*arctan(c/x)*polylog(2,-(1+
I*c/x)/(1+c^2/x^2)^(1/2))-3/2*I*a*b^2*Pi*arctan(c/x)^2-3*I*a*b^2*arctan(c/x
)*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))-3/2*I*a^2*b*ln(c/x)*ln(1+I*c/x)+3/2*I
*a^2*b*ln(c/x)*ln(1-I*c/x)+1/2*I*b^3*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((
1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^3-1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^3-1/2*I*b^3*Pi*c
sgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^
3+6*I*a*b^2*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))+3/2*b^3*arct
an(c/x)*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x
)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2-1/2*I*b^3*P
i*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+
```

```
I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^3+1/2*I*b^
3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn((
(1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^3+1/2
*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^
2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^3+1/2*I*b^3*Pi*csgn(I/((1+
I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1
+c^2/x^2)+1))^2*arctan(c/x)^3+3/2*I*a*b^2*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-
1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-3/2*I*a*b^2*Pi*csgn(((1+I*c
/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2-6*I*b^3*p
olylog(4,(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*I*b^3*polylog(4,-(1+I*c/x)/(1+c^2/x
^2)^(1/2))+3/4*I*b^3*polylog(4,-(1+I*c/x)^2/(1+c^2/x^2))+3/2*a*b^2*polylog(
3,-(1+I*c/x)^2/(1+c^2/x^2))-6*a*b^2*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))
-6*a*b^2*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^3*ln(c/x)*arctan(c/x)^3+b
^3*arctan(c/x)^3*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-b^3*arctan(c/x)^3*ln(1+(1+I*
c/x)/(1+c^2/x^2)^(1/2))-6*b^3*arctan(c/x)*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^
(1/2))-b^3*arctan(c/x)^3*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*b^3*arctan(c/x
)*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))+3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-
1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-3/2*I*a*b^2*Pi*csgn(I*((1+I
*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^
2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\log(x)+\frac{1}{32}\int\frac{28\,b^3\arctan(c,x)^3+3\,b^3\arctan(c,x)\log\left(c^2+x^2\right)^2+96\,ab^2\arctan(c,x)^2+96\,a^2b\arctan(c,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan2(c, x)^3 + 3*b^3*arctan2(c, x)*l
og(c^2 + x^2)^2 + 96*a*b^2*arctan2(c, x)^2 + 96*a^2*b*arctan2(c, x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3/x,x)
```

```
[Out] int((a + b*atan(c/x))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x,x)
```

```
[Out] Integral((a + b*atan(c/x))**3/x, x)
```

3.152 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{3ib^2\text{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x}-\frac{3b\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-$$

```
[Out] -I*(a+b*arccot(x/c))^3/c-(a+b*arccot(x/c))^3/x-3*b*(a+b*arccot(x/c))^2*ln(2
/(1+I*c/x))/c-3*I*b^2*(a+b*arccot(x/c))*polylog(2,1-2/(1+I*c/x))/c-3/2*b^3*
polylog(3,1-2/(1+I*c/x))/c
```

Rubi [B] time = 2.36, antiderivative size = 551, normalized size of antiderivative = 4.05, number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$\frac{3b^2\text{PolyLog}\left(2,-\frac{-x+ic}{2x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,-\frac{-x+ic}{2x}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,\frac{x+ic}{2x}\right)}{2c}-\frac{3b^3\log\left(1+\frac{ic}{x}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] (-3*b*(1 - (I*c)/x)*((2*I)*a - b*Log[1 - (I*c)/x])^2)/(8*c) - (3*b*(1 - (I*
c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^2)/(8*c) - ((I/8)*(1 - (I*c)/x)*(2*a + I
*b*Log[1 - (I*c)/x])^3)/c + (3*b*((2*I)*a - b*Log[1 - (I*c)/x])^2*Log[1 + (
I*c)/x])/(8*c) - (((3*I)/8)*b*((2*I)*a - b*Log[1 - (I*c)/x])^2*Log[1 + (I*c
)/x])/x - (3*b^2*((2*I)*a - b*Log[1 - (I*c)/x])*Log[1 + (I*c)/x]^2)/(8*c) -
 (((3*I)/8)*b^2*((2*I)*a - b*Log[1 - (I*c)/x])*Log[1 + (I*c)/x]^2)/x - (b^3
*(1 + (I*c)/x)*Log[1 + (I*c)/x]^3)/(8*c) - (3*b^3*Log[1 + (I*c)/x]^2*Log[-(
I*c - x)/(2*x)])/(4*c) - (3*b*((2*I)*a - b*Log[1 - (I*c)/x])^2*Log[(I*c + x
)/(2*x)])/(4*c) + (3*b^2*((2*I)*a - b*Log[1 - (I*c)/x])*PolyLog[2, -(I*c -
x)/(2*x)])/(2*c) - (3*b^3*Log[1 + (I*c)/x]*PolyLog[2, (I*c + x)/(2*x)])/(2*
c) + (3*b^3*PolyLog[3, -(I*c - x)/(2*x)])/(2*c) + (3*b^3*PolyLog[3, (I*c +
x)/(2*x)])/(2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8x^2}+\frac{3ib\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x^2}-\frac{3ib^2\left(-2ia\right.}{}\right.$$

$$=\frac{1}{8}\int\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{x^2}\,dx+\frac{1}{8}(3ib)\int\frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{x^2}\,dx$$

$$=-\left(\frac{1}{8}\operatorname{Subst}\left(\int(2a+ib\log(1-icx))^3\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{8}(3ib)\operatorname{Subst}\left(\int(-2ia+b\log(1\right.$$

$$=-\frac{3ib\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x}-\frac{3ib^2\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log^2\left(1+\frac{ic}{x}\right)}{8x}$$

$$=-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c}-\frac{3ib\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x}-\frac{3ib^2\left(\right.}{}$$

$$=-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c}-\frac{3ib\left(2ia-\right.}{}$$

$$=\frac{3ab^2}{2x}+\frac{3ib^3}{4x}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c}$$

$$=\frac{3ab^2}{2x}-\frac{3b^3\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{4c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(\right.}{}$$

$$=-\frac{3ib^3}{4x}-\frac{3b^3\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{4c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)}{}$$

$$=-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)}{}$$

$$=-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)}{}$$

Mathematica [A] time = 0.19, size = 189, normalized size = 1.39

$$\frac{-2\left(a^2\left(ac+3bx\log\left(\frac{1}{\sqrt{\frac{c^2}{x^2}+1}}\right)\right)+3b^2\tan^{-1}\left(\frac{c}{x}\right)^2\left(ac-iax+bx\log\left(1+e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\right)+3ab\tan^{-1}\left(\frac{c}{x}\right)\left(ac+2bx\log\right.\right.}{2cx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] (-2*(b^3*(c - I*x)*ArcTan[c/x]^3 + 3*b^2*ArcTan[c/x]^2*(a*c - I*a*x + b*x*L
og[1 + E^((2*I)*ArcTan[c/x])]) + 3*a*b*ArcTan[c/x]*(a*c + 2*b*x*Log[1 + E^(
(2*I)*ArcTan[c/x])]) + a^2*(a*c + 3*b*x*Log[1/Sqrt[1 + c^2/x^2]])) + (6*I)*
b^2*x*(a + b*ArcTan[c/x])*PolyLog[2, -E^((2*I)*ArcTan[c/x])] - 3*b^3*x*Poly
Log[3, -E^((2*I)*ArcTan[c/x])])/(2*c*x)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(\frac{c}{x}\right)^3+3\,ab^2\arctan\left(\frac{c}{x}\right)^2+3\,a^2b\arctan\left(\frac{c}{x}\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3/x^2, x)
```

maple [B] time = 0.25, size = 306, normalized size = 2.25

$$-\frac{a^3}{x}+\frac{ib^3\arctan\left(\frac{c}{x}\right)^3}{c}-\frac{b^3\arctan\left(\frac{c}{x}\right)^3}{x}-\frac{3b^3\arctan\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)}{c}+\frac{3ib^3\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)}{c}-\frac{3\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x^2,x)
```

```
[Out] -a^3/x+I/c*b^3*arctan(c/x)^3-b^3/x*arctan(c/x)^3-3/c*b^3*arctan(c/x)^2*ln((
1+I*c/x)^2/(1+c^2/x^2)+1)+3*I/c*b^3*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c
^2/x^2))-3/2/c*b^3*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+3*I/c*arctan(c/x)^2*
a*b^2-3/x*a*b^2*arctan(c/x)^2-6/c*ln((1+I*c/x)^2/(1+c^2/x^2)+1)*arctan(c/x)
*a*b^2+3*I/c*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))*a*b^2-3/x*a^2*b*arctan(c/x
)+3/2/c*a^2*b*ln(1+c^2/x^2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3/x^2,x)
[Out] int((a + b*atan(c/x))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x**2,x)
[Out] Integral((a + b*atan(c/x))**3/x**2, x)
```

3.153 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=147

$$\frac{3b^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2}+\frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{3b\left(a+b\cot^{-1}\left(\right.\right.}{2cx}$$

```
[Out] 3/2*I*b*(a+b*arccot(x/c))^2/c^2+3/2*b*(a+b*arccot(x/c))^2/c/x-1/2*(a+b*arcc
ot(x/c))^3/c^2-1/2*(a+b*arccot(x/c))^3/x^2+3*b^2*(a+b*arccot(x/c))*ln(2/(1+
I*c/x))/c^2+3/2*I*b^3*polylog(2,1-2/(1+I*c/x))/c^2
```

Rubi [F] time = 2.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (((3*I)/64)*b^3*(1 - (I*c)/x)^2)/c^2 - (3*a*b^2*(1 + (I*c)/x)^2)/(16*c^2) -
 (((3*I)/64)*b^3*(1 + (I*c)/x)^2)/c^2 - (((3*I)/8)*a^2*b)/x^2 - (3*a*b^2)/(
8*x^2) + (3*a^2*b)/(4*c*x) - (3*b^3)/(2*c*x) + (((3*I)/4)*a^2*b*Log[I - c/x
])/c^2 + (3*a*b^2*Log[I - c/x])/(8*c^2) - (3*a*b^2*(1 - (I*c)/x)*Log[1 - (I
*c)/x])/(4*c^2) + (((3*I)/4)*b^3*(1 - (I*c)/x)*Log[1 - (I*c)/x])/c^2 + (3*a
*b^2*Log[1 - (I*c)/x])/(8*x^2) - (3*b^2*(1 - (I*c)/x)^2*(2*a + I*b*Log[1 -
(I*c)/x]))/(32*c^2) + (((3*I)/8)*b*(1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x
])^2)/c^2 - (((3*I)/32)*b*(1 - (I*c)/x)^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/c
^2 - ((1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^3)/(8*c^2) + ((1 - (I*c)/x
)^2*(2*a + I*b*Log[1 - (I*c)/x])^3)/(16*c^2) - (9*a*b^2*(1 + (I*c)/x)*Log[1
 + (I*c)/x])/(4*c^2) - (((3*I)/4)*b^3*(1 + (I*c)/x)*Log[1 + (I*c)/x])/c^2 +
 (3*a*b^2*(1 + (I*c)/x)^2*Log[1 + (I*c)/x])/(8*c^2) + (((3*I)/32)*b^3*(1 +
(I*c)/x)^2*Log[1 + (I*c)/x])/c^2 + (((3*I)/4)*a^2*b*Log[1 + (I*c)/x])/x^2 +
 (3*a*b^2*Log[1 + (I*c)/x])/(8*x^2) - (3*a*b^2*Log[1 - (I*c)/x]*Log[1 + (I*
c)/x])/(4*x^2) + (3*a*b^2*(1 + (I*c)/x)*Log[1 + (I*c)/x]^2)/(4*c^2) + (((3*
I)/8)*b^3*(1 + (I*c)/x)*Log[1 + (I*c)/x]^2)/c^2 - (3*a*b^2*(1 + (I*c)/x)^2*
Log[1 + (I*c)/x]^2)/(8*c^2) - (((3*I)/32)*b^3*(1 + (I*c)/x)^2*Log[1 + (I*c)
/x]^2)/c^2 - ((I/8)*b^3*(1 + (I*c)/x)*Log[1 + (I*c)/x]^3)/c^2 + ((I/16)*b^3
*(1 + (I*c)/x)^2*Log[1 + (I*c)/x]^3)/c^2 + (3*a*b^2*Log[I + c/x])/(8*c^2) -
 (3*a*b^2*Log[1 - (I*c)/x]*Log[c - I*x])/(4*c^2) - (3*a*b^2*Log[1 + (I*c)/x
]*Log[c + I*x])/(4*c^2) + (3*a*b^2*Log[(c - I*x)/(2*c)]*Log[c + I*x])/(4*c^
2) + (3*a*b^2*Log[c - I*x]*Log[(c + I*x)/(2*c)])/(4*c^2) - (3*a*b^2*Log[c +
 I*x]*Log[((-I)*x)/c])/(4*c^2) - (3*a*b^2*Log[c - I*x]*Log[(I*x)/c])/(4*c^2
) + (3*a*b^2*PolyLog[2, (c - I*x)/(2*c)])/(4*c^2) + (3*a*b^2*PolyLog[2, (c
+ I*x)/(2*c)])/(4*c^2) + (3*a*b^2*PolyLog[2, ((-I)*c)/x])/(4*c^2) + (3*a*b^
2*PolyLog[2, (I*c)/x])/(4*c^2) - (3*a*b^2*PolyLog[2, 1 - (I*x)/c])/(4*c^2)
- (3*a*b^2*PolyLog[2, 1 + (I*x)/c])/(4*c^2) + ((3*I)/8)*b^3*Defer[Int][(Log
[1 - (I*c)/x]^2*Log[1 + (I*c)/x])/x^3, x] - ((3*I)/8)*b^3*Defer[Int][(Log[1
 - (I*c)/x]*Log[1 + (I*c)/x]^2)/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8x^3} + \frac{3ib\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x^3} - \frac{3ib^2\left(-2ia\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{x^3}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{x^3}\,dx$$

$$= -\left(\frac{1}{8}\,\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{8}(3ib)\int\left(-\frac{4a^2\log\left(1+\frac{ic}{x}\right)}{x^3} -\right.$$

$$= -\left(\frac{1}{8}\,\text{Subst}\left(\int\left(-\frac{i(2a+ib\log(1-icx))^3}{c} + \frac{i(1-icx)(2a+ib\log(1-icx))^3}{c}\right)dx, x\right.\right.$$

$$= -\frac{3ab^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + \frac{1}{2}\left(3ia^2b\right)\text{Subst}\left(\int x\log(1+icx)\,dx, x, \frac{1}{x}\right) + \frac{1}{4}\left(\right.$$

$$= \frac{3ia^2b\log\left(1+\frac{ic}{x}\right)}{4x^2} - \frac{3ab^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + \frac{1}{4}\left(3ab^2\right)\text{Subst}\left(\int\left(\frac{i\log^2(1+}{c}\right.\right.$$

$$= -\frac{\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c^2} + \frac{\left(1-\frac{ic}{x}\right)^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{16c^2} + \frac{3ia^2b\log\left(1-\right.}{4x^2}$$

$$= -\frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2} + \frac{3ib\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c^2} - \frac{3ib\left(1-\right.}{}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3iab^2}{2cx} - \frac{3b^3}{4cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2} -$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log}{4c^2}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log}{4c^2}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3i}{}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3i}{}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3i}{}$$

<u>**Mathematica**</u> [A] time = 0.29, size = 178, normalized size = 1.21

$$\frac{a\left(ac(3bx-ac)+6b^2x^2\log\left(\frac{1}{\sqrt{\frac{c^2}{x^2}+1}}\right)\right) - 3b\tan^{-1}\left(\frac{c}{x}\right)\left(a\left(a\left(c^2+x^2\right)-2bcx\right)-2b^2x^2\log\left(1+e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\right) + 3i}{2c^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (3*b^2*(c - I*x)*(-(a*(c + I*x)) + b*x)*ArcTan[c/x]^2 - b^3*(c^2 + x^2)*Arc
Tan[c/x]^3 - 3*b*ArcTan[c/x]*(a*(-2*b*c*x + a*(c^2 + x^2)) - 2*b^2*x^2*Log[
1 + E^((2*I)*ArcTan[c/x])]) + a*(a*c*(-(a*c) + 3*b*x) + 6*b^2*x^2*Log[1/Sqr
t[1 + c^2/x^2]]) - (3*I)*b^3*x^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])])/(2*c^2
*x^2)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \arctan\left(\frac{c}{x}\right)^3 + 3\,ab^2 \arctan\left(\frac{c}{x}\right)^2 + 3\,a^2 b \arctan\left(\frac{c}{x}\right) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \arctan\left(\frac{c}{x}\right) + a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3/x^3, x)
```

maple [B] time = 0.10, size = 396, normalized size = 2.69

$$-\frac{a^3}{2x^2} - \frac{b^3 \arctan\left(\frac{c}{x}\right)^3}{2x^2} - \frac{b^3 \arctan\left(\frac{c}{x}\right)^3}{2c^2} + \frac{3b^3 \arctan\left(\frac{c}{x}\right)^2}{2cx} - \frac{3b^3 \arctan\left(\frac{c}{x}\right) \ln\left(1 + \frac{c^2}{x^2}\right)}{2c^2} - \frac{3ib^3 \ln\left(\frac{c}{x} - i\right) \ln\left(1 + \frac{c^2}{x^2}\right)}{4c^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2-1/2/x^2*b^3*arctan(c/x)^3-1/2/c^2*b^3*arctan(c/x)^3+3/2/c*b^3*
arctan(c/x)^2/x-3/2/c^2*b^3*arctan(c/x)*ln(1+c^2/x^2)-3/4*I/c^2*b^3*ln(c/x-
I)*ln(1+c^2/x^2)-3/4*I/c^2*b^3*ln(I+c/x)*ln(1/2*I*(c/x-I))-3/4*I/c^2*b^3*di
log(1/2*I*(c/x-I))-3/8*I/c^2*b^3*ln(I+c/x)^2+3/4*I/c^2*b^3*ln(c/x-I)*ln(-1/
2*I*(I+c/x))+3/4*I/c^2*b^3*ln(I+c/x)*ln(1+c^2/x^2)+3/8*I/c^2*b^3*ln(c/x-I)^
2+3/4*I/c^2*b^3*dilog(-1/2*I*(I+c/x))-3/2/x^2*a*b^2*arctan(c/x)^2-3/2/c^2*a
*b^2*arctan(c/x)^2+3/c*a*b^2/x*arctan(c/x)-3/2/c^2*a*b^2*ln(1+c^2/x^2)-3/2*
b/x^2*a^2*arctan(c/x)+3/2*a^2*b/c/x+3/2/c^2*a^2*b*arctan(x/c)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3/x^3,x)
```

```
[Out] int((a + b*atan(c/x))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c/x))**3/x**3, x)
```

3.154 $\int x^2 \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=51

$$-\frac{x^{5/2}}{15}+\frac{x^{3/2}}{9}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x^(1/2))-1/3*x^
(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5033, 50, 63, 203}

$$-\frac{x^{5/2}}{15}+\frac{x^{3/2}}{9}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x]/3 + x^(3/2)/9 - x^(5/2)/15 + ArcTan[Sqrt[x]]/3 + (x^3*ArcTan[Sqrt[
x]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tan^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{1+x}\,dx \\
&= -\frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) + \frac{1}{6}\int \frac{x^{3/2}}{1+x}\,dx \\
&= \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{\sqrt{x}}{1+x}\,dx \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) + \frac{1}{6}\int \frac{1}{\sqrt{x}(1+x)}\,dx \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}\tan^{-1}\left(\sqrt{x}\right) + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.67

$$\frac{1}{45}\left(15\left(x^3+1\right)\tan^{-1}\left(\sqrt{x}\right)+\sqrt{x}\left(-3x^2+5x-15\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Sqrt[x]],x]
[Out] (Sqrt[x]*(-15 + 5*x - 3*x^2) + 15*(1 + x^3)*ArcTan[Sqrt[x]])/45
```

fricas [A] time = 0.47, size = 27, normalized size = 0.53

$$\frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{45}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="fricas")
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x)) - 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

giac [A] time = 0.14, size = 31, normalized size = 0.61

$$\frac{1}{3}\,x^3\arctan\left(\sqrt{x}\right)-\frac{1}{15}\,x^{\frac{5}{2}}+\frac{1}{9}\,x^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}+\frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

maple [A] time = 0.03, size = 32, normalized size = 0.63

$$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{x^3\arctan\left(\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x^(1/2)),x)
[Out] 1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x^(1/2))-1/3*x^
(1/2)
```

maxima [A] time = 0.41, size = 31, normalized size = 0.61

$$\frac{1}{3}x^3\arctan\left(\sqrt{x}\right) - \frac{1}{15}x^{\frac{5}{2}} + \frac{1}{9}x^{\frac{3}{2}} - \frac{1}{3}\sqrt{x} + \frac{1}{3}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

mupad [B] time = 0.35, size = 31, normalized size = 0.61

$$\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3} + \frac{x^3\operatorname{atan}\left(\sqrt{x}\right)}{3} - \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(x^(1/2)),x)
```

```
[Out] atan(x^(1/2))/3 + (x^3*atan(x^(1/2)))/3 - x^(1/2)/3 + x^(3/2)/9 - x^(5/2)/1
5
```

sympy [A] time = 1.97, size = 39, normalized size = 0.76

$$-\frac{x^{\frac{5}{2}}}{15} + \frac{x^{\frac{3}{2}}}{9} - \frac{\sqrt{x}}{3} + \frac{x^3\operatorname{atan}\left(\sqrt{x}\right)}{3} + \frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x**(1/2)),x)
```

```
[Out] -x**(5/2)/15 + x**(3/2)/9 - sqrt(x)/3 + x**3*atan(sqrt(x))/3 + atan(sqrt(x)
)/3
```

3.155 $\int x \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=42

$$-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))+1/2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5033, 50, 63, 203}

$$-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/2 - x^(3/2)/6 - ArcTan[Sqrt[x]]/2 + (x^2*ArcTan[Sqrt[x]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}\left(\sqrt{x}\right)dx &= \frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{1+x}\,dx\\ &= -\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{\sqrt{x}}{1+x}\,dx\\ &= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\ &= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.67

$$\frac{1}{6}\left(3\left(x^2-1\right)\tan^{-1}\left(\sqrt{x}\right)-(x-3)\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Sqrt[x]],x]
[Out] (-((-3 + x)*Sqrt[x]) + 3*(-1 + x^2)*ArcTan[Sqrt[x]])/6
```

fricas [A] time = 0.48, size = 20, normalized size = 0.48

$$\frac{1}{2}\left(x^2-1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{6}(x-3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x)) - 1/6*(x - 3)*sqrt(x)
```

giac [A] time = 0.16, size = 26, normalized size = 0.62

$$\frac{1}{2}\,x^2\arctan\left(\sqrt{x}\right)-\frac{1}{6}\,x^{\frac{3}{2}}+\frac{1}{2}\,\sqrt{x}-\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

maple [A] time = 0.02, size = 27, normalized size = 0.64

$$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{x^2\arctan\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x^(1/2)),x)
[Out] -1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))+1/2*x^(1/2)
```

maxima [A] time = 0.41, size = 26, normalized size = 0.62

$$\frac{1}{2}\,x^2\arctan\left(\sqrt{x}\right)-\frac{1}{6}\,x^{\frac{3}{2}}+\frac{1}{2}\,\sqrt{x}-\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

mupad [B] time = 0.37, size = 26, normalized size = 0.62

$$\frac{x^2\,\mathrm{atan}\left(\sqrt{x}\right)}{2}-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x^(1/2)),x)
[Out] (x^2*atan(x^(1/2)))/2 - atan(x^(1/2))/2 + x^(1/2)/2 - x^(3/2)/6
```

sympy [A] time = 1.26, size = 32, normalized size = 0.76

$$-\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{x^2\,\mathrm{atan}\left(\sqrt{x}\right)}{2}-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x**(1/2)),x)
[Out] -x**(3/2)/6 + sqrt(x)/2 + x**2*atan(sqrt(x))/2 - atan(sqrt(x))/2
```

3.156 $\int \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5027, 50, 63, 203}

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(\sqrt{x}\right) dx &= x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx\\
&= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\sqrt{x}+\tan^{-1}\left(\sqrt{x}\right)+x\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.82

$$(x+1)\tan^{-1}\left(\sqrt{x}\right)-\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]],x]
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

fricas [A] time = 0.41, size = 14, normalized size = 0.64

$$(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

giac [A] time = 0.35, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

maple [A] time = 0.02, size = 17, normalized size = 0.77

$$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2)),x)
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

maxima [A] time = 0.41, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

mupad [B] time = 0.07, size = 16, normalized size = 0.73

$$\operatorname{atan}\left(\sqrt{x}\right)+x\operatorname{atan}\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2)),x)
[Out] atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

sympy [A] time = 1.15, size = 19, normalized size = 0.86

$$-\sqrt{x}+x\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2)),x)
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

3.157 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x} \, dx$

Optimal. Leaf size=31

$$i\text{Li}_2\left(-i\sqrt{x}\right) - i\text{Li}_2\left(i\sqrt{x}\right)$$

```
[Out] I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5031, 4848, 2391}

$$i\text{PolyLog}\left(2, -i\sqrt{x}\right) - i\text{PolyLog}\left(2, i\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x,x]
```

```
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x} \, dx &= 2\,\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x} \, dx, x, \sqrt{x}\right)\\
&= i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x} \, dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x} \, dx, x, \sqrt{x}\right)\\
&= i\text{Li}_2\left(-i\sqrt{x}\right) - i\text{Li}_2\left(i\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$i\text{Li}_2\left(-i\sqrt{x}\right) - i\text{Li}_2\left(i\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x,x]
```

```
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(x))/x, x)
```

maple [B] time = 0.04, size = 61, normalized size = 1.97

$$\ln(x)\arctan\left(\sqrt{x}\right)+\frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2}-\frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2}+i\,\mathrm{dilog}\left(1+i\sqrt{x}\right)-i\,\mathrm{dilog}\left(1-i\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x,x)
```

```
[Out] ln(x)*arctan(x^(1/2))+1/2*I*ln(x)*ln(1+I*x^(1/2))-1/2*I*ln(x)*ln(1-I*x^(1/2
))+I*dilog(1+I*x^(1/2))-I*dilog(1-I*x^(1/2))
```

maxima [B] time = 0.42, size = 35, normalized size = 1.13

$$-\frac{1}{2}\pi\log\left(x+1\right)+\arctan\left(\sqrt{x}\right)\log(x)-i\,\mathrm{Li}_2\left(i\sqrt{x}+1\right)+i\,\mathrm{Li}_2\left(-i\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -1/2*pi*log(x + 1) + arctan(sqrt(x))*log(x) - I*dilog(I*sqrt(x) + 1) + I*di
log(-I*sqrt(x) + 1)
```

mupad [B] time = 0.30, size = 24, normalized size = 0.77

$$-\mathrm{Li}_2\left(1-\sqrt{x}\,1\mathrm{i}\right)\,1\mathrm{i}+\mathrm{polylog}\left(2,-\sqrt{x}\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x,x)
```

```
[Out] polylog(2, -x^(1/2)*1i)*1i - dilog(1 - x^(1/2)*1i)*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x,x)
```

```
[Out] Integral(atan(sqrt(x))/x, x)
```

3.158 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^2} \, dx$

Optimal. Leaf size=27

$$-\frac{1}{\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{x}-\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arctan(x^(1/2))-arctan(x^(1/2))/x-1/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5033, 51, 63, 203}

$$-\frac{1}{\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{x}-\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^2,x]
```

```
[Out] -(1/Sqrt[x]) - ArcTan[Sqrt[x]] - ArcTan[Sqrt[x]]/x
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^2} dx = -\frac{\tan^{-1}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\int \frac{1}{x^{3/2}(1+x)} dx$$
$$= -\frac{1}{\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)} dx$$
$$= -\frac{1}{\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x} - \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right)$$
$$= -\frac{1}{\sqrt{x}} - \tan^{-1}\left(\sqrt{x}\right) - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 1.11

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x\right)}{\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^2,x]
[Out] -(ArcTan[Sqrt[x]]/x) - Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

fricas [A] time = 0.43, size = 17, normalized size = 0.63

$$-\frac{(x+1)\arctan\left(\sqrt{x}\right) + \sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -((x + 1)*arctan(sqrt(x)) + sqrt(x))/x
```

giac [A] time = 0.30, size = 21, normalized size = 0.78

$$-\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="giac")
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

maple [A] time = 0.03, size = 22, normalized size = 0.81

$$-\arctan\left(\sqrt{x}\right) - \frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^2,x)
[Out] -arctan(x^(1/2))-arctan(x^(1/2))/x-1/x^(1/2)
```

maxima [A] time = 0.41, size = 21, normalized size = 0.78

$$-\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

mupad [B] time = 0.35, size = 21, normalized size = 0.78

$$-\operatorname{atan}\left(\sqrt{x}\right) - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^2,x)
```

```
[Out] - atan(x^(1/2)) - atan(x^(1/2))/x - 1/x^(1/2)
```

sympy [B] time = 1.92, size = 94, normalized size = 3.48

$$-\frac{x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{x^{2}}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**2,x)
```

```
[Out] -x**(5/2)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*atan(sqrt(x))/(x
**(5/2) + x**(3/2)) - sqrt(x)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - x**2/(x
**(5/2) + x**(3/2)) - x/(x**(5/2) + x**(3/2))
```

3.159 $\int \frac{\tan^{-1}(\sqrt{x})}{x^3} dx$

Optimal. Leaf size=42

$$-\frac{1}{6x^{3/2}}-\frac{\tan^{-1}(\sqrt{x})}{2x^2}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}(\sqrt{x})$$

```
[Out] -1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5033, 51, 63, 203}

$$-\frac{1}{6x^{3/2}}-\frac{\tan^{-1}(\sqrt{x})}{2x^2}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^3,x]
```

```
[Out] -1/(6*x^(3/2)) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 - ArcTan[Sqrt[x]]/(2*x^2
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{x^{5/2}(1+x)}\,dx \\
&= -\frac{1}{6x^{3/2}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx \\
&= -\frac{1}{6x^{3/2}} + \frac{1}{2\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx \\
&= -\frac{1}{6x^{3/2}} + \frac{1}{2\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\
&= -\frac{1}{6x^{3/2}} + \frac{1}{2\sqrt{x}} + \frac{1}{2}\tan^{-1}\left(\sqrt{x}\right) - \frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.81

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-x\right)}{6x^{3/2}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^3,x]
[Out] -1/2*ArcTan[Sqrt[x]]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2))
```

fricas [A] time = 0.46, size = 26, normalized size = 0.62

$$\frac{3\left(x^2-1\right)\arctan\left(\sqrt{x}\right) + (3\,x-1)\sqrt{x}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/6*(3*(x^2 - 1)*arctan(sqrt(x)) + (3*x - 1)*sqrt(x))/x^2
```

giac [A] time = 0.16, size = 26, normalized size = 0.62

$$\frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="giac")
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

maple [A] time = 0.03, size = 27, normalized size = 0.64

$$-\frac{1}{6x^{\frac{3}{2}}} + \frac{\arctan\left(\sqrt{x}\right)}{2} - \frac{\arctan\left(\sqrt{x}\right)}{2x^2} + \frac{1}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^3,x)
[Out] -1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

maxima [A] time = 0.41, size = 26, normalized size = 0.62

$$\frac{3\,x-1}{6\,x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x}\right)}{2\,x^2}+\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

mupad [B] time = 0.35, size = 24, normalized size = 0.57

$$\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}+\frac{x-\frac{1}{3}}{2\,x^{3/2}}-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^3,x)
```

```
[Out] atan(x^(1/2))/2 + (x - 1/3)/(2*x^(3/2)) - atan(x^(1/2))/(2*x^2)
```

sympy [B] time = 4.92, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^3}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{2x^2}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**3,x)
```

```
[Out] 3*x**(7/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*atan(sqrt(x
))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**
(5/2)) - 3*sqrt(x)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**3/(6*x**(
7/2) + 6*x**(5/2)) + 2*x**2/(6*x**(7/2) + 6*x**(5/2)) - x/(6*x**(7/2) + 6*x
**(5/2))
```

3.160 $\int x^{3/2} \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=36

$$\frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{x^2}{10}+\frac{x}{5}-\frac{1}{5}\log(x+1)$$

```
[Out] 1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(1+x)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 43}

$$-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)+\frac{x}{5}-\frac{1}{5}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTan[Sqrt[x]],x]
```

```
[Out] x/5 - x^2/10 + (2*x^(5/2)*ArcTan[Sqrt[x]])/5 - Log[1 + x]/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tan^{-1}\left(\sqrt{x}\right)dx &= \frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1+x}\,dx\\ &= \frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+x+\frac{1}{1+x}\right)dx\\ &= \frac{x}{5}-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.83

$$\frac{1}{10}\left(4x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-(x-2)x-2\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTan[Sqrt[x]],x]
```

```
[Out] (-((-2 + x)*x) + 4*x^(5/2)*ArcTan[Sqrt[x]] - 2*Log[1 + x])/10
```

fricas [A] time = 0.48, size = 24, normalized size = 0.67

$$\frac{2}{5}x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}x^2+\frac{1}{5}x-\frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

giac [A] time = 0.17, size = 24, normalized size = 0.67

$$\frac{2}{5}x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}x^2+\frac{1}{5}x-\frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

maple [A] time = 0.02, size = 25, normalized size = 0.69

$$\frac{x}{5}-\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)}{5}-\frac{\ln\left(x+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctan(x^(1/2)),x)
```

```
[Out] 1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(x+1)
```

maxima [A] time = 0.31, size = 24, normalized size = 0.67

$$\frac{2}{5}x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}x^2+\frac{1}{5}x-\frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

mupad [B] time = 0.35, size = 24, normalized size = 0.67

$$\frac{x}{5}-\frac{\ln\left(x+1\right)}{5}+\frac{2\,x^{5/2}\,\mathrm{atan}\left(\sqrt{x}\right)}{5}-\frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atan(x^(1/2)),x)
```

```
[Out] x/5 - log(x + 1)/5 + (2*x^(5/2)*atan(x^(1/2)))/5 - x^2/10
```

sympy [B] time = 4.79, size = 85, normalized size = 2.36

$$\frac{4x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10}+\frac{4x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10}-\frac{x^3}{10x+10}+\frac{x^2}{10x+10}-\frac{2x\log\left(x+1\right)}{10x+10}-\frac{2\log\left(x+1\right)}{10x+10}-\frac{2}{10x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atan(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*atan(sqrt(x))/(10*x + 10) + 4*x**(5/2)*atan(sqrt(x))/(10*x + 10)
 - x**3/(10*x + 10) + x**2/(10*x + 10) - 2*x*log(x + 1)/(10*x + 10) - 2*log
(x + 1)/(10*x + 10) - 2/(10*x + 10)
```

3.161 $\int \sqrt{x} \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=29

$$\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

```
[Out] -1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 43}

$$\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

```
[Out] -x/3 + (2*x^(3/2)*ArcTan[Sqrt[x]])/3 + Log[1 + x]/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tan^{-1}\left(\sqrt{x}\right)dx &= \frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{1+x}\,dx\\ &= \frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \left(1+\frac{1}{-1-x}\right)dx\\ &= -\frac{x}{3}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{3}\left(2x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-x+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

```
[Out] (-x + 2*x^(3/2)*ArcTan[Sqrt[x]] + Log[1 + x])/3
```

fricas [A] time = 0.47, size = 19, normalized size = 0.66

$$\frac{2}{3}x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}x+\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

giac [A] time = 0.18, size = 19, normalized size = 0.66

$$\frac{2}{3} \, x^{\frac{3}{2}} \arctan\left(\sqrt{x}\right) - \frac{1}{3} \, x + \frac{1}{3} \, \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

maple [A] time = 0.03, size = 20, normalized size = 0.69

$$-\frac{x}{3} + \frac{2x^{\frac{3}{2}} \arctan\left(\sqrt{x}\right)}{3} + \frac{\ln\left(x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctan(x^(1/2)),x)
```

```
[Out] -1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(x+1)
```

maxima [A] time = 0.31, size = 19, normalized size = 0.66

$$\frac{2}{3} \, x^{\frac{3}{2}} \arctan\left(\sqrt{x}\right) - \frac{1}{3} \, x + \frac{1}{3} \, \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{x} \, \mathrm{atan}\left(\sqrt{x}\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atan(x^(1/2)),x)
```

```
[Out] int(x^(1/2)*atan(x^(1/2)), x)
```

sympy [A] time = 1.24, size = 24, normalized size = 0.83

$$\frac{2x^{\frac{3}{2}} \operatorname{atan}\left(\sqrt{x}\right)}{3} - \frac{x}{3} + \frac{\log\left(x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atan(x**(1/2)),x)
```

```
[Out] 2*x**(3/2)*atan(sqrt(x))/3 - x/3 + log(x + 1)/3
```

3.162 $\int \frac{\tan^{-1}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$2\sqrt{x}\,\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

```
[Out] -ln(1+x)+2*x^(1/2)*arctan(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 31}

$$2\sqrt{x}\,\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\,\tan^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1+x}\,dx\\ &= 2\sqrt{x}\,\tan^{-1}\left(\sqrt{x}\right)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$2\sqrt{x}\,\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

fricas [A] time = 0.44, size = 16, normalized size = 0.80

$$2\,\sqrt{x}\arctan\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

giac [A] time = 0.18, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

maple [A] time = 0.02, size = 17, normalized size = 0.85

$$-\ln\left(x+1\right) + 2\sqrt{x}\ \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(1/2),x)
[Out] -ln(x+1)+2*x^(1/2)*arctan(x^(1/2))
```

maxima [A] time = 0.31, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

mupad [B] time = 0.35, size = 16, normalized size = 0.80

$$2\sqrt{x}\ \mathrm{atan}\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^(1/2),x)
[Out] 2*x^(1/2)*atan(x^(1/2)) - log(x + 1)
```

sympy [A] time = 0.33, size = 17, normalized size = 0.85

$$2\sqrt{x}\ \mathrm{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(1/2),x)
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

3.163 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=22

$$\log(x)-\log(x+1)-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[Out] ln(x)-ln(1+x)-2*arctan(x^(1/2))/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5033, 36, 29, 31}

$$\log(x)-\log(x+1)-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{x(1+x)}\,dx\\
&= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{x}\,dx-\int \frac{1}{1+x}\,dx\\
&= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\log(x) - \log(x+1) - \frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(3/2),x]
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

fricas [A] time = 0.43, size = 26, normalized size = 1.18

$$-\frac{x\log(x+1) - x\log(x) + 2\sqrt{x}\arctan\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] -(x*log(x + 1) - x*log(x) + 2*sqrt(x)*arctan(sqrt(x)))/x
```

giac [A] time = 0.17, size = 18, normalized size = 0.82

$$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

maple [A] time = 0.04, size = 19, normalized size = 0.86

$$\ln(x) - \ln(x+1) - \frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(3/2),x)
[Out] ln(x)-ln(x+1)-2*arctan(x^(1/2))/x^(1/2)
```

maxima [A] time = 0.31, size = 18, normalized size = 0.82

$$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

mupad [B] time = 0.36, size = 22, normalized size = 1.00

$$2\ln\left(\sqrt{x}\right) - \ln(x+1) - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^(3/2),x)
[Out] 2*log(x^(1/2)) - log(x + 1) - (2*atan(x^(1/2)))/x^(1/2)
```

sympy [**A**] time = 1.16, size = 20, normalized size = 0.91

$$\log(x) - \log(x+1) - \frac{2\operatorname{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(3/2),x)
[Out] log(x) - log(x + 1) - 2*atan(sqrt(x))/sqrt(x)
```

3.164 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3x}-\frac{\log(x)}{3}+\frac{1}{3}\log(x+1)$$

```
[Out] -1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 44}

$$-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3x}-\frac{\log(x)}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^(5/2),x]
[Out] -1/(3*x) - (2*ArcTan[Sqrt[x]])/(3*x^(3/2)) - Log[x]/3 + Log[1 + x]/3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int \frac{1}{x^2(1+x)}\,dx\\ &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx\\ &= -\frac{1}{3x}-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{\log(x)}{3}+\frac{1}{3}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.84

$$\frac{1}{3}\left(-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{1}{x}-\log(x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(5/2),x]
[Out] (-x^(-1) - (2*ArcTan[Sqrt[x]])/x^(3/2) - Log[x] + Log[1 + x])/3
```

fricas [A] time = 0.48, size = 33, normalized size = 0.89

$$\frac{x^2 \log(x+1) - x^2 \log(x) - 2\sqrt{x} \arctan\left(\sqrt{x}\right) - x}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="fricas")
[Out] 1/3*(x^2*log(x + 1) - x^2*log(x) - 2*sqrt(x)*arctan(sqrt(x)) - x)/x^2
```

giac [A] time = 0.17, size = 28, normalized size = 0.76

$$\frac{x-1}{3x} - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}} + \frac{1}{3}\log(x+1) - \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="giac")
[Out] 1/3*(x - 1)/x - 2/3*arctan(sqrt(x))/x^(3/2) + 1/3*log(x + 1) - 1/3*log(x)
```

maple [A] time = 0.03, size = 26, normalized size = 0.70

$$-\frac{1}{3x} - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}} - \frac{\ln(x)}{3} + \frac{\ln(x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(5/2),x)
[Out] -1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(x+1)
```

maxima [A] time = 0.31, size = 25, normalized size = 0.68

$$-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}} - \frac{1}{3x} + \frac{1}{3}\log(x+1) - \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="maxima")
[Out] -2/3*arctan(sqrt(x))/x^(3/2) - 1/3/x + 1/3*log(x + 1) - 1/3*log(x)
```

mupad [B] time = 0.35, size = 27, normalized size = 0.73

$$\frac{\ln(x+1)}{3} - \frac{2\ln\left(\sqrt{x}\right)}{3} - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{3x^{3/2}} - \frac{1}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^(5/2),x)
[Out] log(x + 1)/3 - (2*log(x^(1/2)))/3 - (2*atan(x^(1/2)))/(3*x^(3/2)) - 1/(3*x)
```

sympy [B] time = 4.90, size = 143, normalized size = 3.86

$$-\frac{2x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{x^3\log(x)}{3x^3+3x^2} + \frac{x^3\log(x+1)}{3x^3+3x^2} - \frac{x^2\log(x)}{3x^3+3x^2} + \frac{x^2\log(x+1)}{3x^3+3x^2} - \frac{x^2}{3x^3+3x^2} - \frac{x}{3x^3+3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(5/2),x)
```

```
[Out] -2*x**(3/2)*atan(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*atan(sqrt(x))/(3*x*
*3 + 3*x**2) - x**3*log(x)/(3*x**3 + 3*x**2) + x**3*log(x + 1)/(3*x**3 + 3*
x**2) - x**2*log(x)/(3*x**3 + 3*x**2) + x**2*log(x + 1)/(3*x**3 + 3*x**2) -
 x**2/(3*x**3 + 3*x**2) - x/(3*x**3 + 3*x**2)
```

3.165 $\int \frac{\tan^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=33

$$\frac{1}{10}i\text{Li}_2\left(-iax^5\right) - \frac{1}{10}i\text{Li}_2\left(iax^5\right)$$

```
[Out] 1/10*I*polylog(2,-I*a*x^5)-1/10*I*polylog(2,I*a*x^5)
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5031, 4848, 2391}

$$\frac{1}{10}i\text{PolyLog}\left(2, -iax^5\right) - \frac{1}{10}i\text{PolyLog}\left(2, iax^5\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x^5]/x,x]
```

```
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(ax^5\right)}{x} dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\tan^{-1}(ax)}{x} dx, x, x^5\right)\\ &= \frac{1}{10}i\,\text{Subst}\left(\int \frac{\log(1-iax)}{x} dx, x, x^5\right) - \frac{1}{10}i\,\text{Subst}\left(\int \frac{\log(1+iax)}{x} dx, x, x^5\right)\\ &= \frac{1}{10}i\text{Li}_2\left(-iax^5\right) - \frac{1}{10}i\text{Li}_2\left(iax^5\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{10}i\text{Li}_2\left(-iax^5\right) - \frac{1}{10}i\text{Li}_2\left(iax^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x^5]/x,x]
```

```
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\arctan\left(ax^5\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x^5)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax^5\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x^5)/x, x)
```

maple [C] time = 0.09, size = 57, normalized size = 1.73

$$\ln(x)\arctan\left(a\,x^5\right) - \frac{\sum_{\_R1=\mathrm{RootOf}\left(a^2\_Z^{10}+1\right)} \frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x^5)/x,x)
```

```
[Out] ln(x)*arctan(a*x^5)-1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^10*a^2+1))
```

maxima [B] time = 0.42, size = 50, normalized size = 1.52

$$-\frac{1}{20}\,\pi\log\left(a^2x^{10}+1\right)+\frac{1}{5}\,\arctan\left(ax^5\right)\log\left(ax^5\right)-\frac{1}{10}i\,\mathrm{Li}_2\left(i\,ax^5+1\right)+\frac{1}{10}i\,\mathrm{Li}_2\left(-i\,ax^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/20*pi*log(a^2*x^10 + 1) + 1/5*arctan(a*x^5)*log(a*x^5) - 1/10*I*dilog(I*
a*x^5 + 1) + 1/10*I*dilog(-I*a*x^5 + 1)
```

mupad [B] time = 0.34, size = 25, normalized size = 0.76

$$\frac{\mathrm{polylog}\left(2,-a\,x^5\,1\mathrm{i}\right)\,1\mathrm{i}}{10}-\frac{\mathrm{polylog}\left(2,a\,x^5\,1\mathrm{i}\right)\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x^5)/x,x)
```

```
[Out] (polylog(2, -a*x^5*1i)*1i)/10 - (polylog(2, a*x^5*1i)*1i)/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atan}\left(ax^5\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x**5)/x,x)
```

```
[Out] Integral(atan(a*x**5)/x, x)
```

3.166 $\int \frac{\tan^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=39

$$\frac{i\text{Li}_2(-iax^n)}{2n} - \frac{i\text{Li}_2(iax^n)}{2n}$$

```
[Out] 1/2*I*polylog(2,-I*a*x^n)/n-1/2*I*polylog(2,I*a*x^n)/n
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5031, 4848, 2391}

$$\frac{i\text{PolyLog}(2, -iax^n)}{2n} - \frac{i\text{PolyLog}(2, iax^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x^n]/x,x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*a*x^n])/n - ((I/2)*PolyLog[2, I*a*x^n])/n
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax^n)}{x} \, dx &= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(ax)}{x} \, dx, x, x^n\right)}{n} \\
&= \frac{i \,\text{Subst}\left(\int \frac{\log(1-iax)}{x} \, dx, x, x^n\right)}{2n} - \frac{i \,\text{Subst}\left(\int \frac{\log(1+iax)}{x} \, dx, x, x^n\right)}{2n} \\
&= \frac{i\text{Li}_2(-iax^n)}{2n} - \frac{i\text{Li}_2(iax^n)}{2n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.82

$$\frac{i\left(\text{Li}_2(-iax^n) - \text{Li}_2(iax^n)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x^n]/x,x]
```

```
[Out] ((I/2)*(PolyLog[2, (-I)*a*x^n] - PolyLog[2, I*a*x^n]))/n
```

fricas [B] time = 0.47, size = 63, normalized size = 1.62

$$\frac{2\,n\arctan\left(ax^{n}\right)\log(x)+i\,n\log\left(i\,ax^{n}+1\right)\log(x)-i\,n\log\left(-i\,ax^{n}+1\right)\log(x)-i\,\mathrm{Li}_2\left(i\,ax^{n}\right)+i\,\mathrm{Li}_2\left(-i\,ax^{n}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*n*arctan(a*x^n)*log(x) + I*n*log(I*a*x^n + 1)*log(x) - I*n*log(-I*a*
x^n + 1)*log(x) - I*dilog(I*a*x^n) + I*dilog(-I*a*x^n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\arctan\left(ax^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x^n)/x, x)
```

maple [B] time = 0.05, size = 94, normalized size = 2.41

$$\frac{\ln\left(a\,x^{n}\right)\arctan\left(a\,x^{n}\right)}{n}+\frac{i\ln\left(a\,x^{n}\right)\ln\left(1+ia\,x^{n}\right)}{2n}-\frac{i\ln\left(a\,x^{n}\right)\ln\left(1-ia\,x^{n}\right)}{2n}+\frac{i\,\mathrm{dilog}\left(1+ia\,x^{n}\right)}{2n}-\frac{i\,\mathrm{dilog}\left(1-ia\,x^{n}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x^n)/x,x)
```

```
[Out] 1/n*ln(a*x^n)*arctan(a*x^n)+1/2*I/n*ln(a*x^n)*ln(1+I*a*x^n)-1/2*I/n*ln(a*x^
n)*ln(1-I*a*x^n)+1/2*I/n*dilog(1+I*a*x^n)-1/2*I/n*dilog(1-I*a*x^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-an\int\frac{x^{n}\log(x)}{a^{2}xx^{2\,n}+x}\,dx+\arctan\left(ax^{n}\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(a*x^n)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\mathrm{atan}\left(a\,x^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x^n)/x,x)
```

```
[Out] int(atan(a*x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{atan}\left(ax^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x**n)/x,x)
```

```
[Out] Integral(atan(a*x**n)/x, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```